Compact Layered Drawings of General Directed Graphs

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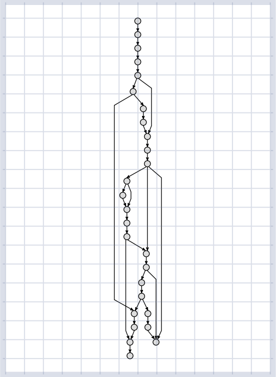
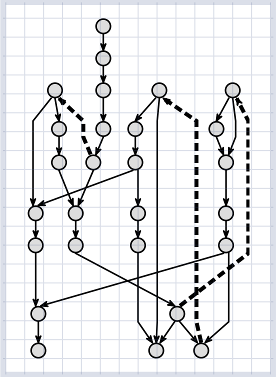
24th International Symposium on
Graph Drawing & Network Visualization
19-21 September 2016

Compact Graph Layering for Digraphs

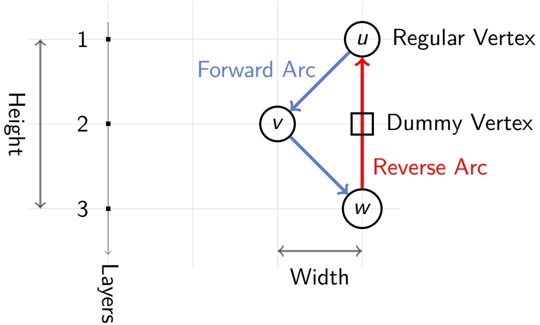


Classic Layering on A4



Compact Layering on A4

Layering for Digraphs: Notation



- Adjacent vertices on different layers
- total arc lengths = #arcs + #dummy vertices

Compact vs Classic Layering for Digraphs

Classic Layering [Sugiyama et. al, 1981]

- Preprocessing: cycle breaking $\rightarrow$ DAG
  - minimize: **number of reverse arcs** *Rev*
- Layering of the DAG
  - minimize: **total arc lengths** *Len*

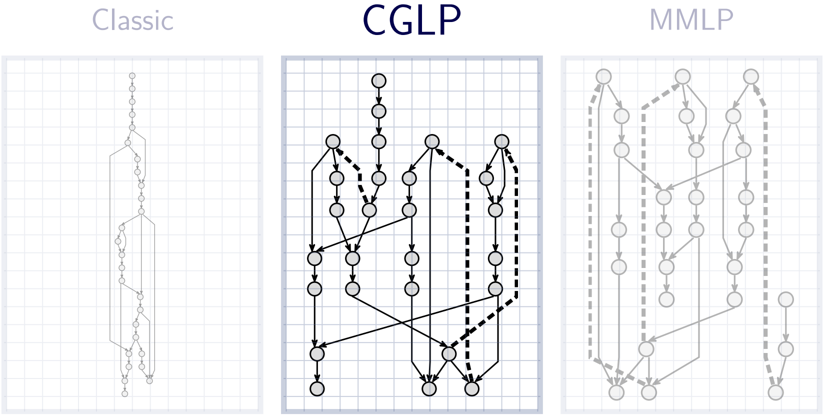
Compact Layering

- integrates cycle breaking & layering

Compact Layering Models for Digraphs

- **Generalised Layering (GLP)** [Rüegg et. al, 2016]
  - minimize: weighted sum of Rev, Len for given Height H
- **Compact Generalised Layering (CGLP)**[this talk]
  - GLP + Width (including dummy vertices)
  - minimize: weighted sum of Rev, Len, Width for given Height H
- **Min+Max Length Layering (MMLP)**[this talk]
  - slight modification of CGLP

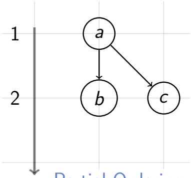
Compact Generalised Layering Problem (CGLP)



Mixed Integer Linear Program CGL

- models CGLP as Partial Ordering Problem (**POP**)

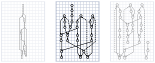
Let $\ell(v)$ the position of vertex v in ordering



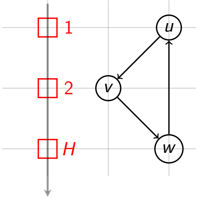
POP variables

$$\text{for each pair } u \neq v \in V: \quad y_{u,v} = \begin{cases} 1 & \ell(u) < \ell(v) \\ 0 & \text{otherwise} \end{cases}$$

$$\text{for each pair } u \neq v \in V: \quad y_{u,v} + y_{v,u} \leq 1$$



Mixed Integer Linear Program CGL



- Consider layers as help vertices $\{1, 2, \cdots, H\}$
- with $\ell(k) = k$ for each $k \in \{1, 2, \cdots, H\}$

Variables for

Layering: for each $v \in V$, $k \in \{1, 2, \cdots, H\}$

- POP variable $y_{v,k}$, $y_{k,v}$

Reverse arcs: for each arc (u, v)

- POP variable $y_{u,v}$, $y_{v,u}$

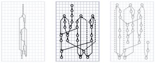
> POP Variables
>
> $$y_{u,v} = \begin{cases} 1 & \ell(u) < \ell(v) \\ 0 & \text{otherwise} \end{cases}$$

Dummy vertices: for each $(u, v) \in A$, $k \in \{1, 2, \cdots, H\}$

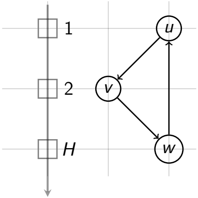
- $z_{uv,k} = \begin{cases} 1 & \text{arc } (u, v) \text{ causes dummy on layer } k \\ 0 & \text{otherwise} \end{cases}$

Width:

- $W \in \mathbb{R}$



Mixed Integer Linear Program CGL



$$\min \; \omega_{rev} \sum_{(u,v)\in A} y_{v,u} + \omega_{len} \sum_{(u,v)\in A} \sum_{k=1}^{H} z_{uv,k} + \omega_{wid} \, W$$

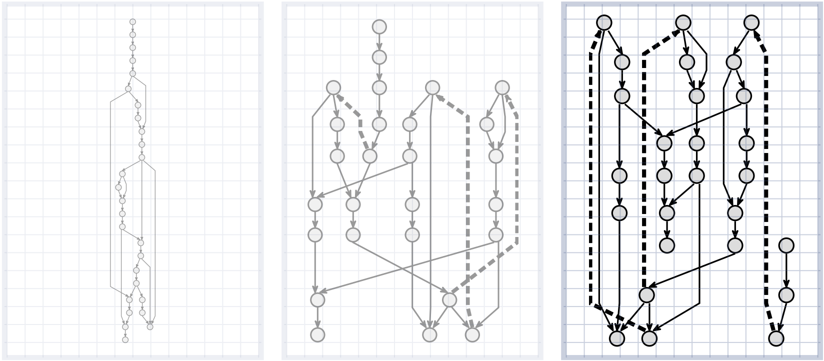
$$
\begin{aligned}
s.t. \quad & y_{v,1} && = 0 && \forall v \in V \\
& y_{H,v} && = 0 && \forall v \in V \\
& y_{k,v} + y_{v,k+1} && = 1 && \forall v \in V, \quad 1 \le k \le H-1 \\
& y_{k,v} - y_{k+1,v} && \ge 0 && \forall v \in V, \quad 1 \le k \le H-1 \\
& y_{u,v} + y_{v,u} && = 1 && \forall (u,v) \in A \\
& y_{v,k} + y_{k,u} - y_{v,u} && \ge 0 && \forall (u,v) \in A, \quad 1 \le k \le H \\
& y_{u,k} + y_{k,v} - y_{u,v} && \ge 0 && \forall (u,v) \in A, \quad 1 \le k \le H \\
& y_{k,u} + y_{v,k} - z_{uv,k} && \le 1 && \forall (u,v) \in A, \quad 1 \le k \le H \\
& y_{k,v} + y_{u,k} - z_{uv,k} && \le 1 && \forall (u,v) \in A, \quad 1 \le k \le H \\
& \sum_{v\in V} (1 - y_{v,k} - y_{k,v}) + \sum_{(u,v)\in A} z_{uv,k} && \le W && 1 \le k \le H \\
& y \in \{0,1\}, \; z \in [0,1], \; W \in \mathbb{R}
\end{aligned}
$$

Mixed Integer Linear Program EXT

- extends DAG Layering Model [Healy and Nikolov, 2002]
- models CGLP as **Assignment Problem (AP)**
- uses so called **assignment variables** for layering of vertices
- much slower than CGL

Min+Max Length Problem (MMLP)

Classic | CGLP | **MMLP**



Mixed Integer Linear Program MML

- slight modification of CGL
- without variables and constraints corresponding to dummies
- MML objective vs CGL objective
  - CGL

$$\min \omega_{rev} \sum_{(u,v)\in A} y_{v,u} + \omega_{len} \sum_{(u,v)\in A} \sum_{k=1}^{H} z_{uv,k} + \omega_{wid} \; W$$

  - MML

$$\min \omega_{rev} \sum_{(u,v)\in A} y_{v,u} + \omega_{len} \sum_{(u,v)\in A} \sum_{k=1}^{H} (y_{k,v} - y_{k,u}) + \omega_{wid} \; W_r$$

Evaluation

Used Hard/Software

System: Intel Core i7-4790, 3.6 GHz with 32 GB RAM, Linux

Solver: MIP Solver Gurobi 6.5

- Number of used threads: 1

Parameters

- $H = \lceil \frac{\sqrt{|V|}}{0.6} \rceil$
- $\omega_{rev} = |A| \cdot H$
- $\omega_{len} = 1$
- $\omega_{wid} = 1$

Evaluation

Benchmark Sets:

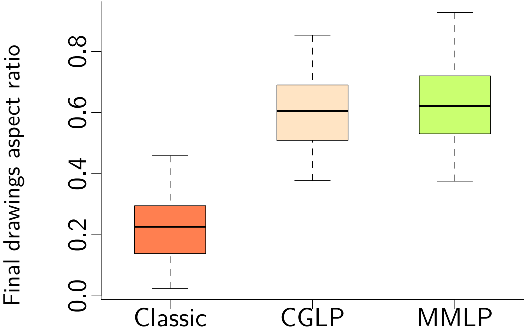
ATTar: extraction of 146 acyclic AT&T graphs with

- final drawings aspect ratio with classic layout:
  - $\frac{\text{final drawings Width}}{\text{final drawings Height}} \leq 0.5$
- Vertices: 20-99
- Arcs: 20-168
- On average: $\frac{|A|}{|V|} = 1.5$

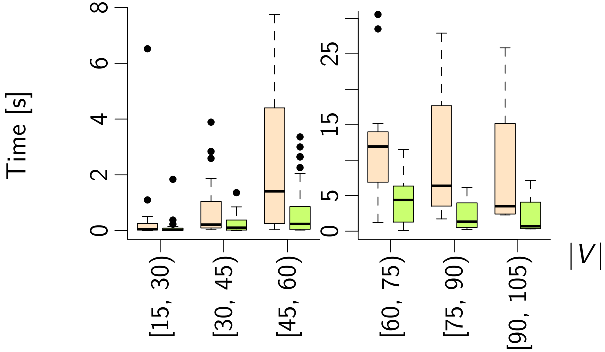
Random: 340 Random (non acyclic) directected graphs

- Vertices: 17-100
- Arcs: 30-158
- On average: $\frac{|A|}{|V|} = 1.5$

Classic vs CGLP vs MMLP (ATTar Graphs)

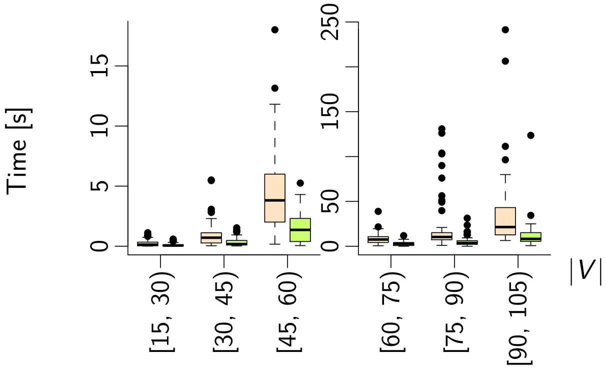


CGL vs MML (ATTar Graphs)



EXT: 29 timeouts (time limit 1h)

CGL vs MML (Random Graphs)



EXT: 143 timeouts (time limit 1h)

Conclusion

We introduced:

- Two Compact Layout Problems: **CGLP** and **MMLP**
- Two new ILP models based on POP variables: **CGL** and **MML**

Our experiments showed:

- Both models can improve aspect ratio
- Both ILP formulations can be solved for each ATTar instance within 30 seconds.