A GENERALIZATION
OF THE
DIRECTED GRAPH LAYERING PROBLEM

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Problem

Diagrams in practice often have

an unfortunate aspect ratio

resulting in

poor utilization of the drawing area

small diagram elements

a lot of whitespace
SUGIYAMA LAYOUT

P1 Cycle Removal
P2 Layer Assignment
P3 Crossing Minimization
P4 Node Placement
P5 Edge Routing

Topological Phases

Geometrical Phases

K. Sugiyama, S. Tagawa, M. Toda

C. D. Schulze, M. Spönemann, R. v. Hanxleden
Drawing layered graphs with port constraints
Journal of Visual Languages and Computing, 2013
ALTERING THE ASPECT RATIO

Which phase has influence?

- **P3 Crossing Minimization**: (almost) nothing to do with it
- **P4 Node Placement**: certainly have an impact, still, the topology is already fixed
- **P5 Edge Routing**: edges may occupy space between layers

This leaves us with...

- **P1 Cycle Removal**
- **P2 Layer Assignment**
Layering

Height

Estimated vs effective

Width
EXISTING WORK

LAYERING

P. Eades and K. Sugiyama,
*How to draw a directed graph*, Journal of Information Processing, 1990.

E. R. Gansner, E. Koutsofios, S. C. North, and K.-P. Vo,

E. G. Coffman and R. L. Graham,

"MINIMUM WIDTH"

P. Healy and N. Nikolov,
*How to layer a directed acyclic graph*, Graph Drawing, 2001.

P. Healy and N. Nikolov,

N. Nikolov, A. Tarassov, and J. Branke,
*In search for efficient heuristics for minimum-width graph layering with consideration of dummy nodes*, Journal of Experimental Algorithmics, 2005.

ASPECT RATIO

L. Nachmanson, G. Robertson, and B. Lee,
*Drawing graphs with GLEE*, Graph Drawing, 2008.
Input graph must be acyclic

1. Height bound by the longest path

2. Width bound by highest node degree
"We have found that allowing small deviation sometimes gives us the necessary freedom for overcoming local inefficiencies in the layout, without affecting visualization of the overall directionality."

T. Dwyer and Y. Koren,

**DIG-COLA: Directed Graph Layout through Constrained Energy Minimization,** INFOVIS, 2005.

- All edges point downwards
- Some edges point upwards, less crossings
SOMETHING NEW
WARNING

From now on, drawing are **left to right** instead of **top down**.

Hence ...

**Height is Width**

and

**Width is Height**

(I will most definitely use them wrongly at some point ...)

**Observation**

0 reversed edges, 71 edge length

2 reversed edges, 35 edge length

6 reversed edges, 16 edge length
Given a directed acyclic graph $G = (V, E)$, find a valid layering $L: V \rightarrow \mathbb{N}$.

**Minimize**

$$\sum_{(v, w) \in E} L(w) - L(v)$$

**Subject to**

$$L(w) - L(v) \geq 1 \quad \forall (v, w) \in E$$

**Generalized Layering (GLP)**

Given a directed acyclic graph $G = (V, E)$, find a feasible layering $L: V \to \mathbb{N}$.

**Minimize**

$$
\text{minimize} \left( \sum_{(v,w)\in E} |L(w) - L(v)| \right) + w_{rev}|\{(v,w) \in E: L(v) > L(w)\}|
$$

**Subject to**

$$
|L(w) - L(v)| \geq 11 \quad \forall (v,w) \in E
$$
IP MODEL

INPUT

Graph $G = (V, E)$  

INTEGER VARIABLES

$l(v)$  

$\forall v \in V$  

takes values in $\{1, \ldots, n\} (|V| = n)$ indicating node $v$ is placed in layer $l(v)$

BOOLEAN VARIABLES

$r(u, v)$  

$\forall (u, v) \in E$  

1 iff $e(u, v) \in E$ reversed, 0 otherwise

MINIMIZE

$\omega_{\text{len}} \sum_{(u,v)\in E} |l(u) - l(v)| + \omega_{\text{rev}} \sum_{(u,v)\in E} r(u,v)$

SUBJECT TO

$1 \leq l(v) \leq n$  

$\forall v \in V$

$|l(u) - l(v)| \geq 1$  

$\forall (u, v) \in E$

$n \cdot r(u, v) + l(v) \geq l(u) + 1$  

$\forall (u, v) \in E$
HEURISTIC

1. Remove leaf nodes iteratively
2. Construct initial feasible layering, deduce edge directions
3. Run DLP
4. Greedily improve result
5. Reattach leaf nodes
**Layering Construction**

**Step 2**

**Idea**

Place nodes on a line (assign indexes),
try to keep edges short
and the number of left edges small

**Loop**

select candidate with smallest $|u|$ (start with random node)
if $|a_i| < |a_o|$
    assign index to the left of currently placed nodes
else
    assign to the right

---

A. J. McAllister,

J. Pantrigo, R. Mart, A. Duarte, and E. Pardo,
Layering Improvement

Idea

Identify badly placed nodes and move them to a different layer

Move

\[
\text{move}(v) = \begin{cases} 
0, & \text{if } WS = \emptyset \\
  l(v) - \min(l(WS)) + 1, & \text{if } WP = \emptyset \\
  l(v) - \max(l(WP)) - 1, & \text{else}
\end{cases}
\]

Profit

\[
\text{profit}(v, m, x) = \begin{cases} 
0, & \text{if } m \leq 1 \\
  m \cdot (|W_x| - |E|) + |WS_x|, & \text{else}
\end{cases}
\]

\[
\text{move}(n4) = 2 \quad \text{profit}(n4, 2, 2) = 2 + 1 = 3
\]
EXAMPLE

ORIGINAL

CONSTRUCTION

DLP & IMPROVEMENT

7 left edges, 34 edge length

6 left edges, 32 edge length
RESULTS
# Topological Metrics

<table>
<thead>
<tr>
<th></th>
<th>1-10</th>
<th>1-20</th>
<th>1-30</th>
<th>1-40</th>
<th>1-50</th>
<th>EA GA</th>
<th>Heur</th>
<th>Heur*</th>
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<tbody>
<tr>
<td><strong>Dummy Nodes</strong></td>
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<td>46.73</td>
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<td>North</td>
<td>39.91</td>
<td>55.47</td>
<td>65.73</td>
<td>75.66</td>
<td>82.47</td>
<td>141.3</td>
<td>53.53</td>
<td>68.91</td>
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<tr>
<td><strong>Reversed Edges</strong></td>
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<tr>
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<td>North</td>
<td>2.74</td>
<td>1.47</td>
<td>1.02</td>
<td>0.72</td>
<td>0.56</td>
<td>0</td>
<td>7.07</td>
<td>8.55</td>
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# Geometrical Metrics

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<th>EA GA</th>
<th>Heur</th>
<th>Heur*</th>
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<tbody>
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<td>943</td>
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<td>874,374</td>
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WRAP UP
NOTES

GLP does not consider node sizes and is too slow

Heuristic fails for paths and trees

Averages of estimated and effective values show the same tendency
Still, for 64% of the graphs: estimated increased and effective decreased

QUESTIONS

Do people accept reversed edges?

Are there better metrics for compactness?

Does an iterative procedure/portfolio of algorithms help?
APPENDIX
## Different Phase Strategies

<table>
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<th>Width (px)</th>
<th>Polyline</th>
<th>Orthogonal</th>
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<td><strong>LS</strong></td>
<td><strong>BK</strong></td>
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<tr>
<td>Heur</td>
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<td>652</td>
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<table>
<thead>
<tr>
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<table>
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<tr>
<td>0.67</td>
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<td>0.67</td>
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