WHAT IS A STABLE DISK TREE?

Tree embeddings. Given an abstract tree G, an embedding of G as coin tree is a set of disks whose contact graph is equal to G.

One tree may have many different embeddings. Or it may have no embedding at all.

Disk contact graphs. Given a set of interior-disjoint disks, their contact graph is induced by kissing pairs of disks.

If the disks all have the same size, they are known as unit disk contact graphs, or coin graphs.

Free embedding. We can choose the combinatorial embedding of the tree: cyclic order of the neighbours around each vertex.

If the embedding is fixed, stable shapes can be made.

ε-Stability. A graph G is ε-stable if for any two embeddings of G as coin-tree graphs of unit disks, there is a rigid transformation of one such that there is a matching between the resulting embeddings where the distance between matched disks is at most ε.

Every graph is stable for some ε. The challenge is to make ε independent of the size of the graph.

HOW IS THIS USEFUL?

Four-hex gadget. We can create arbitrarily stable hexagons of any size. We need to arrange our stable shapes into larger shapes, which have controlled flexibility.

Four hexagons and a path of disks can be arranged into a shape that is rigid but which allows flipping horizontally and/or vertically.

Stability. For the reduction to work, we need to make hexagons that are sufficiently stable.

We need to make the construction rigid. We can do this using polynomially many disks.

NP-hardness. Using a stable hexagon, we can prove it is NP-hard to test whether a given tree has an embedding as a kissing unit disk tree.

Logic engine construction. Each column can be flipped vertically. Within each column, each row can be flipped horizontally. Deciding if there is a non-intersecting state is NP-complete.

WHY IS THIS HEXAGON STABLE?

The construction. We place two disks (the hubs) at distance 7 from each other. From each hub, we grow four feathers, consisting of a shaft and many individual barbs. The feathers form a hexagonal circle packing around each hub.

We connect the hubs with a straight path to make sure they cannot drift farther than 7 from each other. We then connect two arms to the central path, which connect to the two hands at the far ends of the construction. Finally, we fill the remaining space in the heart of the construction.

Stable hexagon. The tree, shown as a kissing disk contact tree in the background of this poster, is 1-stable. It can be made arbitrarily large and arbitrarily stable (but to increase the stability, we also need to increase the size).

Arms & hands. The arms & hands are not necessary for a stable configuration. They are used to ensure two opposite corners of the hexagon are fixed.

Is this really a tree? We can perturb the disks in such a way that the contact graph is exactly the tree drawn in brown.

The perturbation angles propagate quadratically. The smallest angles will be in the order of $1/n^2$.

Why distance 7? We take hub distance 7 because this is an integer, which is slightly larger than an integer multiple of $\sqrt{3}$. This means the two hexagonal packings with almost, but not quite, fit each other. We could make it more stable by choosing 26, or 97...

$7 = 4\sqrt{3}$

Straight feathers. All feathers must be straight. We can annotate the disks by graph distance from their hub.

As soon as we bend any feather, too many disks are too close to hub.

No zigzagging. Given sufficient empty space, paths can be compressed slightly by "zigzagging".

$1 \to 1$

For this to be possible, they need an empty strip of height strictly greater than 1.

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WHERE?

The 24th International Symposium on Graph Drawing & Network Visualization, 19-21 Sept. 2016, **Athens, Greece**. Poster session.

grim

shaft

barb

hub

feather