Work started at the Bertinoro Workshop on Graph Drawing, BWGD 2016

Visibility Representations of Boxes in 2.5 Dimensions

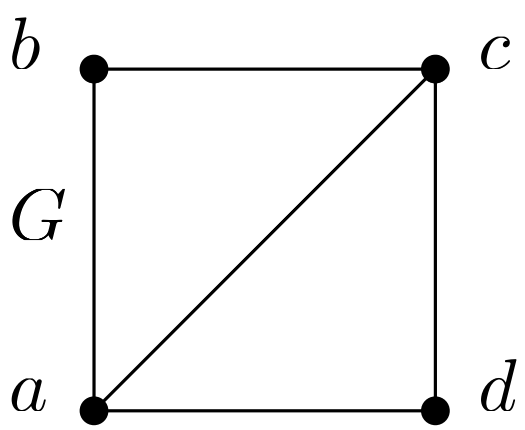
Alessio Arleo[1], Carla Binucci[1],
Emilio Di Giacomo[1], William S. Evans[2],
Luca Grilli[1], Giuseppe Liotta[1],
Henk Meijer[3], Fabrizio Montecchiani[1],
Sue Whitesides[4], Stephen Wismath[5]

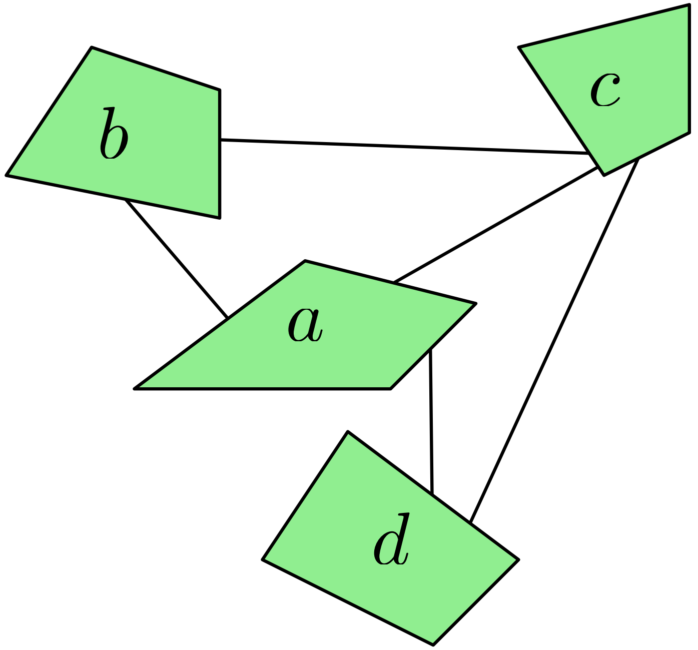
[1]Università degli Studi di Perugia, Italy
[2]University of British Columbia, Canada
[3]University College Roosevelt, the Netherlands
[4]University of Victoria, Canada
[5]University of Lethbridge, Canada

Visibility Representations (VR)

A *Visibility Representation (VR)* of a graph G maps

- the vertices of G to non-overlapping geometric objects
- the edges of G to segments that do not intersect any geometric object other than at their end-points (*visibilities*)



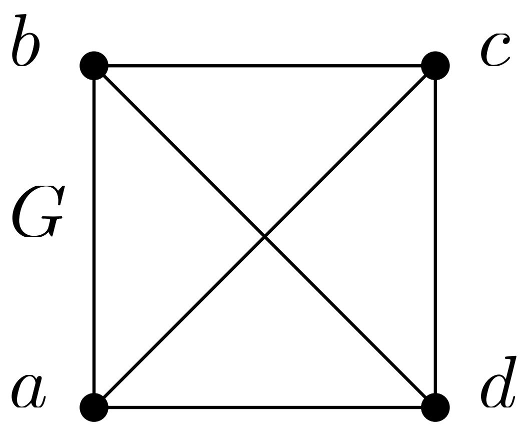


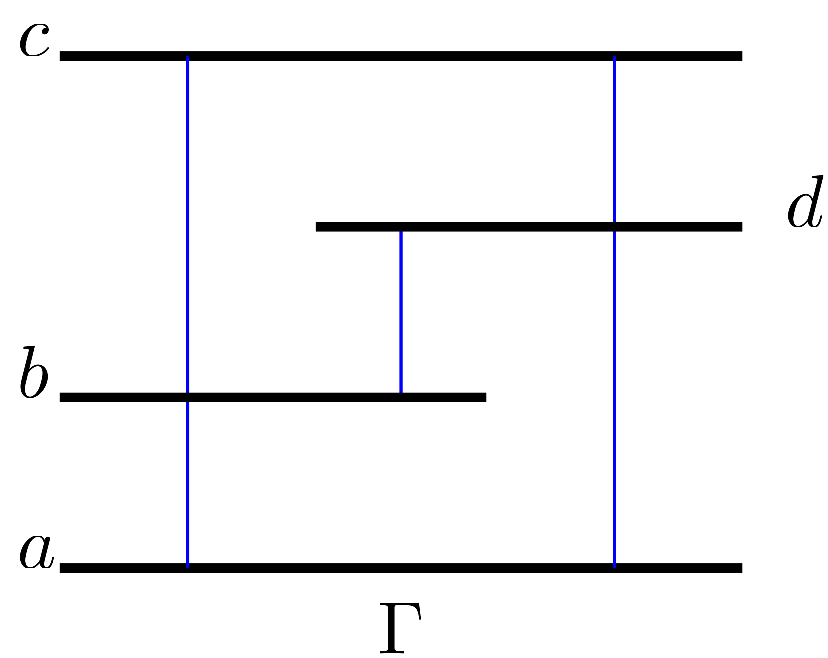
Γ

Bar Visibility Representations (BVR)

In a *Bar Visibility Representation (BVR)*

- vertices $\rightarrow$ horizontal segments (*bars*)
- edges $\rightarrow$ vertical segments





Every planar graph admits a (weak) BVR

Graphs that admit a (strong) BVR have been characterized

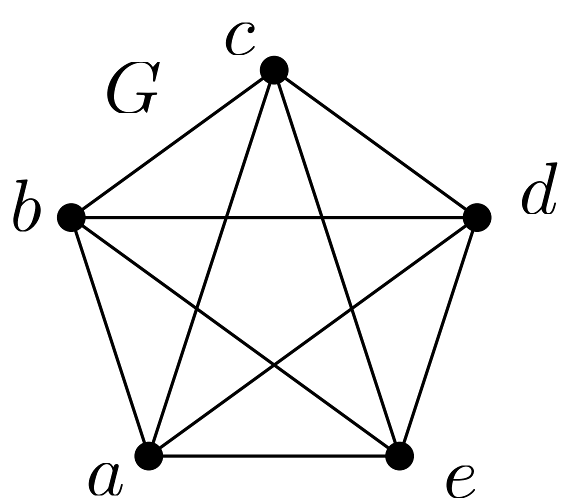
Duchet et al. 1983, Rosenstiehl & Tarjan 1986,

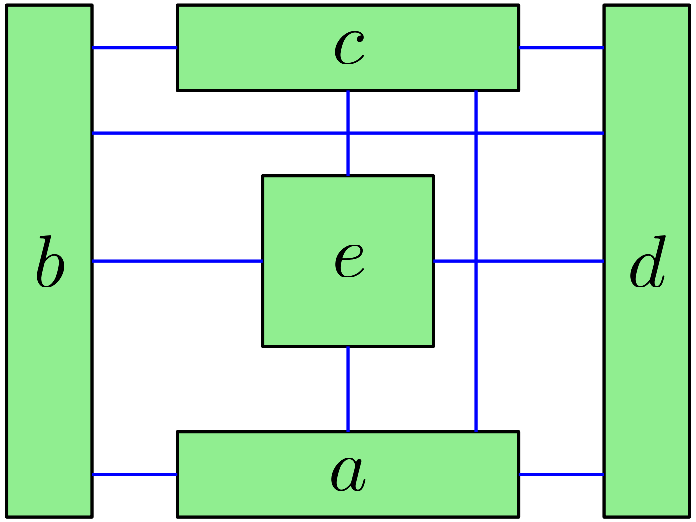
Tamassia & Tollis 1991, Wismath 1985

Rectangle Visibility Representations (RVR)

In a *Rectangle Visibility Representation (RVR)*

- vertices $\rightarrow$ axis-aligned rectangles
- edges $\rightarrow$ horizontal or vertical segments





Γ

At most $6n - 20$ edges Hutchinson et al. 1999

Recognition is NP-hard in general Shermer 1996

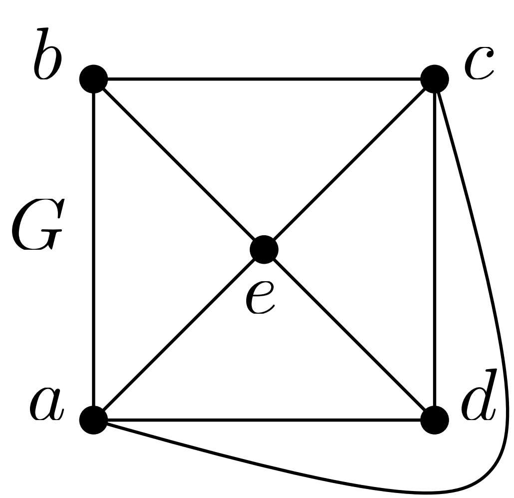
and polynomial time solvable in some restricted cases

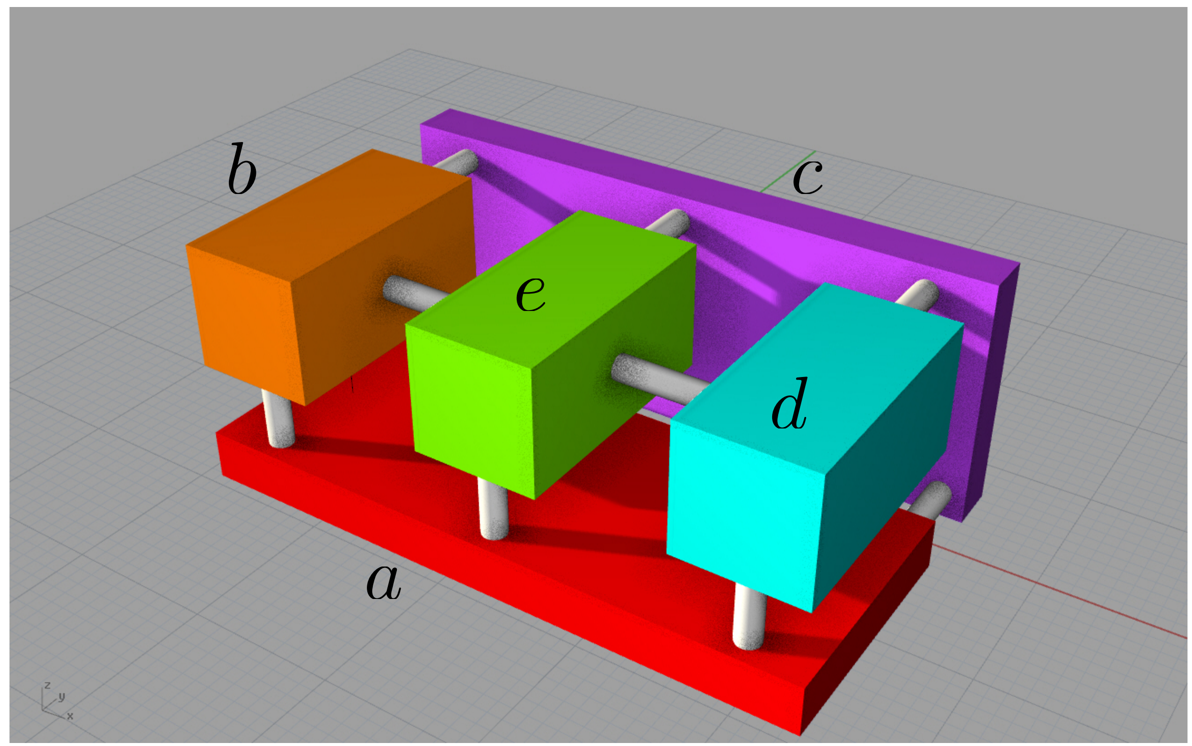
Biedl et al. 2016, Streinu & Whitesides 2003

Box Visibility Representations (BR)

In a *Box Visibility Representation (BR)*

- vertices $\rightarrow$ axis-aligned 3D boxes
- edges $\rightarrow$ segments parallel to the x-, y-, and z-axis





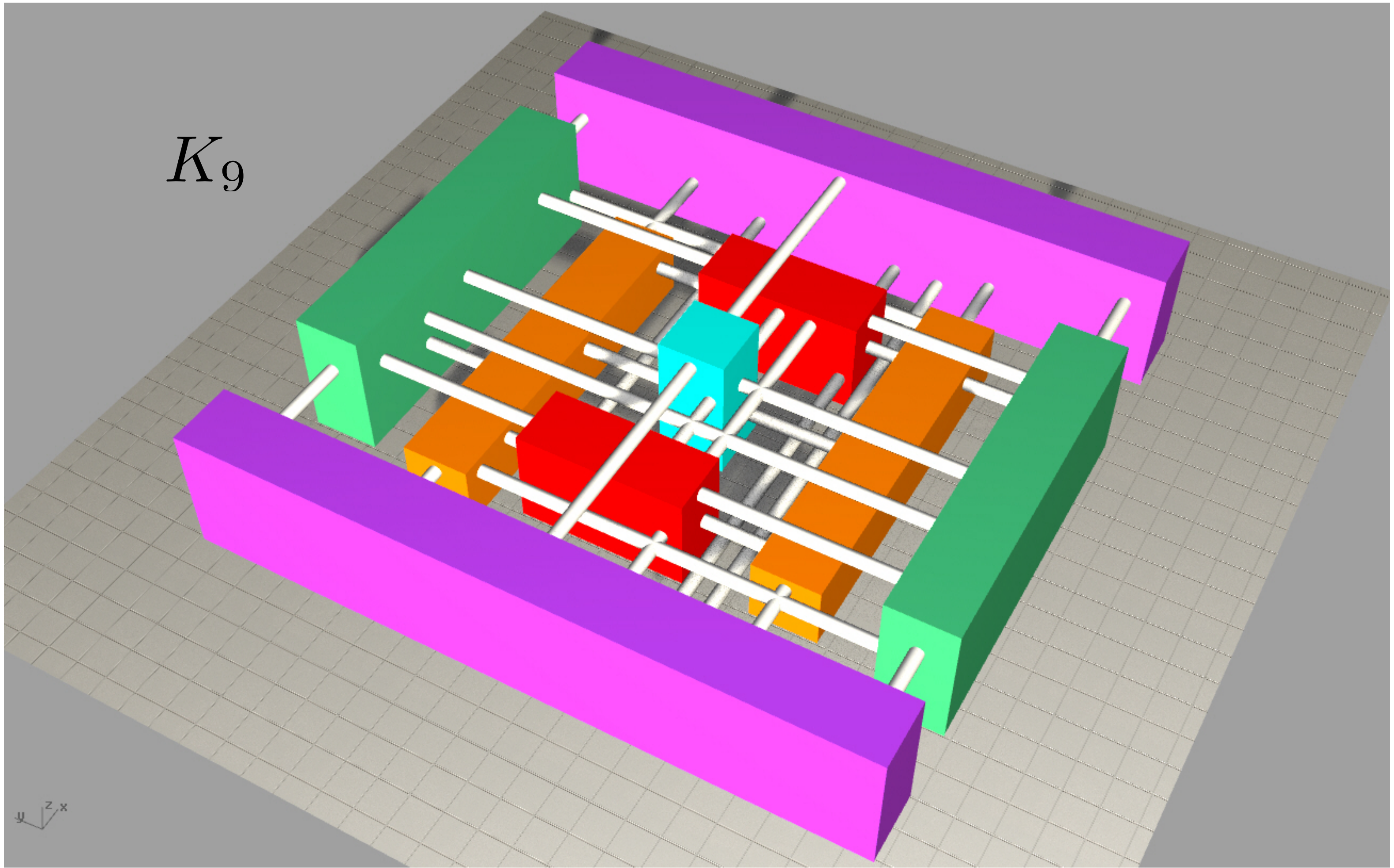
K_{56} admits a BR, while K_{184} does not

Fekete & Meijer 1999

2.5D Box Visibility Representation (2.5D-BR)

2.5D Box Visibility Representations (2.5D-BR):

- vertices $\rightarrow$ axis-aligned 3D boxes whose bottom faces lie in the plane $z = 0$
- edges $\rightarrow$ segments parallel to the x- and y-axis



Our results 1/2

- Every complete bipartite graph admits a 2.5D-BR

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- The complete graph K_n admits a 2.5D-BR if and only if $n \leq 19$.

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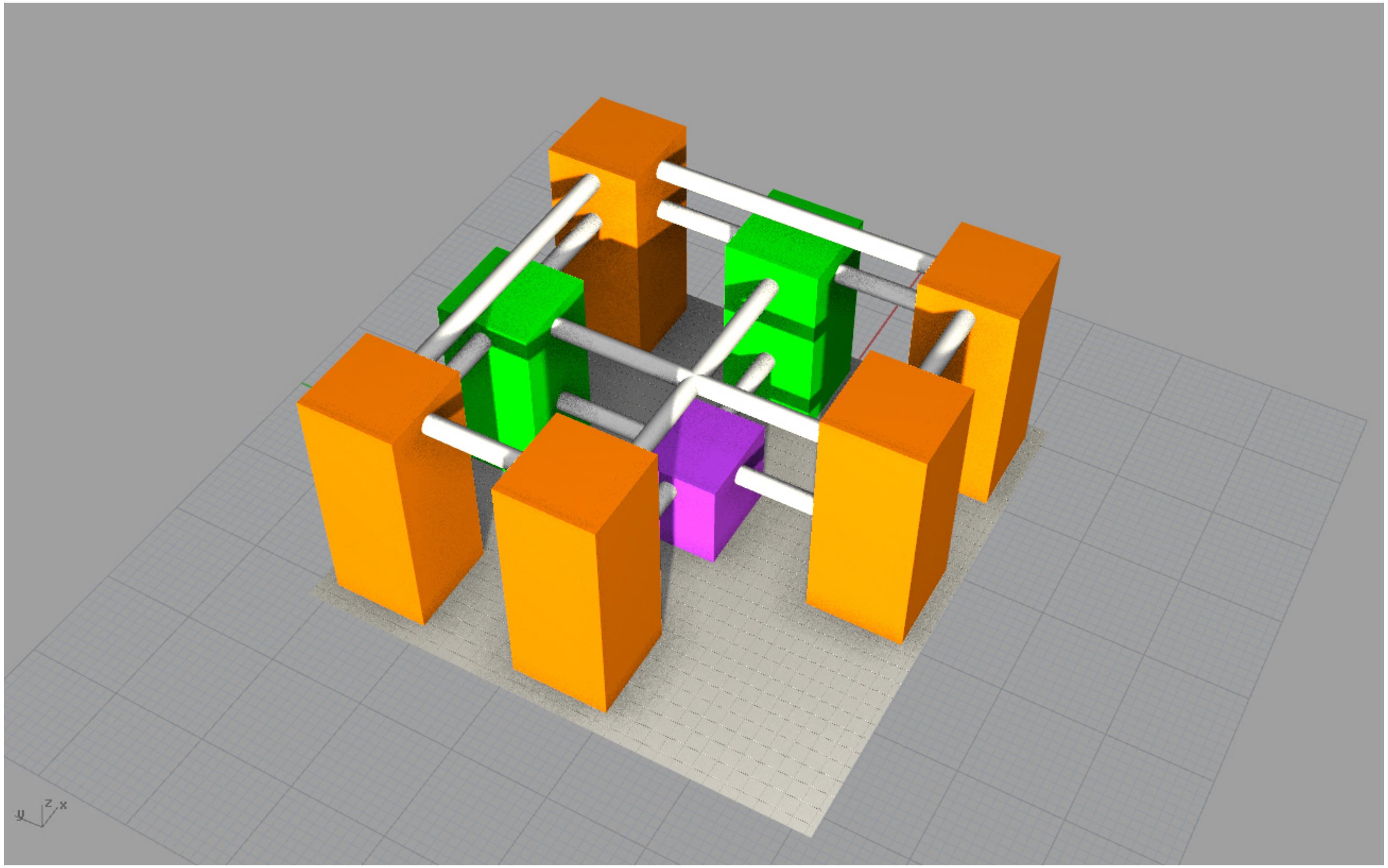
- Every complete bipartite graph admits a 2.5D-BR
- The complete graph K_n admits a 2.5D-BR if and only if $n \leq 19$.
- Every graph with pathwidth at most 7 admits a 2.5D-BR, which can be computed in linear time

2.5D Grid Box Representations (2.5D-GBR)

2.5D Grid Box Visibility Representations (2.5D-GBR)
2.5D-BR such that the bottom face of each box is a unit square with corners at integer coordinates



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- An n-vertex graph that admits a 2.5D-GBR has at most $4n - 6\sqrt{n}$ edges and that this bound is tight.

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- An n-vertex graph that admits a 2.5D-GBR has at most $4n - 6\sqrt{n}$ edges and that this bound is tight.

- Deciding whether a given graph G admits a 2.5D-GBR with a given footprint is NP-complete
  - The *footprint* of a 2.5D-BR Γ is the set of bottom faces of the boxes in Γ.

2.5D-BR of K_n

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Lemma 1 *A complete graph admits a 2.5D-BR only if it has at most* 19 *vertices.*

Given a 2.5D-BR Γ of K_n, we show that:

- There is one line ℓ_h parallel to the x-axis and one line ℓ_v parallel to the y-axis whose union intersect the footprints of all boxes
- ℓ_h and ℓ_v intersect at most 10 boxes each
  $\Rightarrow$ there can be at most 20 boxes in Γ
- There is a box that is intersected by both ℓ_h and ℓ_v
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Two observations

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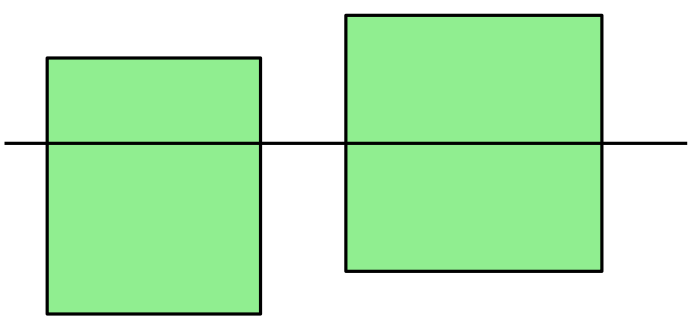
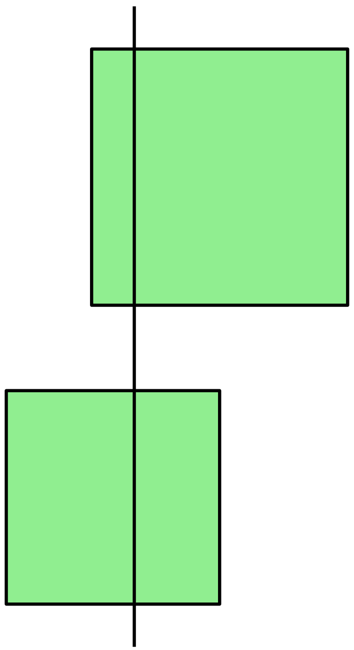
- We can assume that every box in Γ has a distinct integer height in the range $[1, n]$

- For any pair of rectangles in the footprint of Γ there exists a line that intersects both rectangles and that is parallel to the x- or to the y-axis





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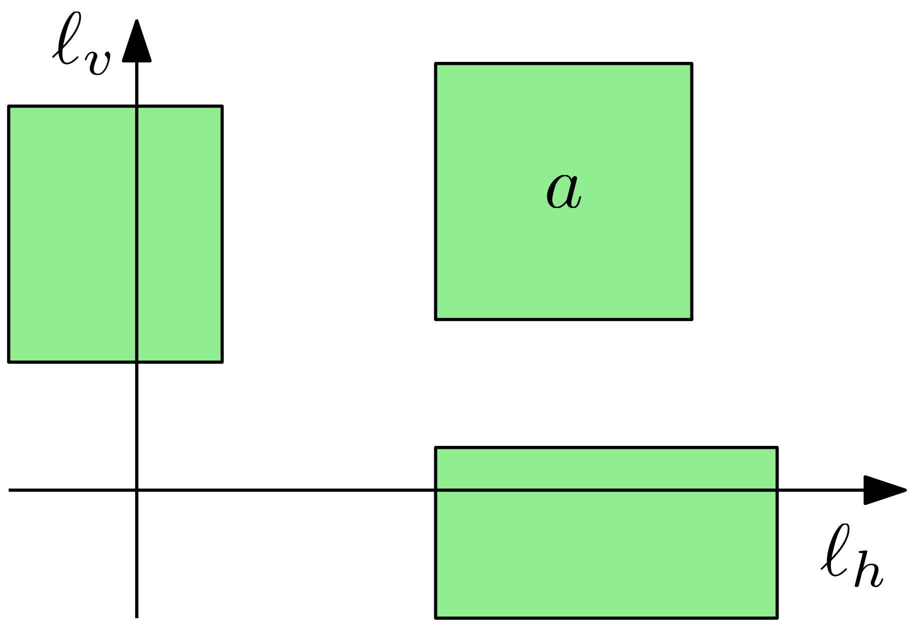
2.5D-BR of K_n

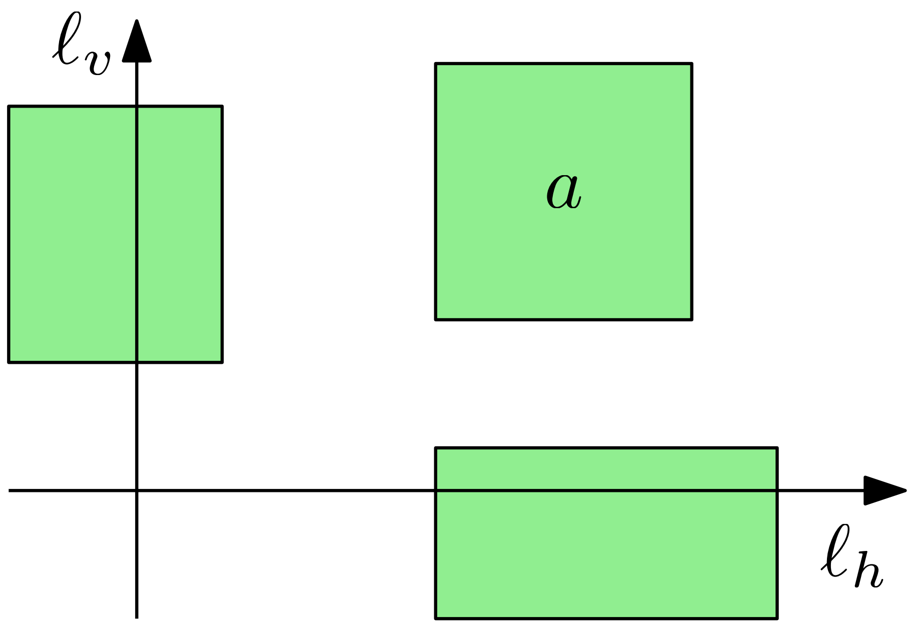
Given a 2.5D-BR Γ of K_n, let $\mathcal{R}$ be the set of rectangles in the footprint of Γ

- Let ℓ_h and ℓ_v be a horizontal and a vertical line whose union intersects the maximum number of rectangles in $\mathcal{R}$
- Suppose that a rectangle $a \in \mathcal{R}$ is not instersected



2.5D-BR of K_n

Move ℓ_h and ℓ_v as close as possible to a without changing the set of instersected rectangles



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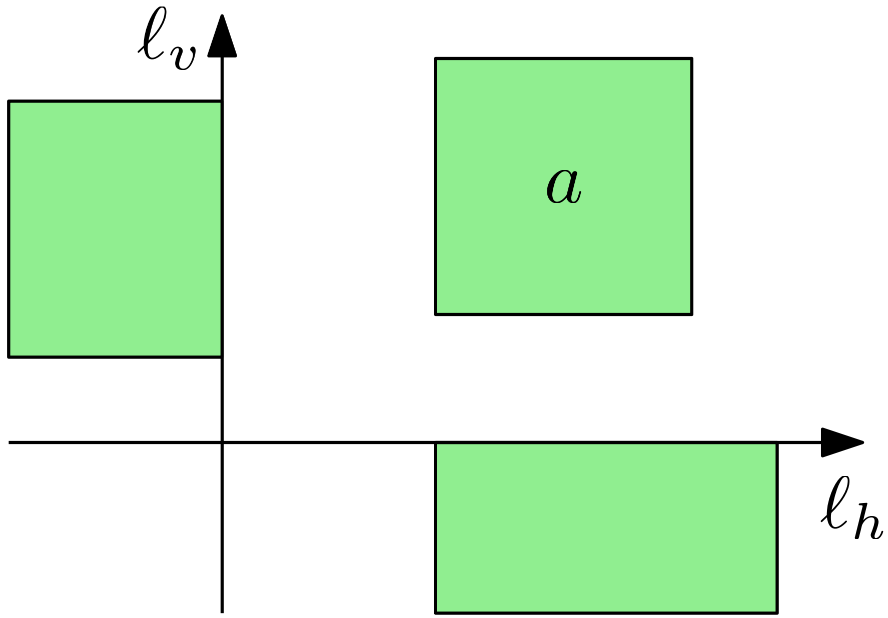
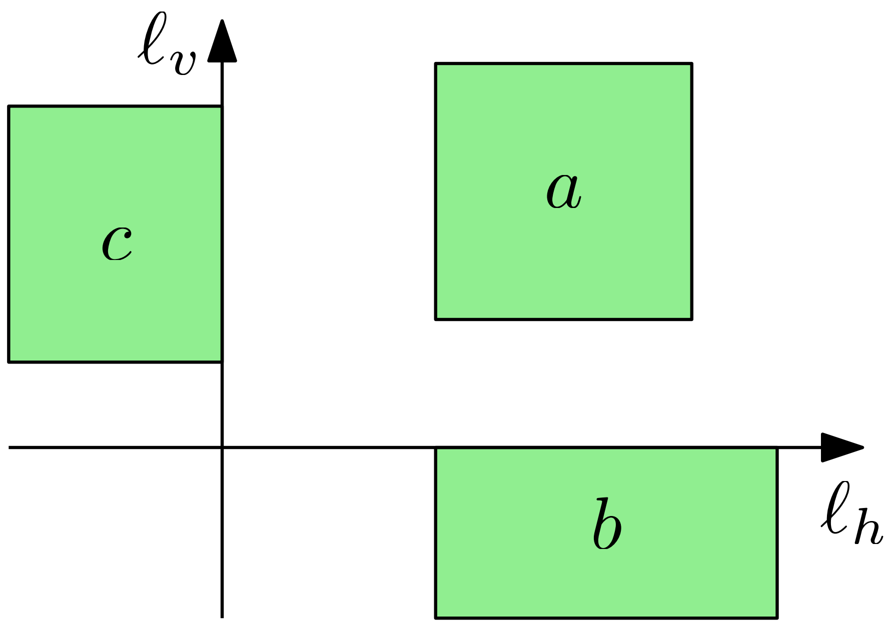


2.5D-BR of K_n

Move ℓ_h and ℓ_v as close as possible to a without changing the set of instersected rectangles

- Let c be the rectangle that prevents ℓ_v to move closer to a
- Let b be the rectangle that prevents ℓ_h to move closer to a



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- Let c be the rectangle that prevents ℓ_v to move closer to a
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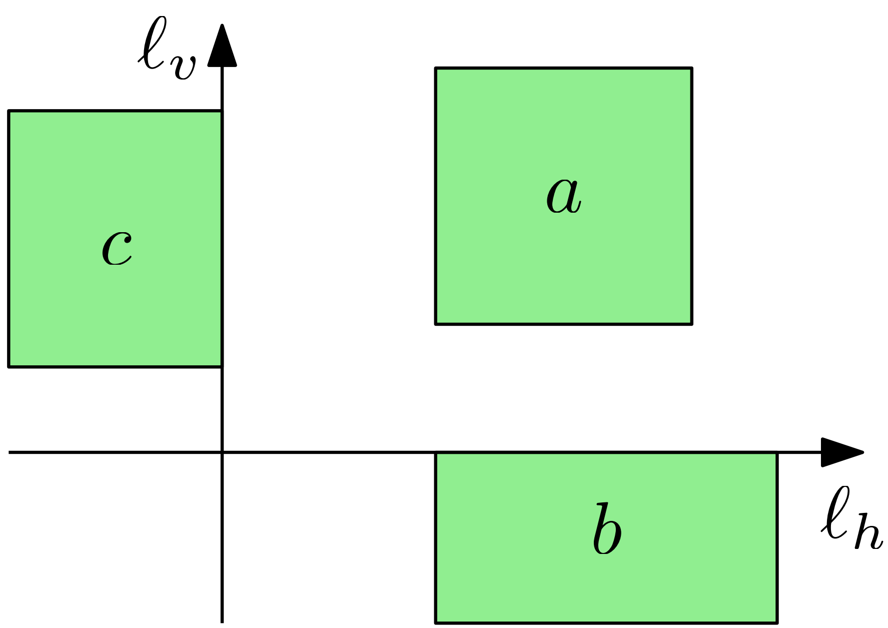
- there must be either a horizontal or a vertical line that intersects both b and c



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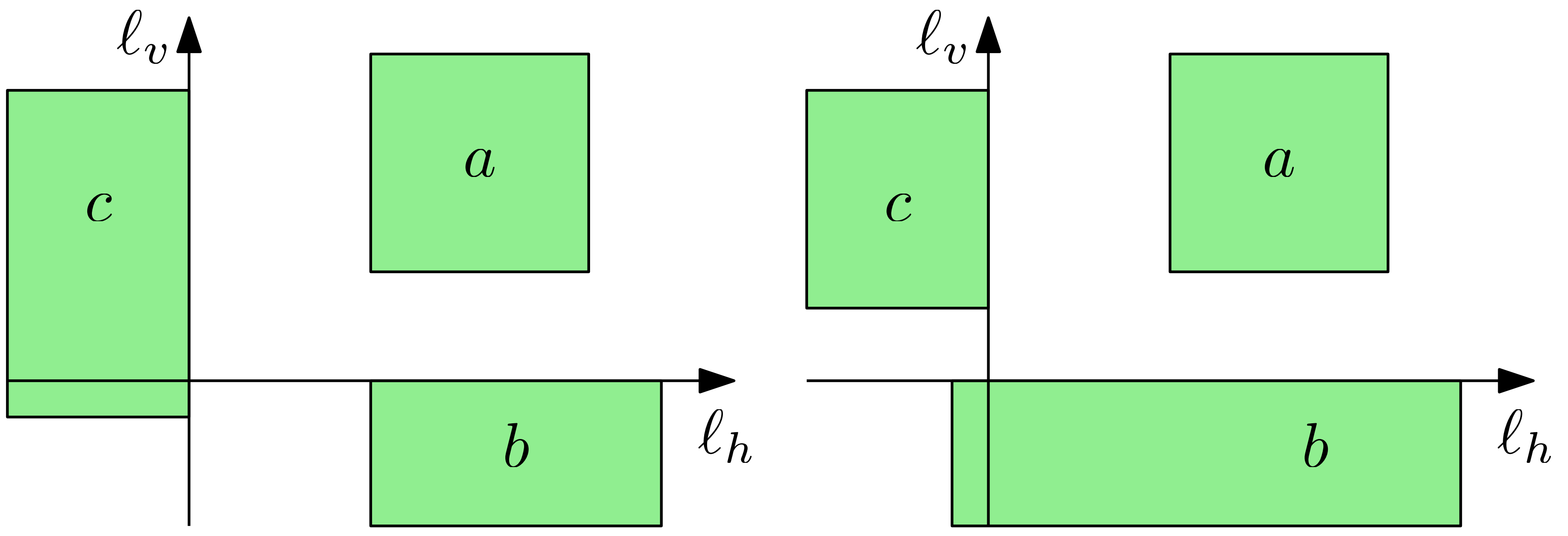

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- c is intersected by both ℓ_v and ℓ_h
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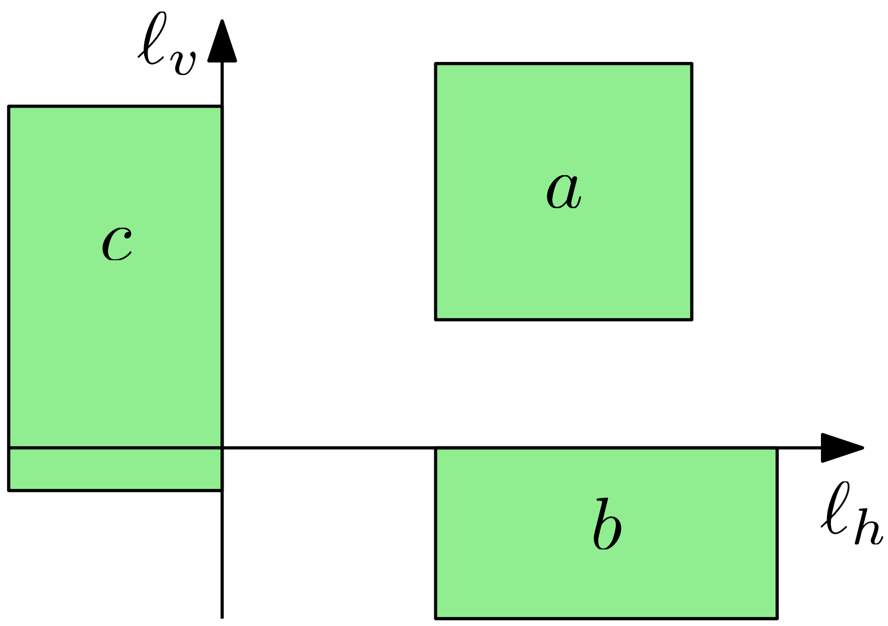

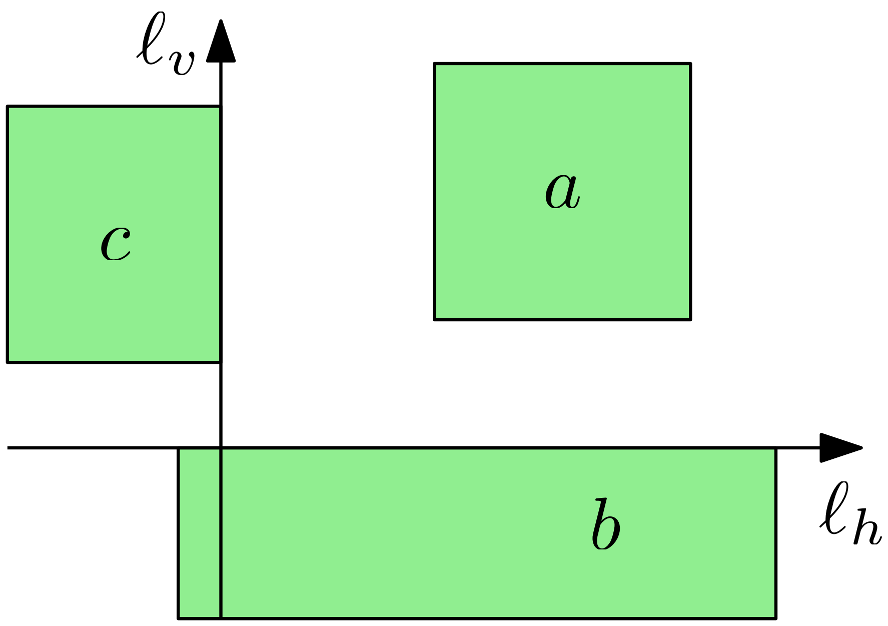

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- c is intersected by both ℓ_v and ℓ_h $\Rightarrow$ ℓ_v can be moved closer to a

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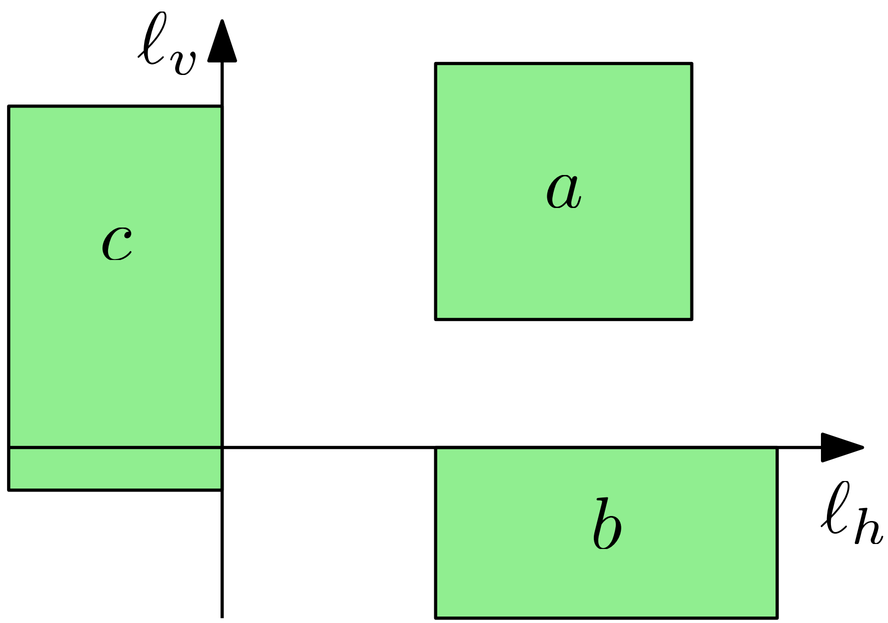

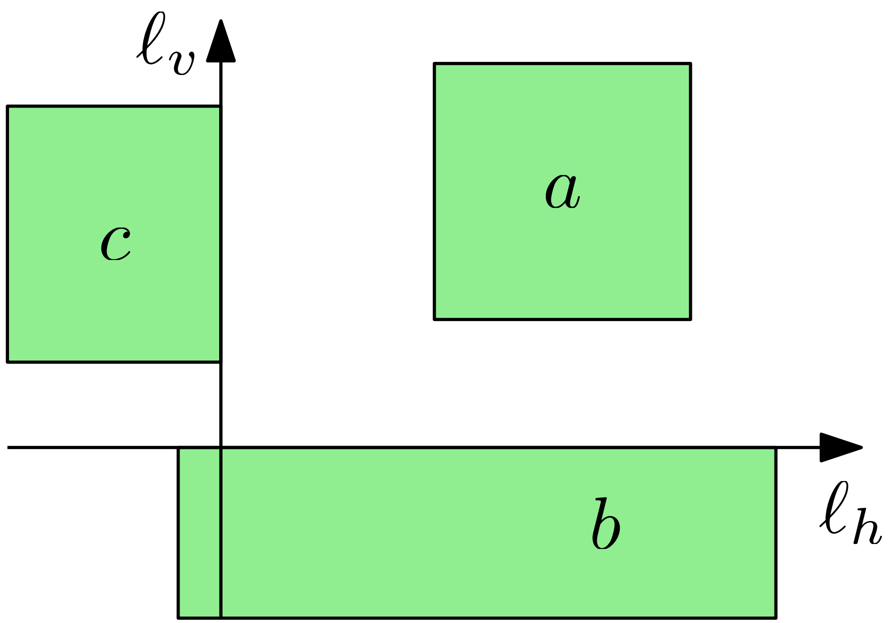

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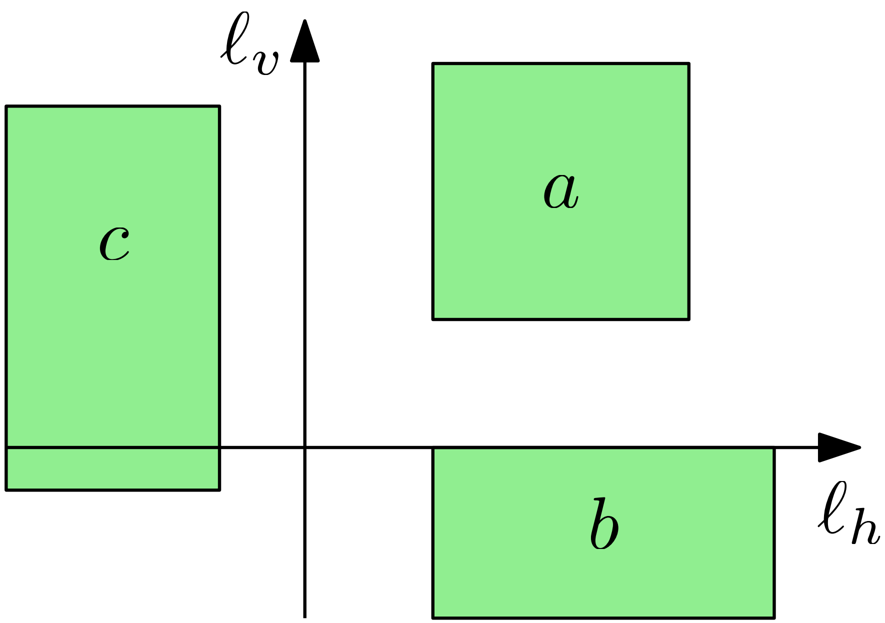
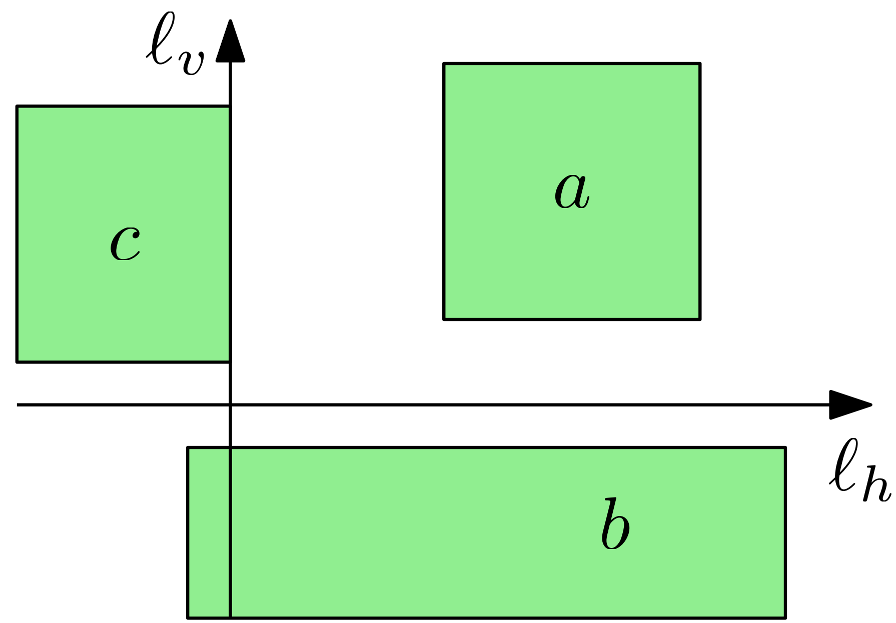


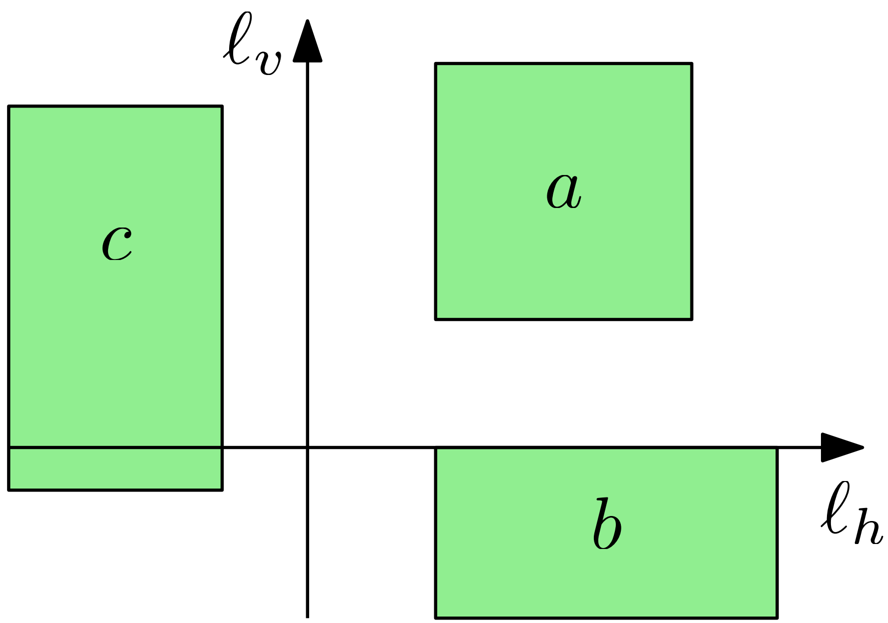
- b is intersected by both ℓ_v and ℓ_h $\Rightarrow$ ℓ_h can be moved closer to a

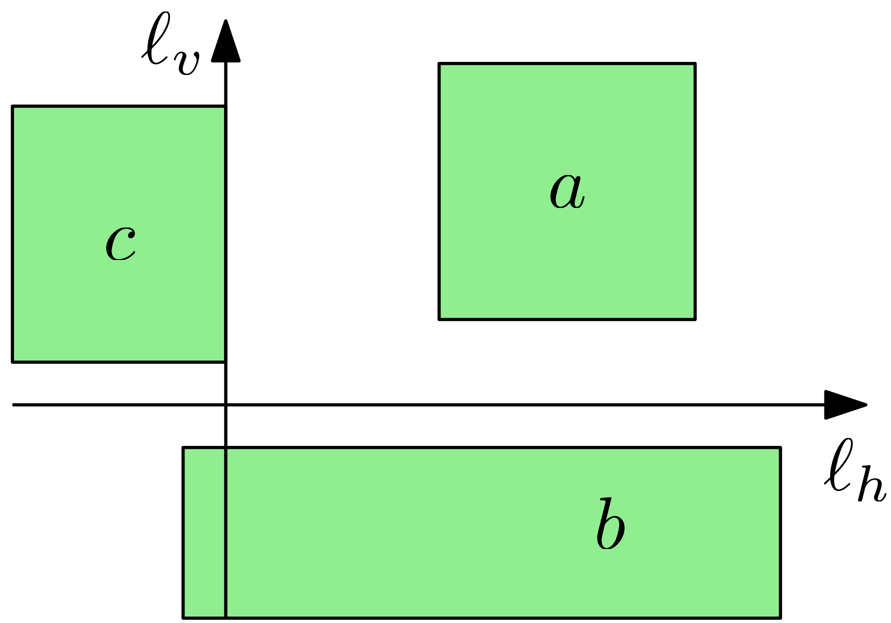


2.5D-BR of K_n

- c is intersected by both ℓ_v and ℓ_h $\Rightarrow$ ℓ_v can be moved closer to a **A contradiction**

- b is intersected by both ℓ_v and ℓ_h $\Rightarrow$ ℓ_h can be moved closer to a **A contradiction**





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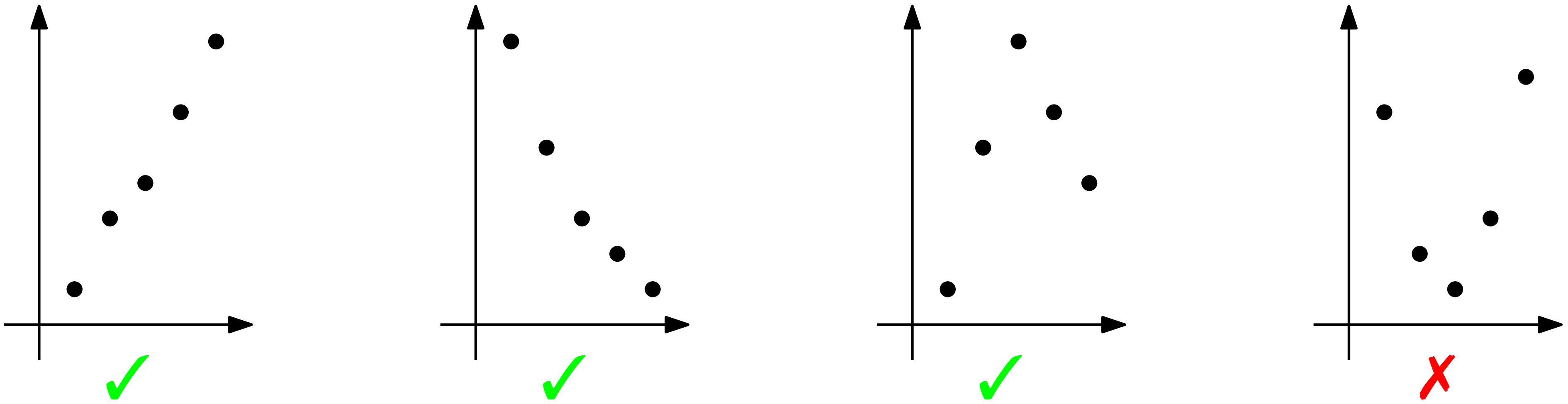
Unimaximal sequences

A sequence of distinct integers is *unimaximal* if no element of the sequence is smaller than both its predecessor and successor.



Unimaximal sequences

Lemma 4 *For all $m > 1$, in every sequence of $\binom{m}{2} + 1$ distinct integers, there exists at least one unimaximal sequence of length m.* *Fekete et al. 1995*

In a sequence of 11 distinct integers, there is a unimaximal sequence of length 5

Unimaximal sequences

Suppose that ℓ_h (or ℓ_v) intersects 11 boxes

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The heights of these boxes form a sequence of distinct integers

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The heights of these boxes form a sequence of distinct integers

$\Rightarrow$ There exist five boxes whose heights form a unimaximal sequence

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Since the heights form a unimaximal sequence, no two boxes see each other "above" a third one.

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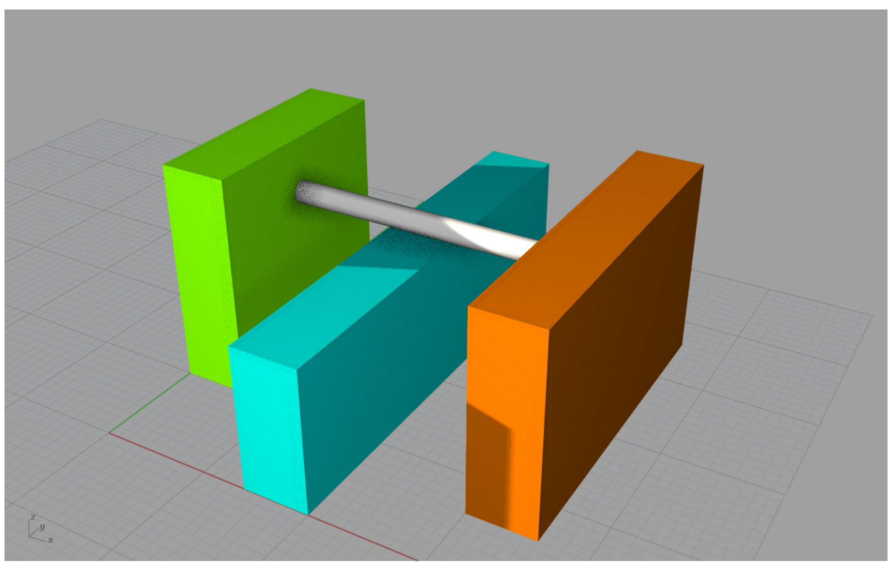

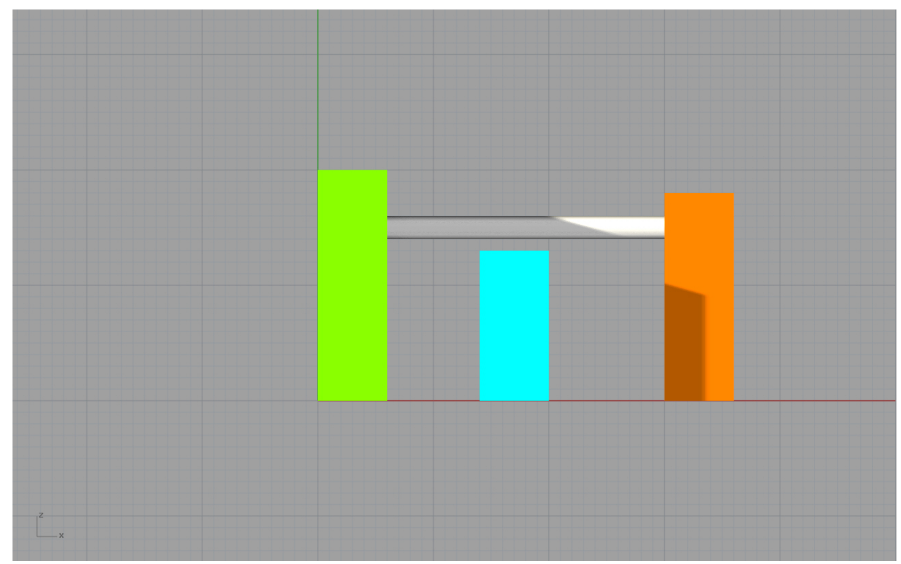

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Since the heights form a unimaximal sequence, no two boxes see each other “above” a third one.

$\Rightarrow$ Boxes must see each other at level $z = 0$

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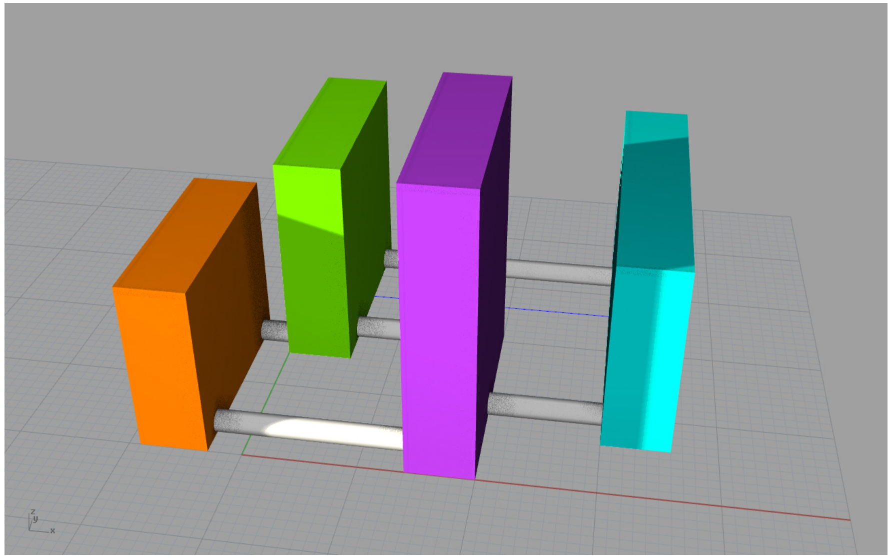

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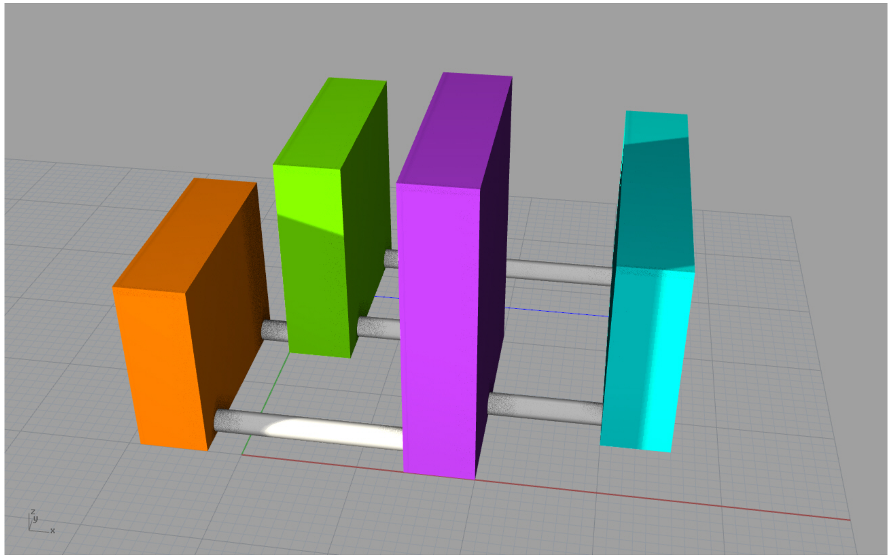
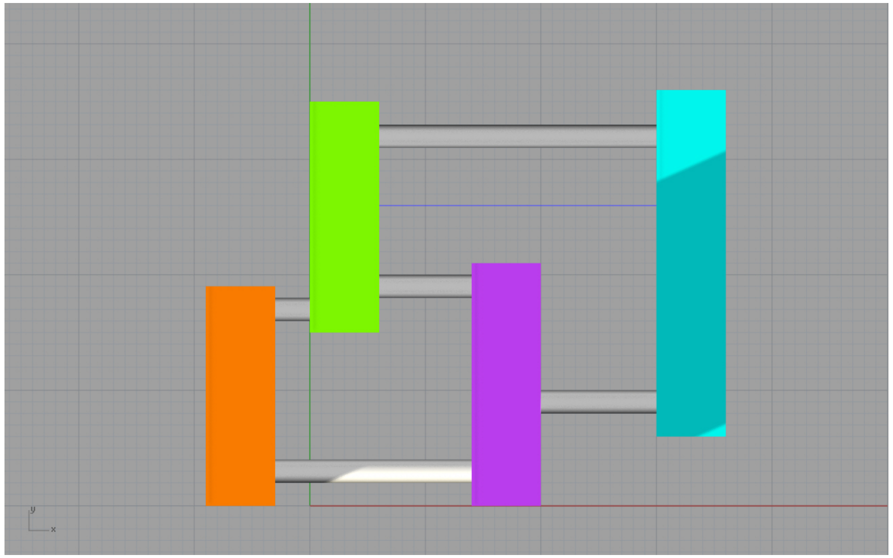


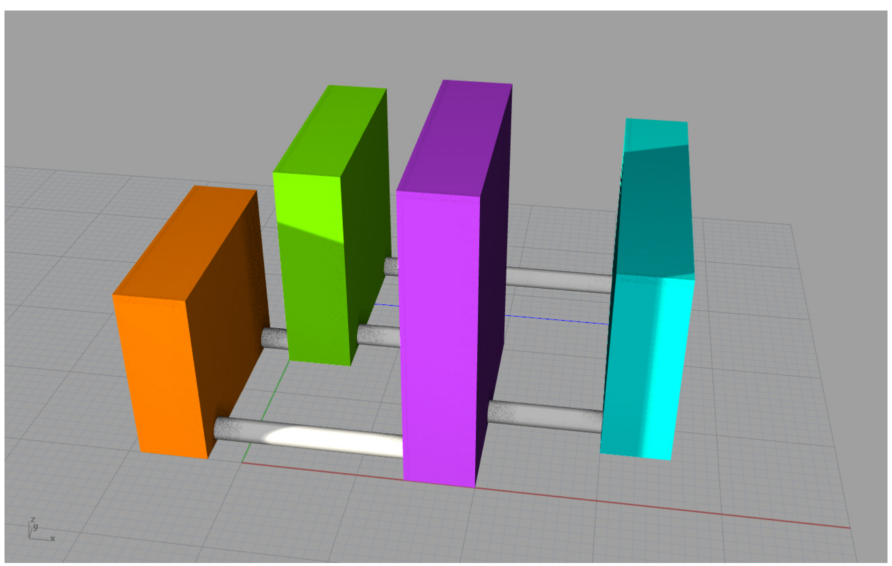


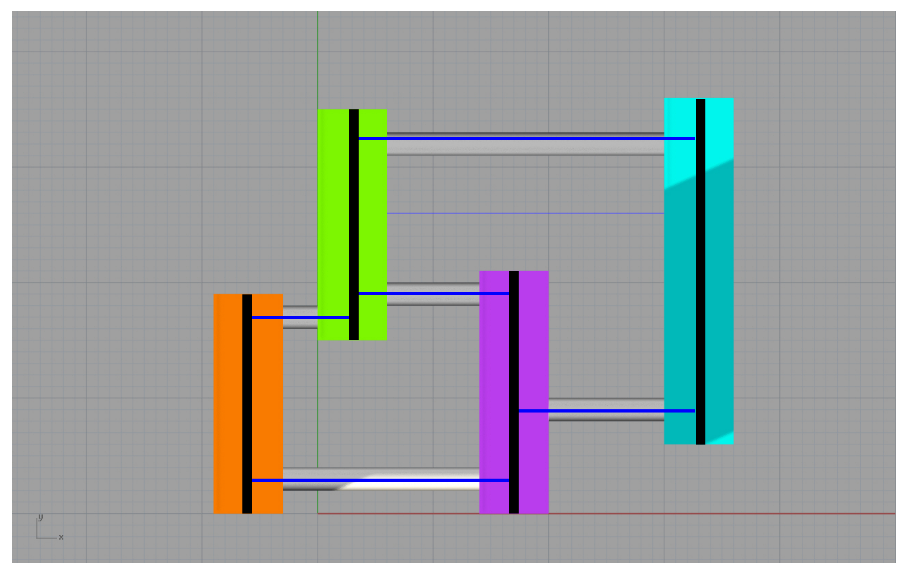
Unimaximal sequences

Suppose that ℓ_h (or ℓ_v) intersects 11 boxes

$\Rightarrow$ There exist five boxes whose heights form a unimaximal sequence

$\Rightarrow$ Boxes must see each other at level $z = 0$
$\Rightarrow$ The five boxes and their visibilities form a BVR





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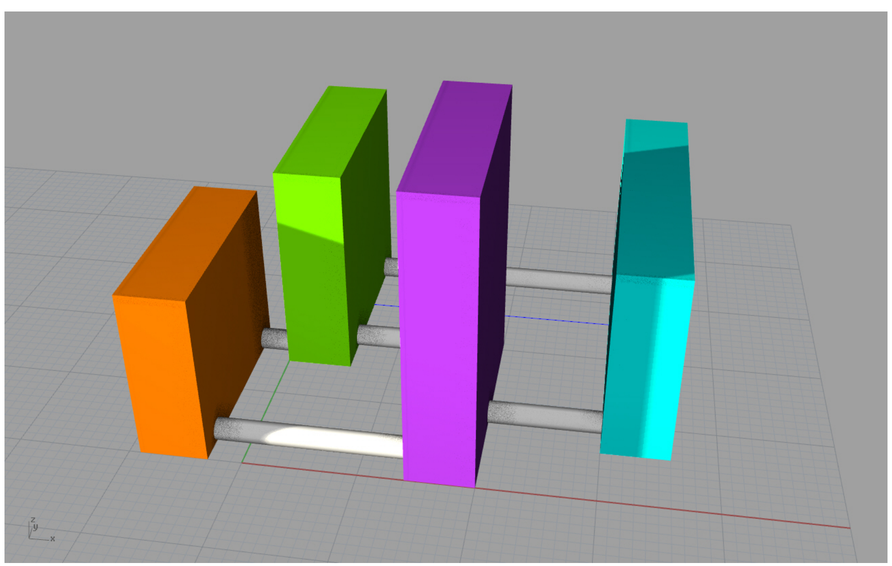

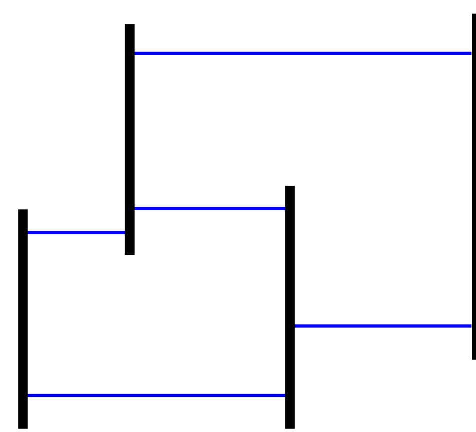

Unimaximal sequences

Suppose that ℓ_h (or ℓ_v) intersects 11 boxes

$\Rightarrow$ There exist five boxes whose heights form a unimaximal sequence

$\Rightarrow$ Boxes must see each other at level $z = 0$
$\Rightarrow$ The five boxes and their visibilities form a BVR
$\Rightarrow$ There is a BVR of K_5 – **A contradiction!**

2.5D-BR of K_n

Lemma 1 *A complete graph admits a 2.5D-BR only if it has at most* 19 *vertices.*

Given a 2.5D-BR Γ of K_n, we show that:

- There is one line ℓ_h parallel to the x-axis and one line ℓ_v parallel to the y-axis whose union intersect the footprints of all boxes ✓
- ℓ_h and ℓ_v intersect at most 10 boxes each
  $\Rightarrow$ there can be at most 20 boxes in Γ ✓
- There is a box that is intersected by both ℓ_h and ℓ_v
  $\Rightarrow$ there can be at most 19 boxes in Γ

2.5D-BR of K_n

Suppose that no box is intersected by both ℓ_h and ℓ_v

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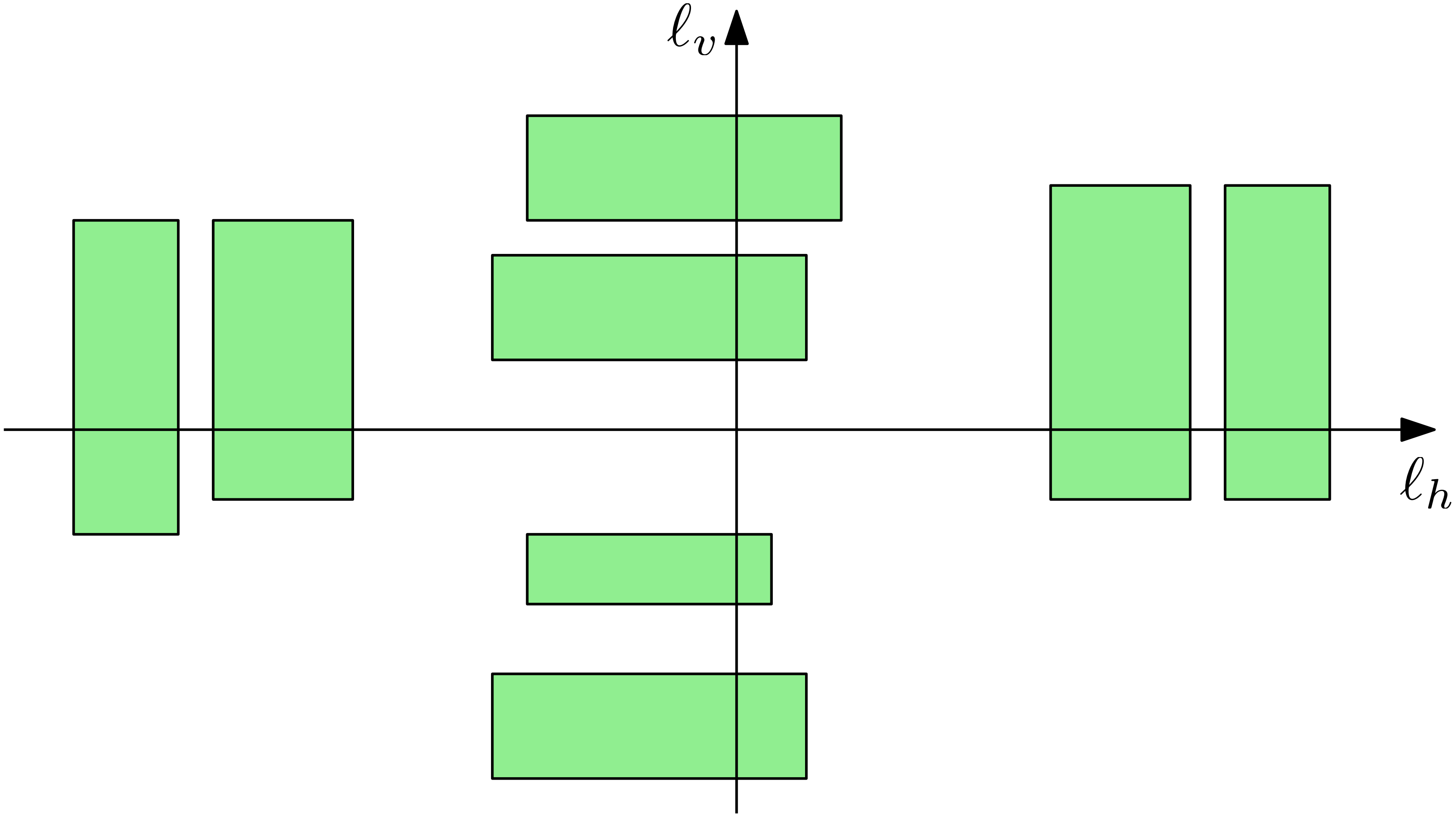

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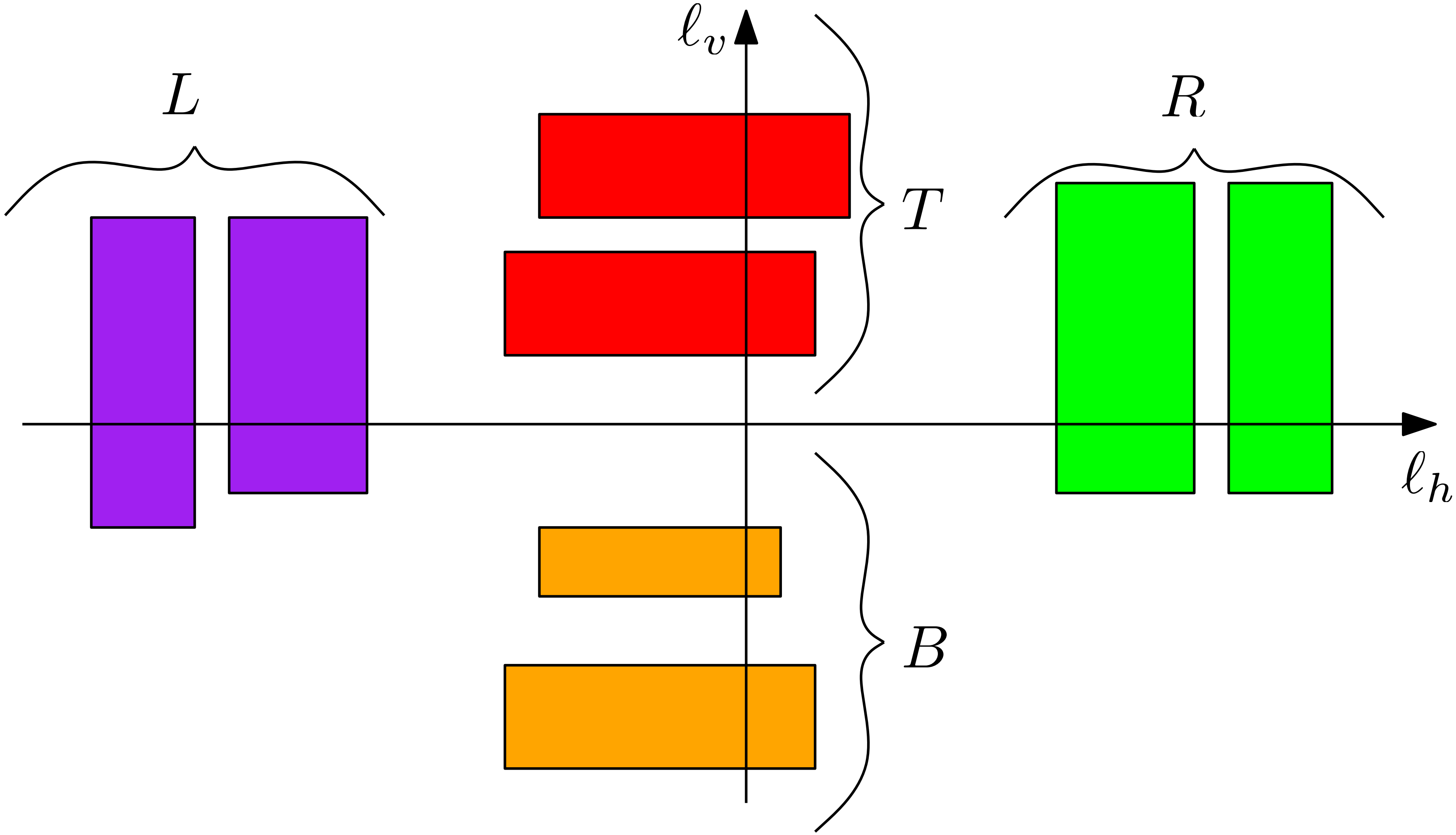

2.5D-BR of K_n

Suppose that no box is intersected by both ℓ_h and ℓ_v

Case 1. $|L| \neq |R|$ and $|B| \neq |T|$
Case 2. $|L| \neq |R|$ and $|B| = |T|$
Case 3. $|L| = |R|$ and $|B| = |T|$

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2.5D-BR of K_n

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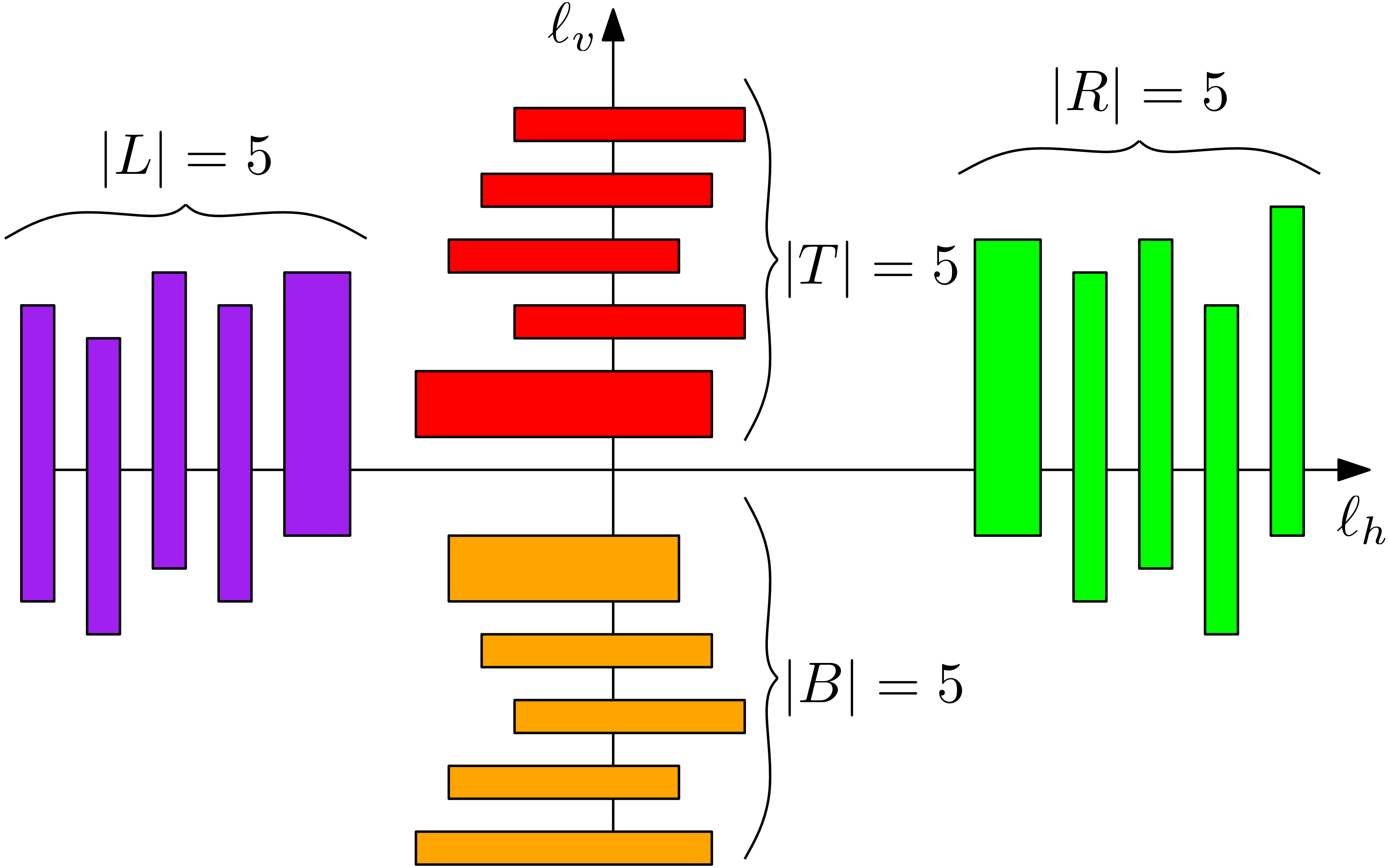

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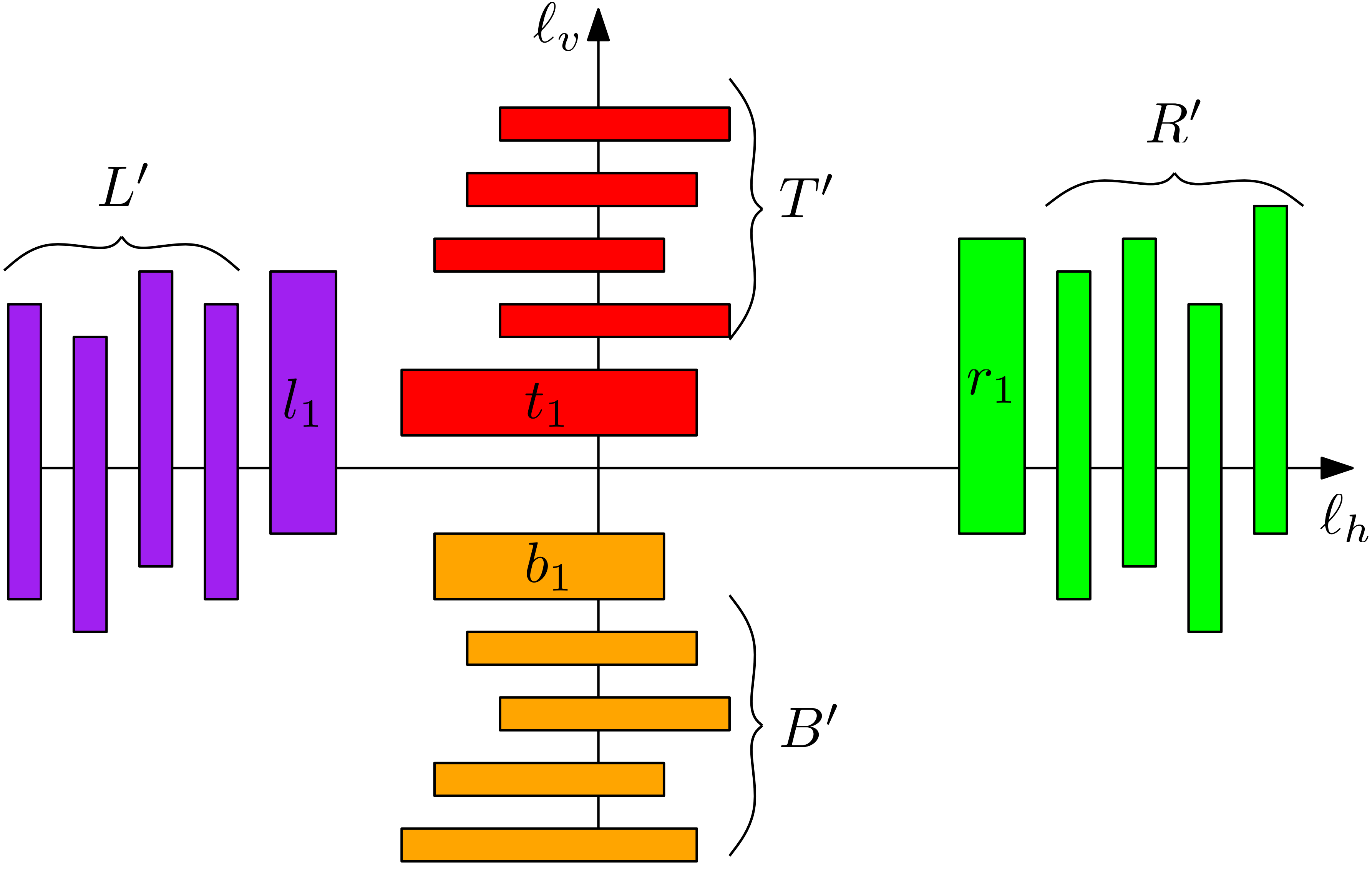
Case 3. $|L| = |R|$ and $|B| = |T|$



2.5D-BR of K_n

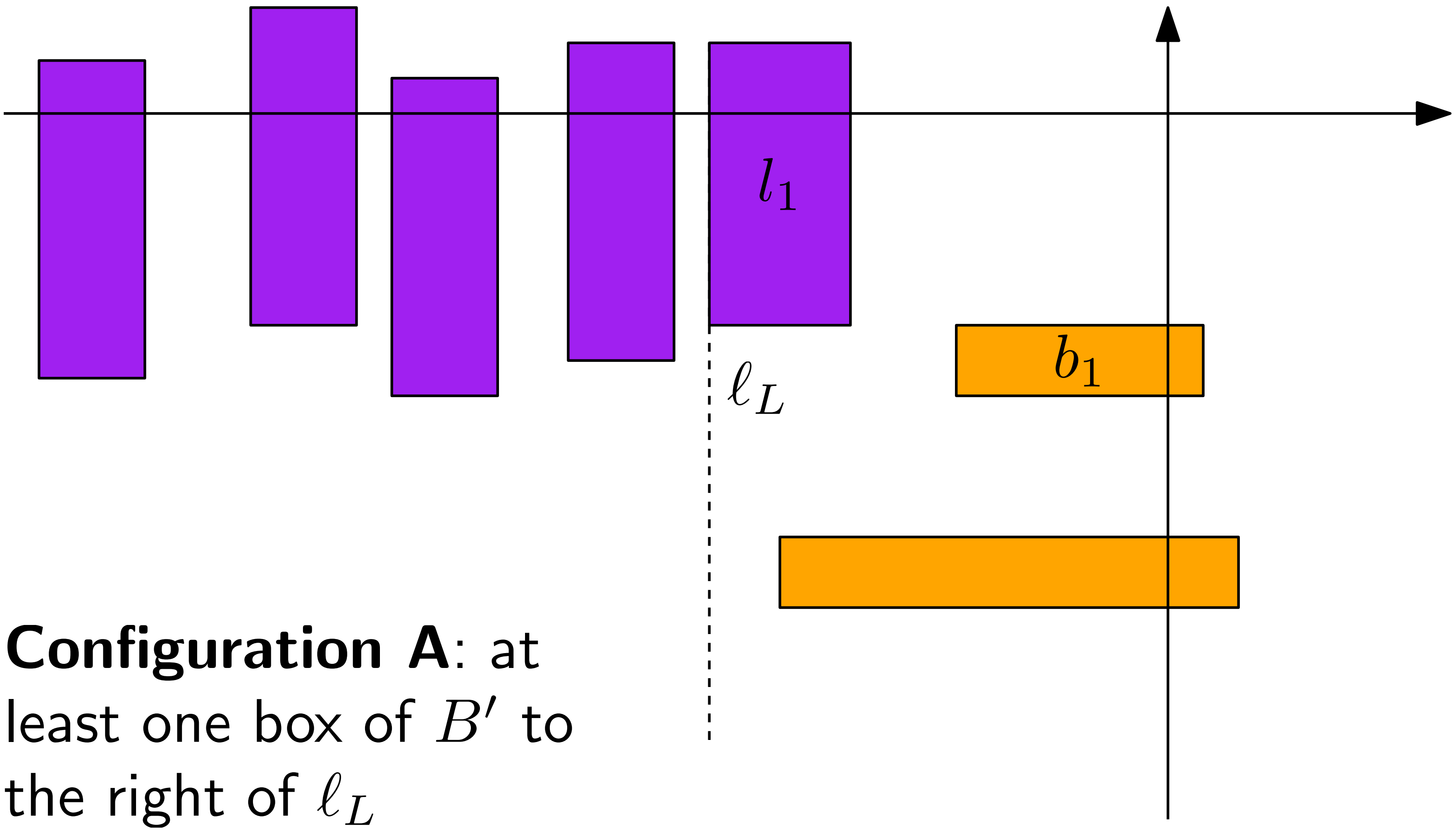
Case 3. $|L| = |R|$ and $|B| = |T|$

Two possible configurations for L and B

2.5D-BR of K_n

Case 3. $|L| = |R|$ and $|B| = |T|$

Two possible configurations for L and B



Configuration A: at least one box of B' to the right of ℓ_L

2.5D-BR of K_n

Case 3. $|L| = |R|$ and $|B| = |T|$

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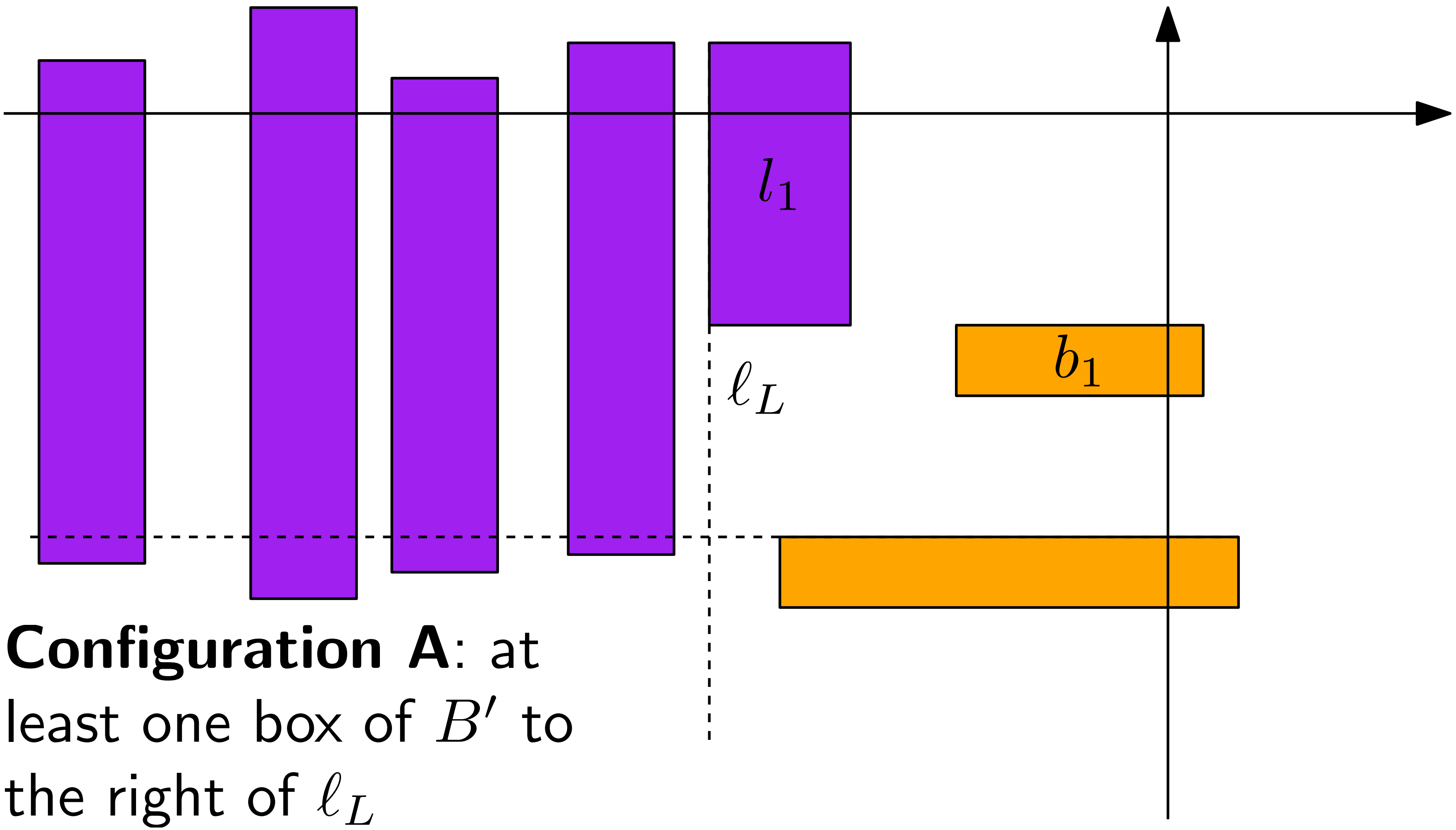

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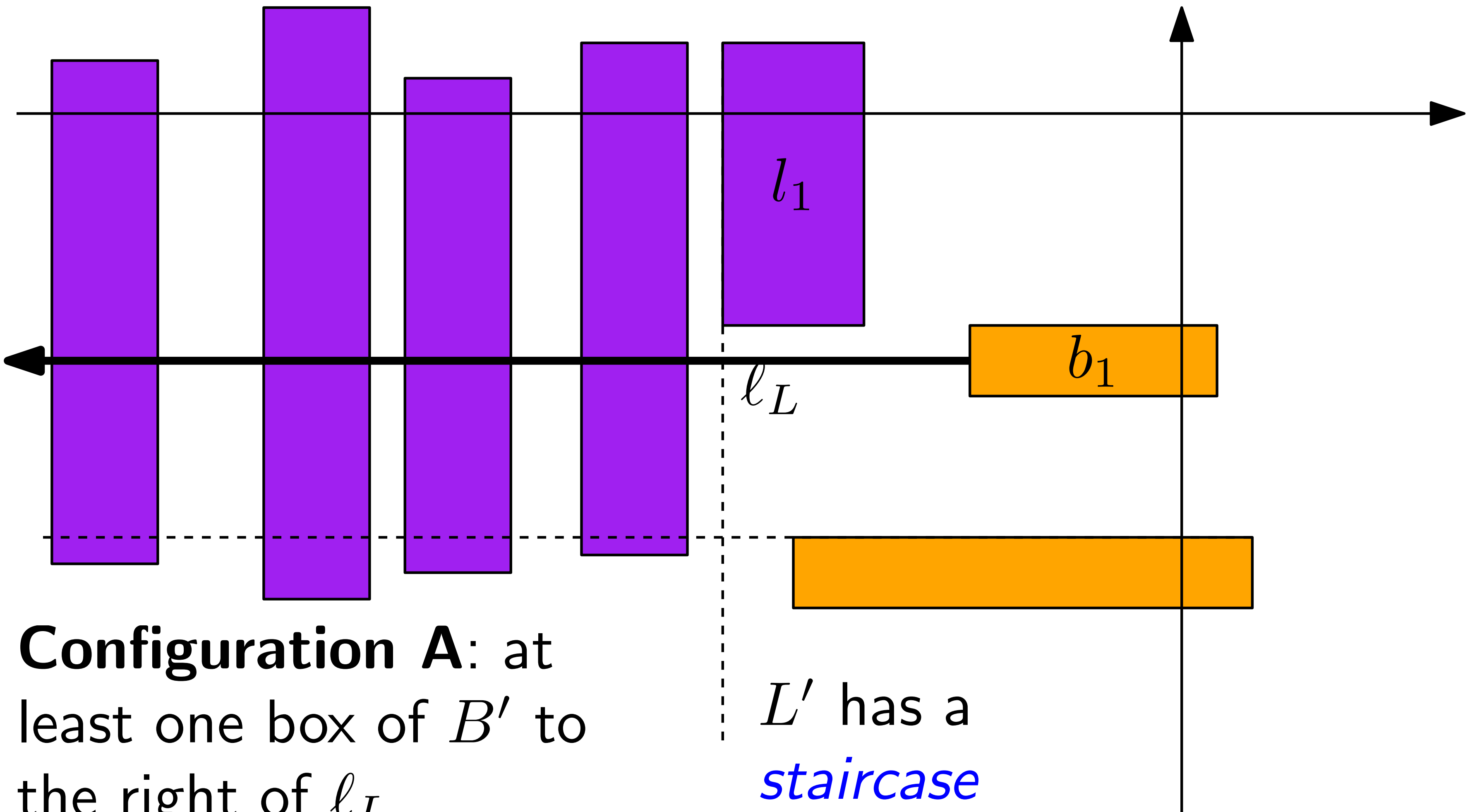

Configuration A: at least one box of B' to the right of ℓ_L

L' has a *staircase layout (SL)*

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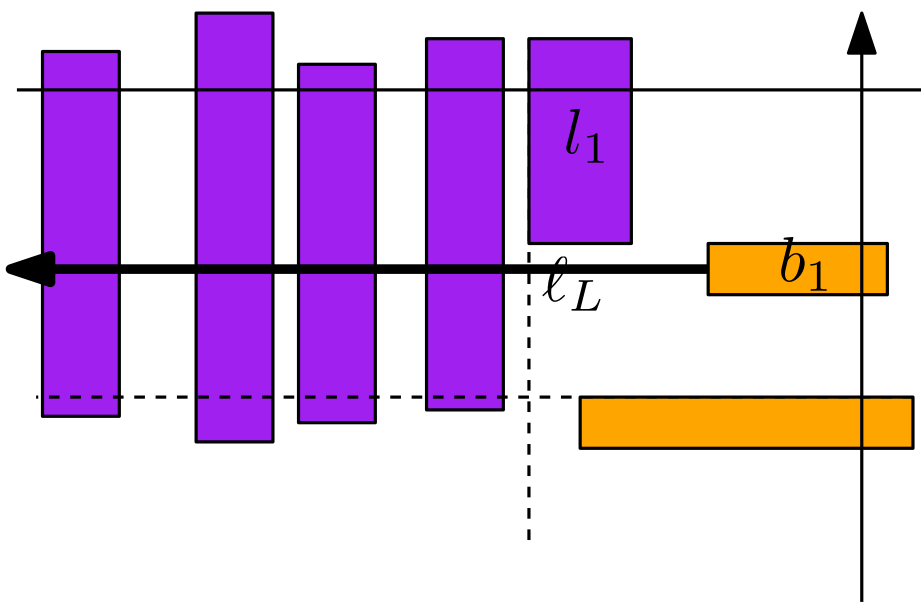

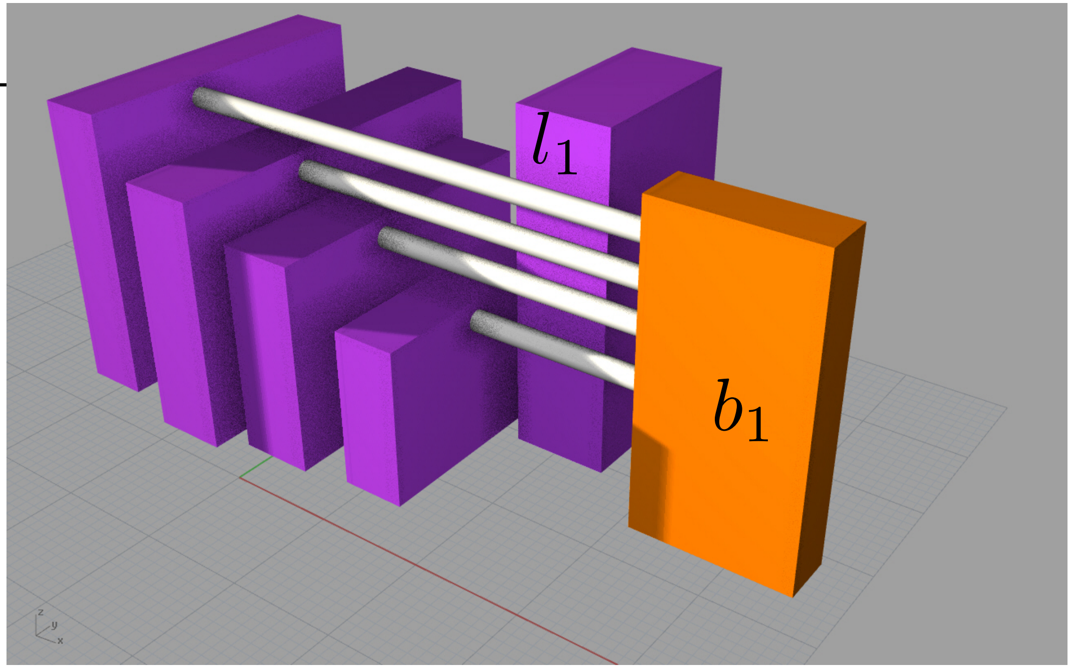

Configuration A: at least one box of B' to the right of ℓ_L

L' has a *staircase layout (SL)*

2.5D-BR of K_n

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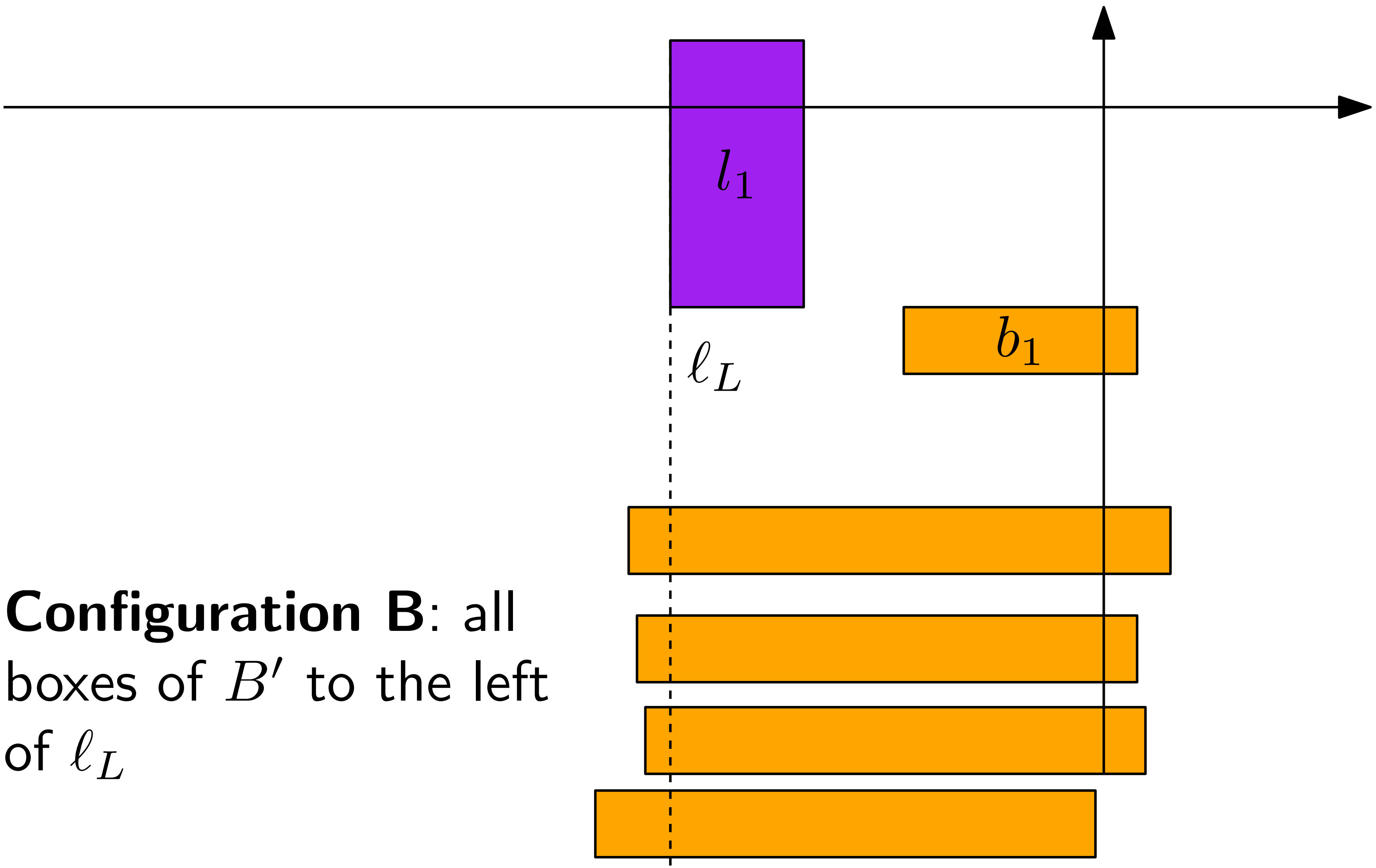
Two possible configurations for L and B

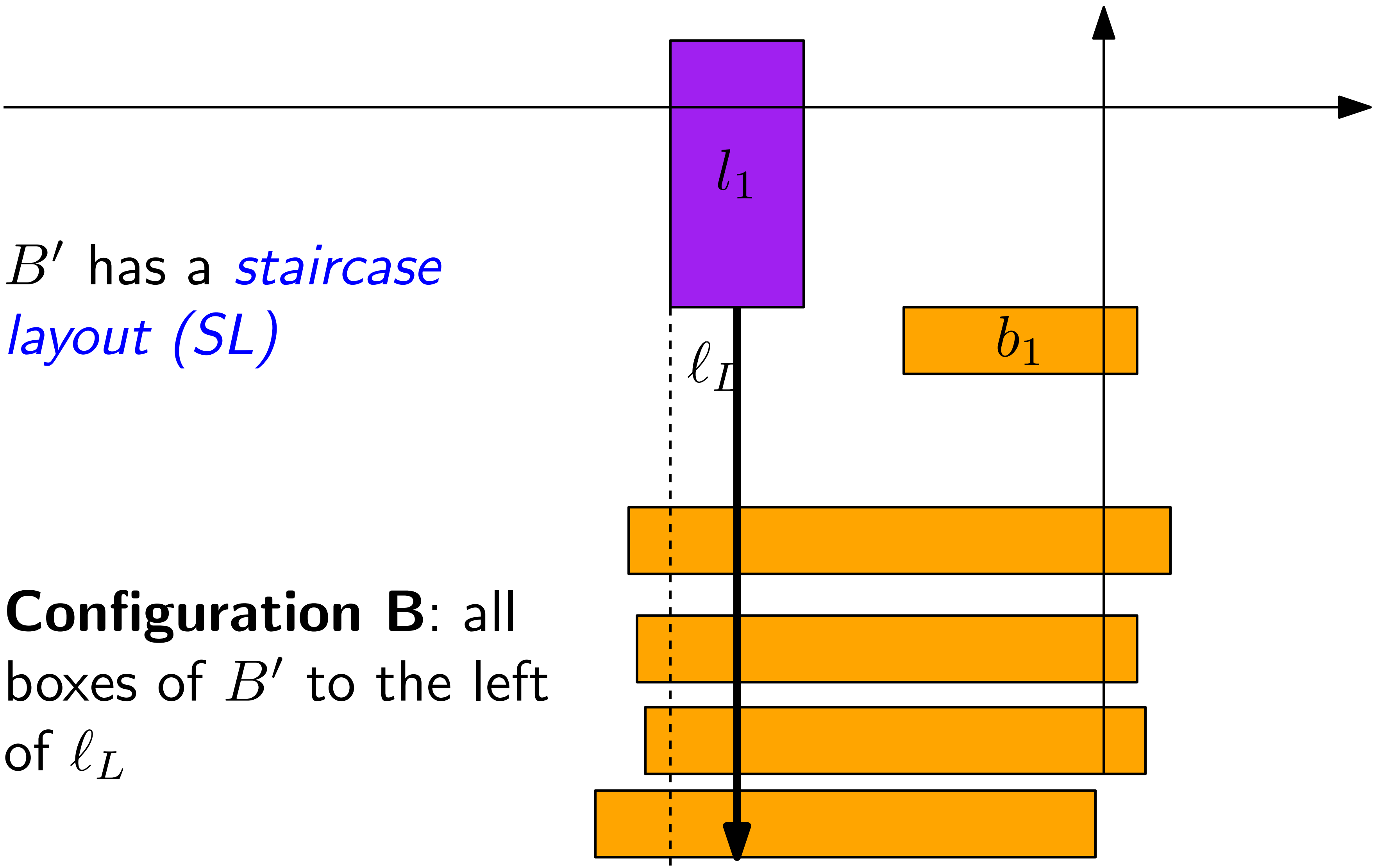


Configuration B: all boxes of B' to the left of ℓ_L

2.5D-BR of K_n

Case 3. $|L| = |R|$ and $|B| = |T|$

Two possible configurations for L and B



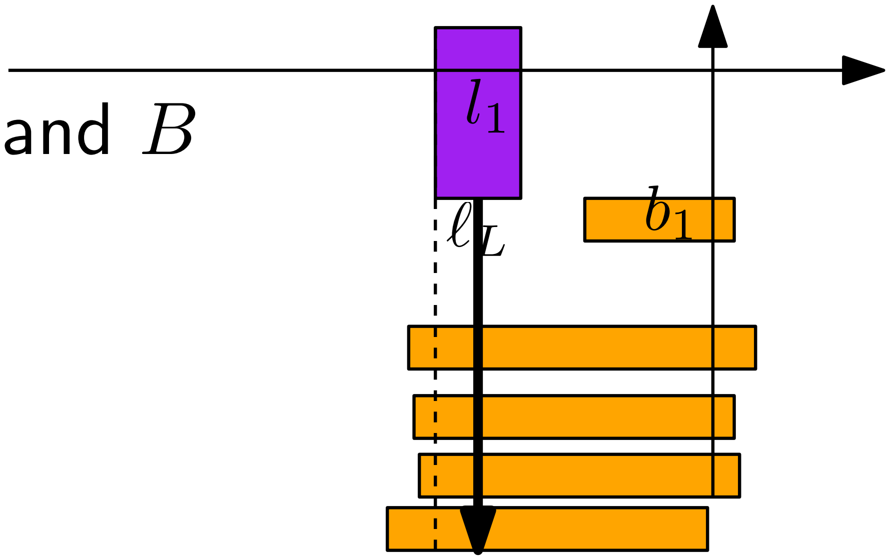
B' has a *staircase layout (SL)*

Configuration B: all boxes of B' to the left of ℓ_L

2.5D-BR of K_n

Case 3. $|L| = |R|$ and $|B| = |T|$

Two possible configurations for L and B



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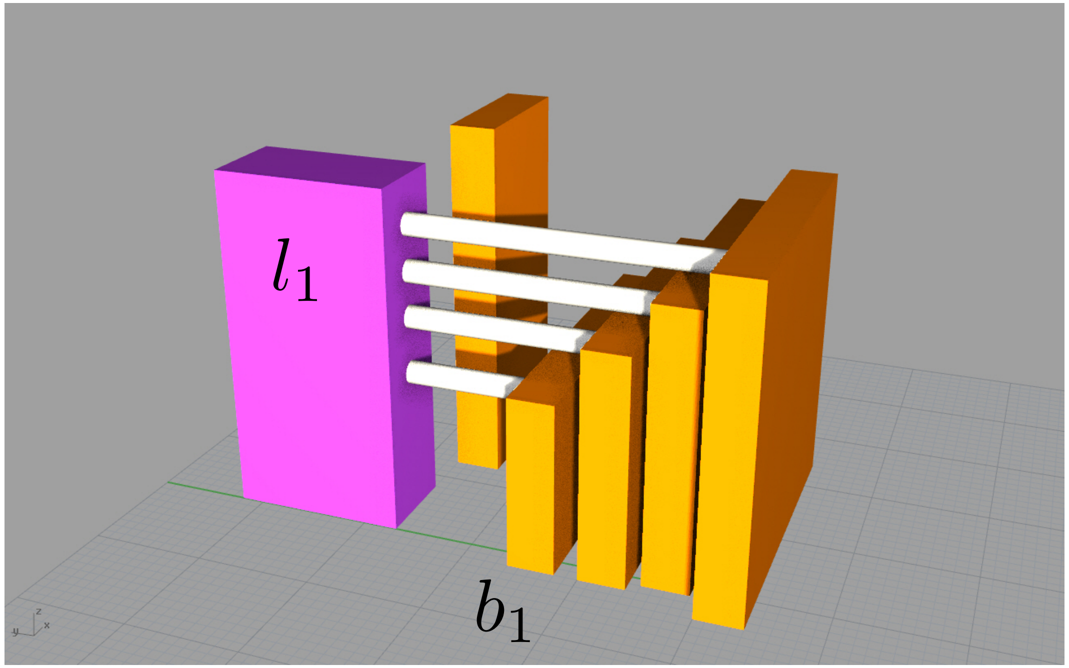


Configuration B: all boxes of B' to the left of ℓ_L

2.5D-BR of K_n

Two possible configurations for B and L:

- Configuration A $\Rightarrow L'$ has a SL
- Configuration B $\Rightarrow B'$ has a SL

2.5D-BR of K_n

Two possible configurations for B and L:

- Configuration A $\Rightarrow L'$ has a SL
- Configuration B $\Rightarrow B'$ has a SL

The same holds for L and T, for T and R, and for R and B

2.5D-BR of K_n

Two possible configurations for B and L:

- Configuration A $\Rightarrow L'$ has a SL
- Configuration B $\Rightarrow B'$ has a SL

The same holds for L and T, for T and R, and for R and B

	Conf. A	Conf. B
B and L	SL of L'	SL of B'
L and T	SL of T'	SL of L'
T and R	SL of R'	SL of T'
R and B	SL of B'	SL of R'

2.5D-BR of K_n

Two possible configurations for B and L:

- Configuration A $\Rightarrow L'$ has a SL
- Configuration B $\Rightarrow B'$ has a SL

The same holds for L and T, for T and R, and for R and B

	Conf. A	Conf. B
B and L	SL of L'	SL of B'
L and T	SL of T'	SL of L'
T and R	SL of R'	SL of T'
R and B	SL of B'	SL of R'

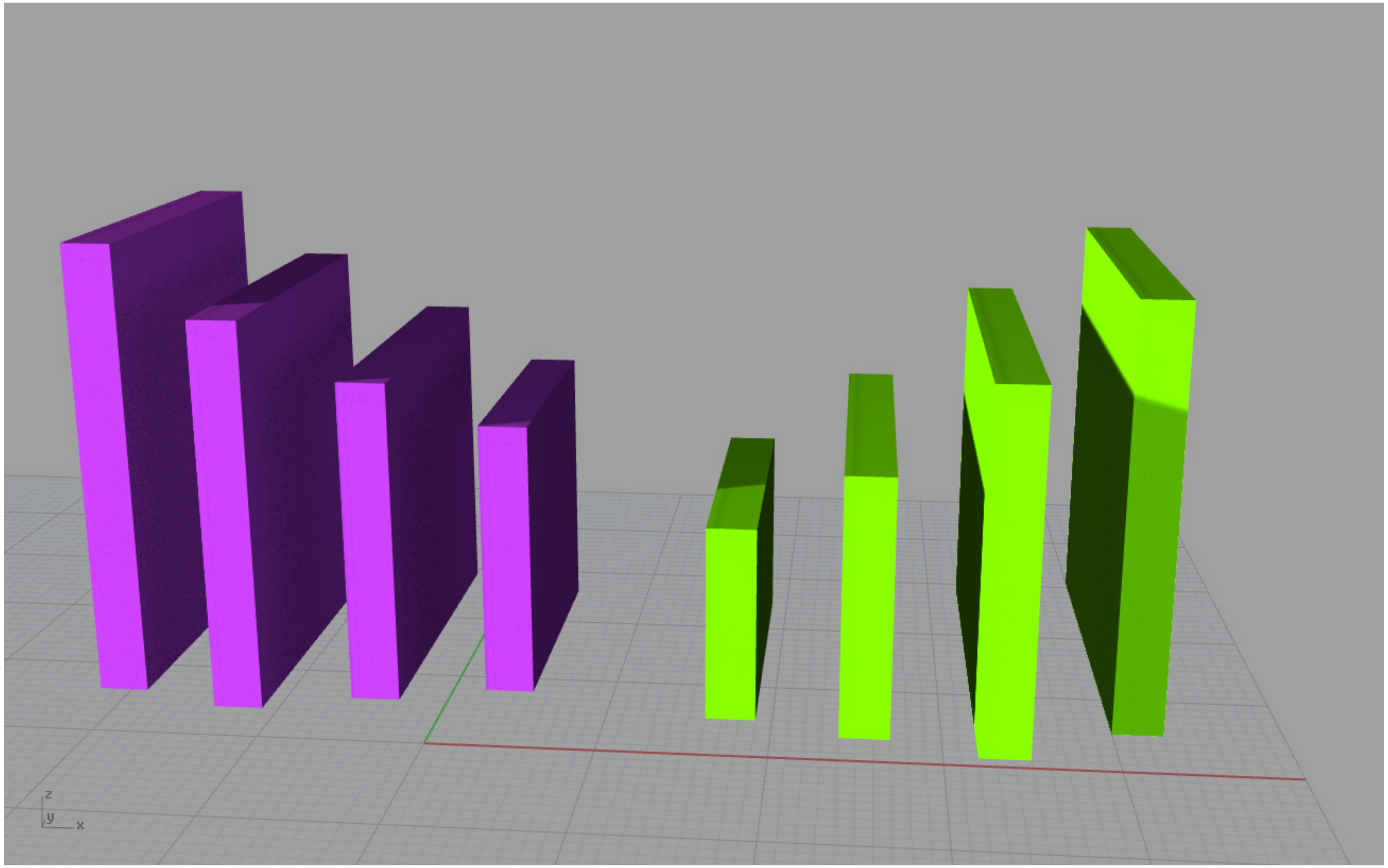
$\Rightarrow B'$ and T' have both a SL or L' and R' have both a SL

2.5D-BR of K_n

Suppose that L' and R' have both a SL

2.5D-BR of K_n

Suppose that L' and R' have both a SL



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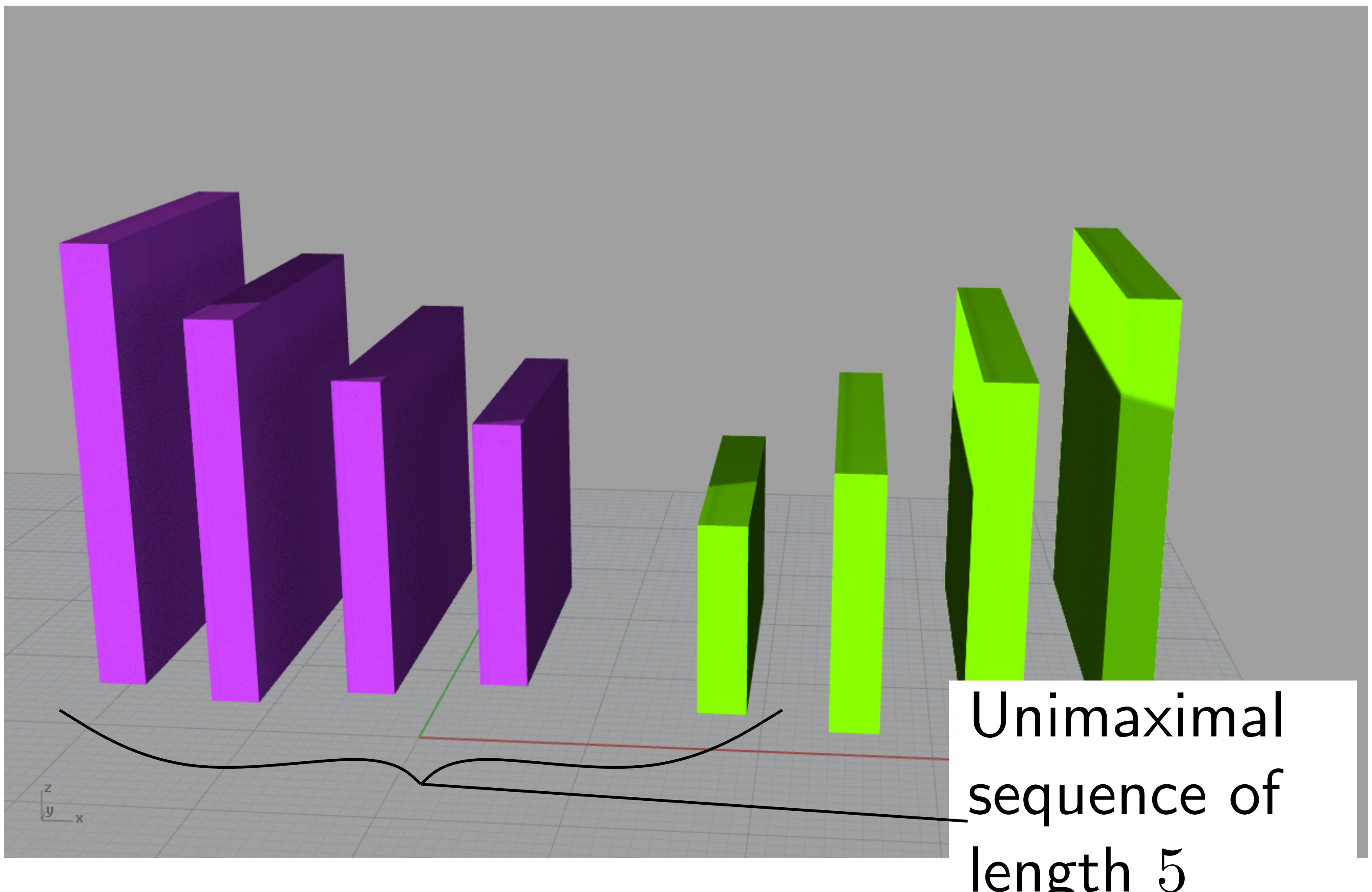


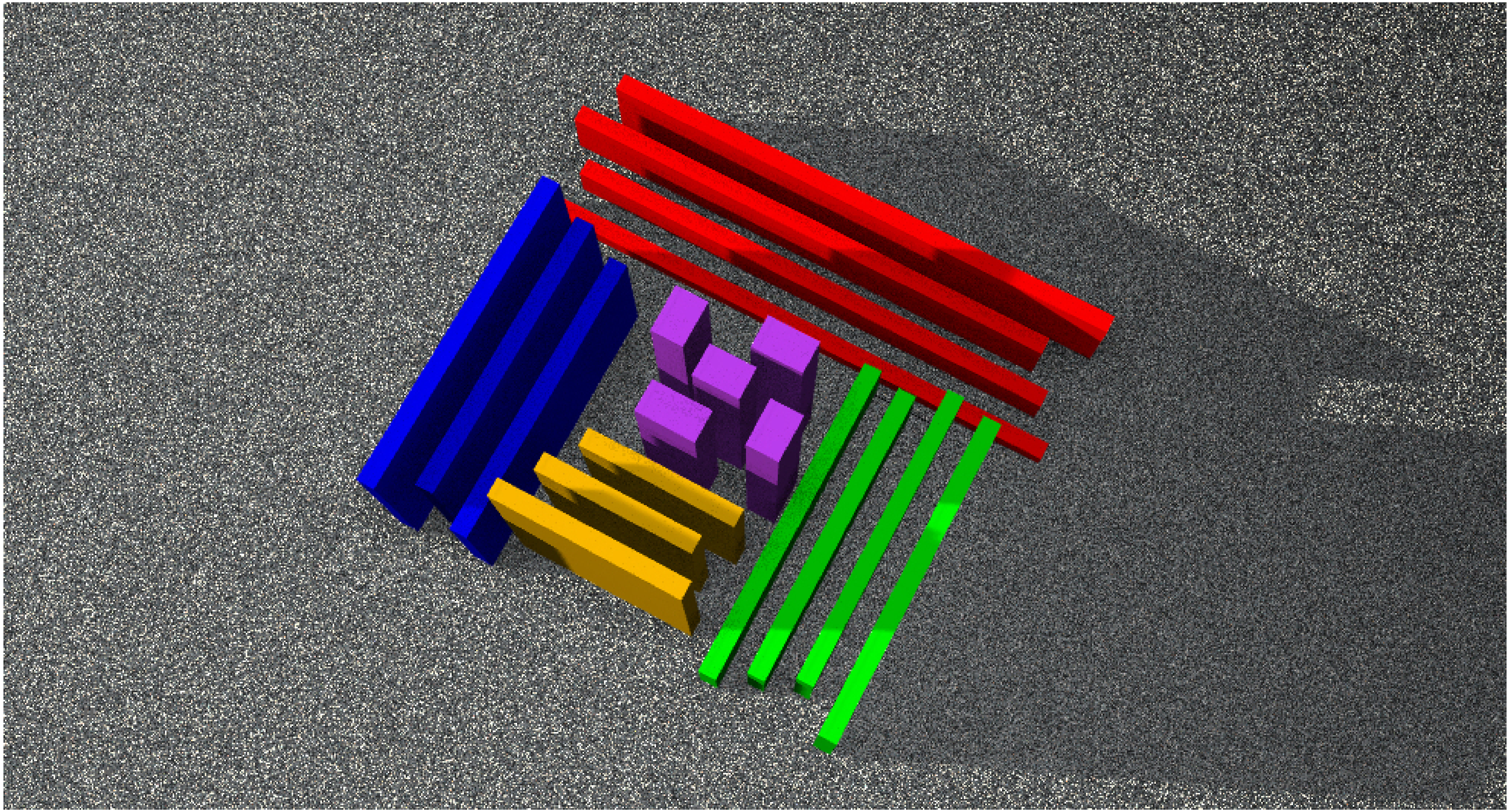
2.5D-BR of K_n

Lemma 1 *A complete graph admits a 2.5D-BR only if it has at most* 19 *vertices.*

Given a 2.5D-BR Γ of K_n, we show that:

- There is one line ℓ_h parallel to the x-axis and one line ℓ_v parallel to the y-axis whose union intersect the footprints of all boxes ✓
- ℓ_h and ℓ_v intersect at most 10 boxes each $\Rightarrow$ there can be at most 20 boxes in Γ ✓
- There is a box that is intersected by both ℓ_h and ℓ_v $\Rightarrow$ there can be at most 19 boxes in Γ ✓

A 2.5D-BR of K_{19}



2.5D-GBR with a given footprint: NP-hardness

2.5D-GBR: NP-hardness

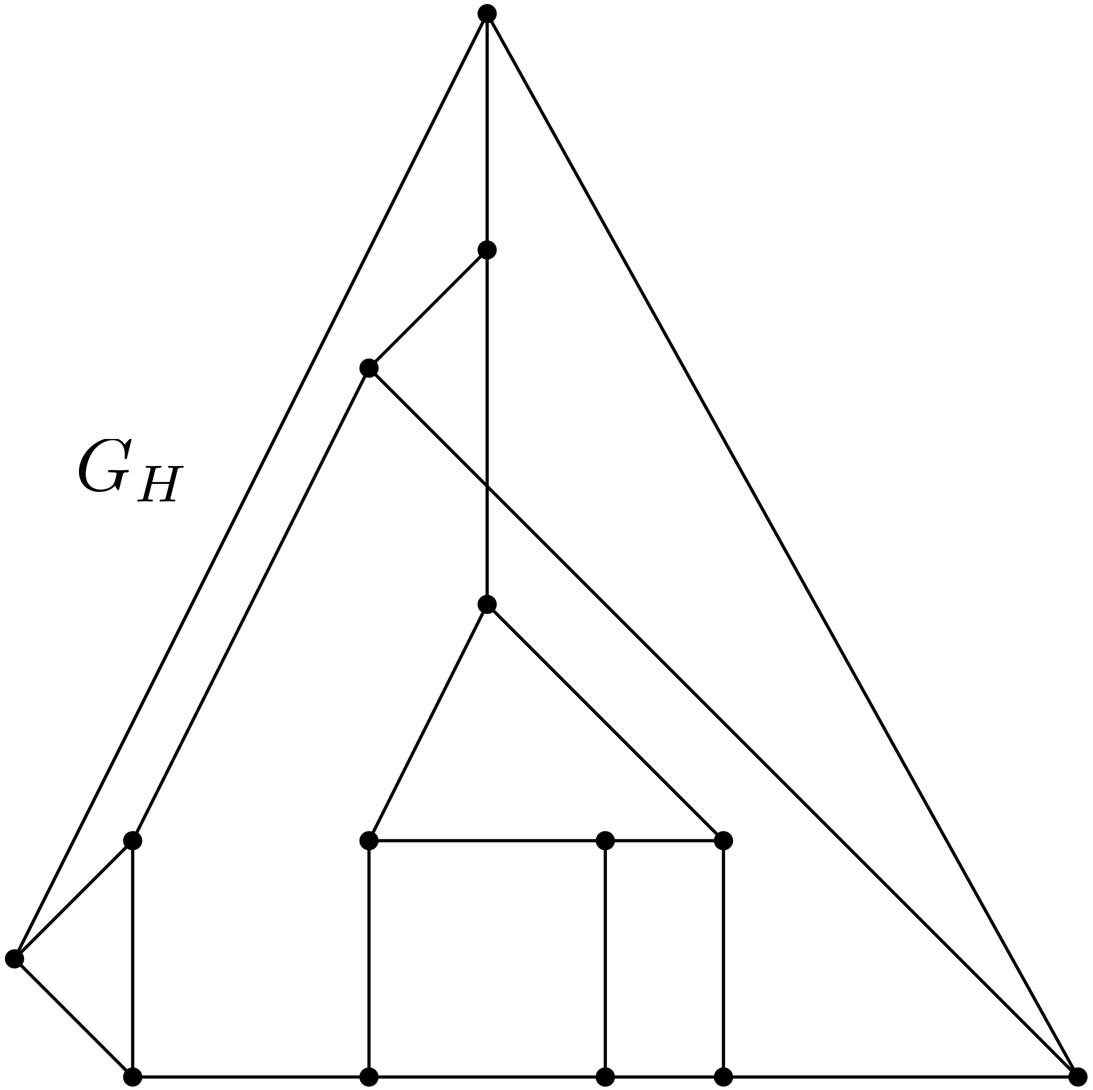
Theorem 1 *Deciding whether a given graph G admits a 2.5D-GBR with a given footprint (2.5DGBR-WGF) is NP-complete, even if G is a path.*

2.5D-GBR: NP-hardness

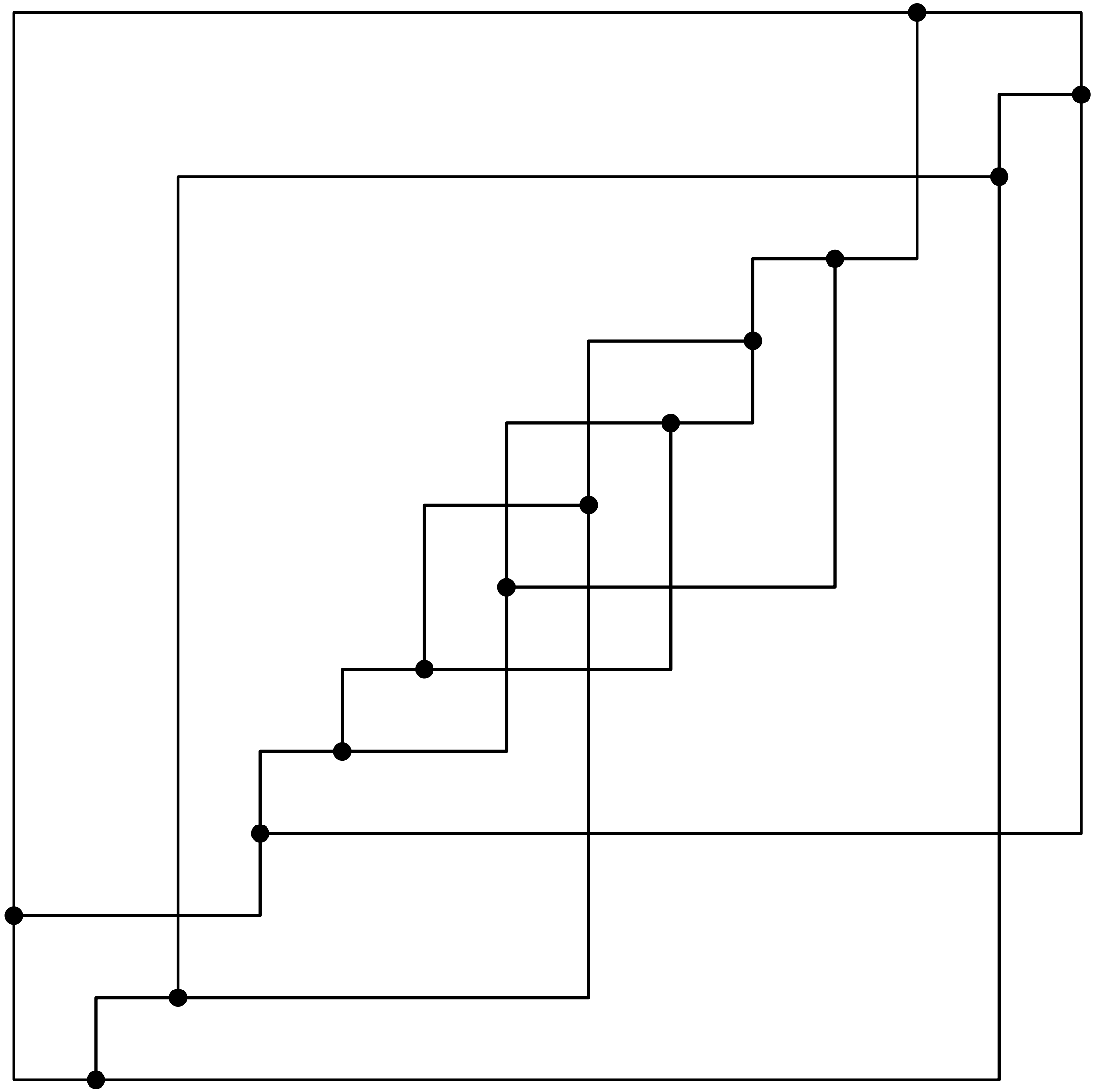
Theorem 1 *Deciding whether a given graph G admits a 2.5D-GBR with a given footprint (2.5DGBR-WGF) is NP-complete, even if G is a path.*

- Reduction from HAMILTONIAN-PATH-FOR-CUBIC-GRAPHS (HPCG)
- Let G_H be an instance of HPCG
- We construct an instance $\langle G, F \rangle$ of 2.5DGBR-WGF, where G is a path

2.5D-GBR: NP-hardness



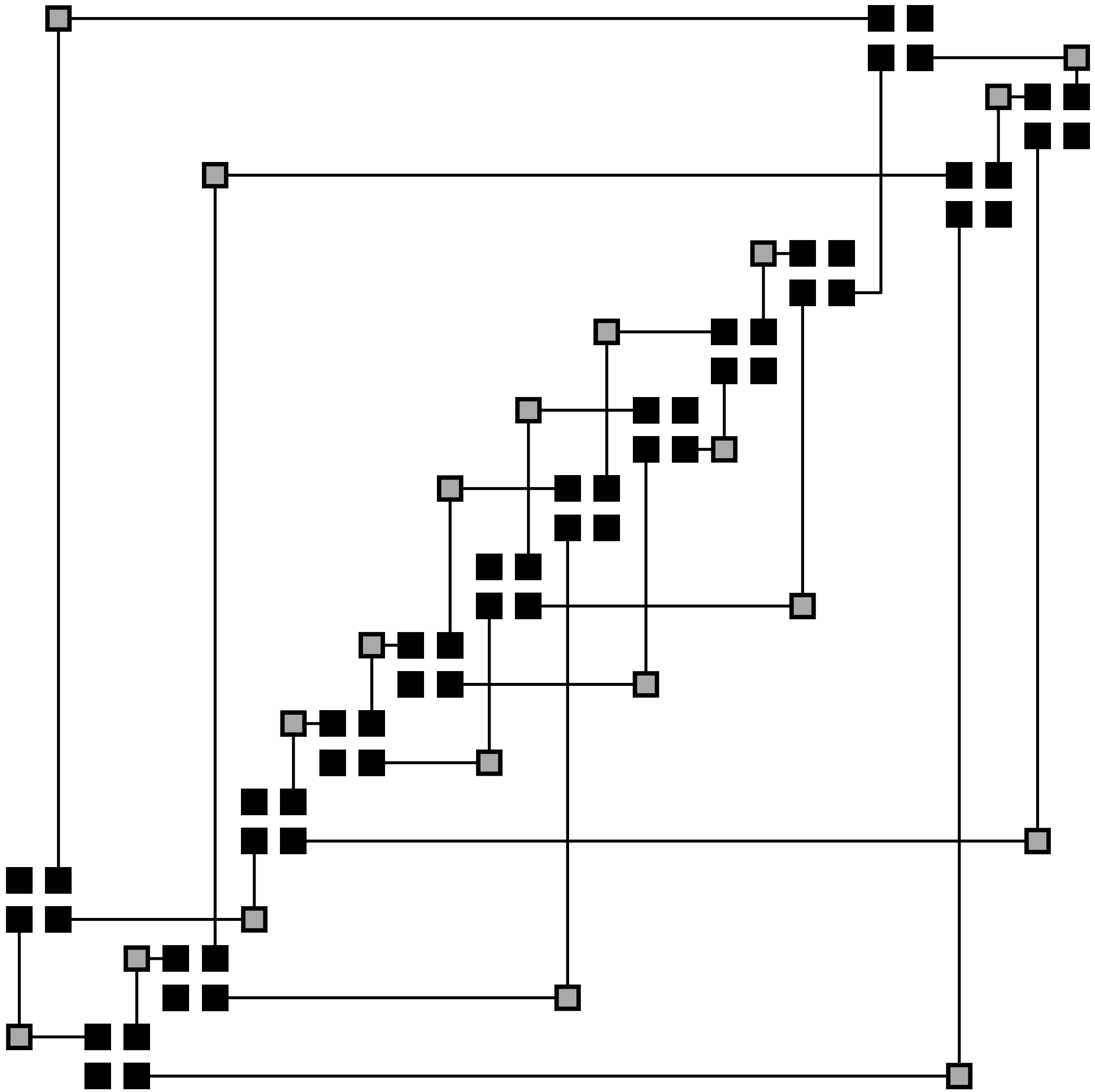
2.5D-GBR: NP-hardness



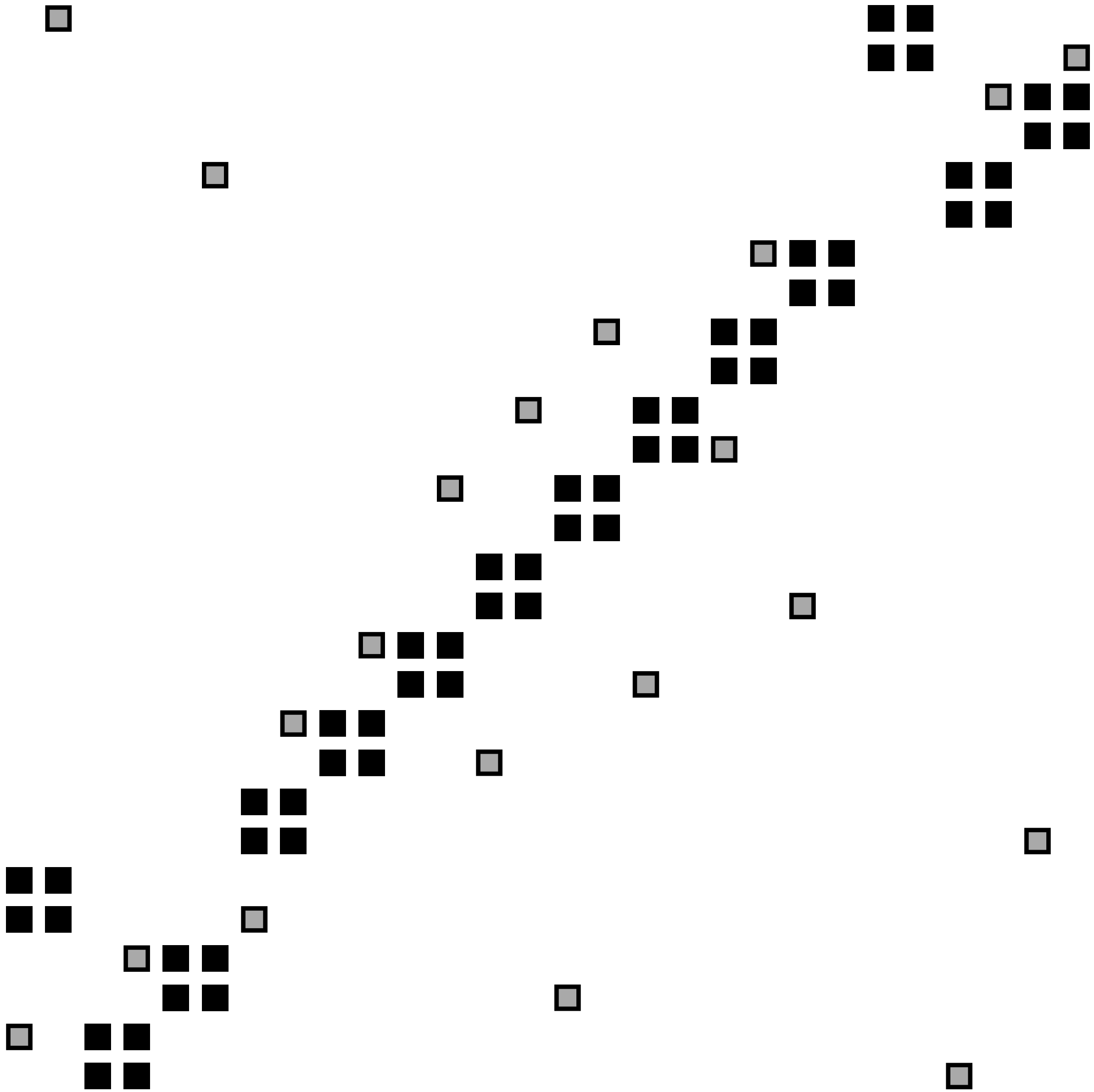
Orthogonal drawing of G_H s.t. every edge has exactly 1 bend and no two vertices share the same x- or y-coordinate.

Bruckdorfer et al. 2014

2.5D-GBR: NP-hardness

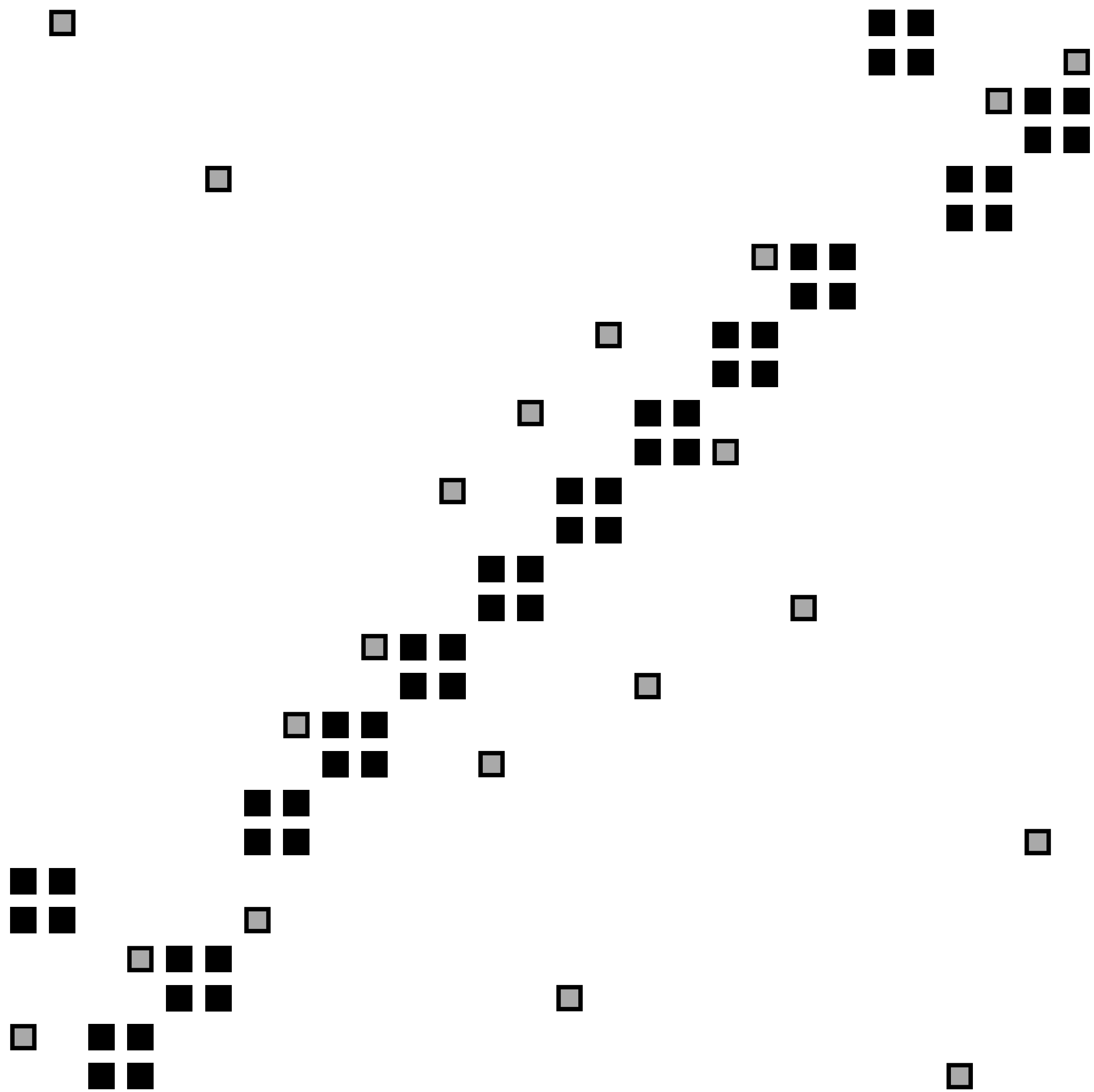


2.5D-GBR: NP-hardness



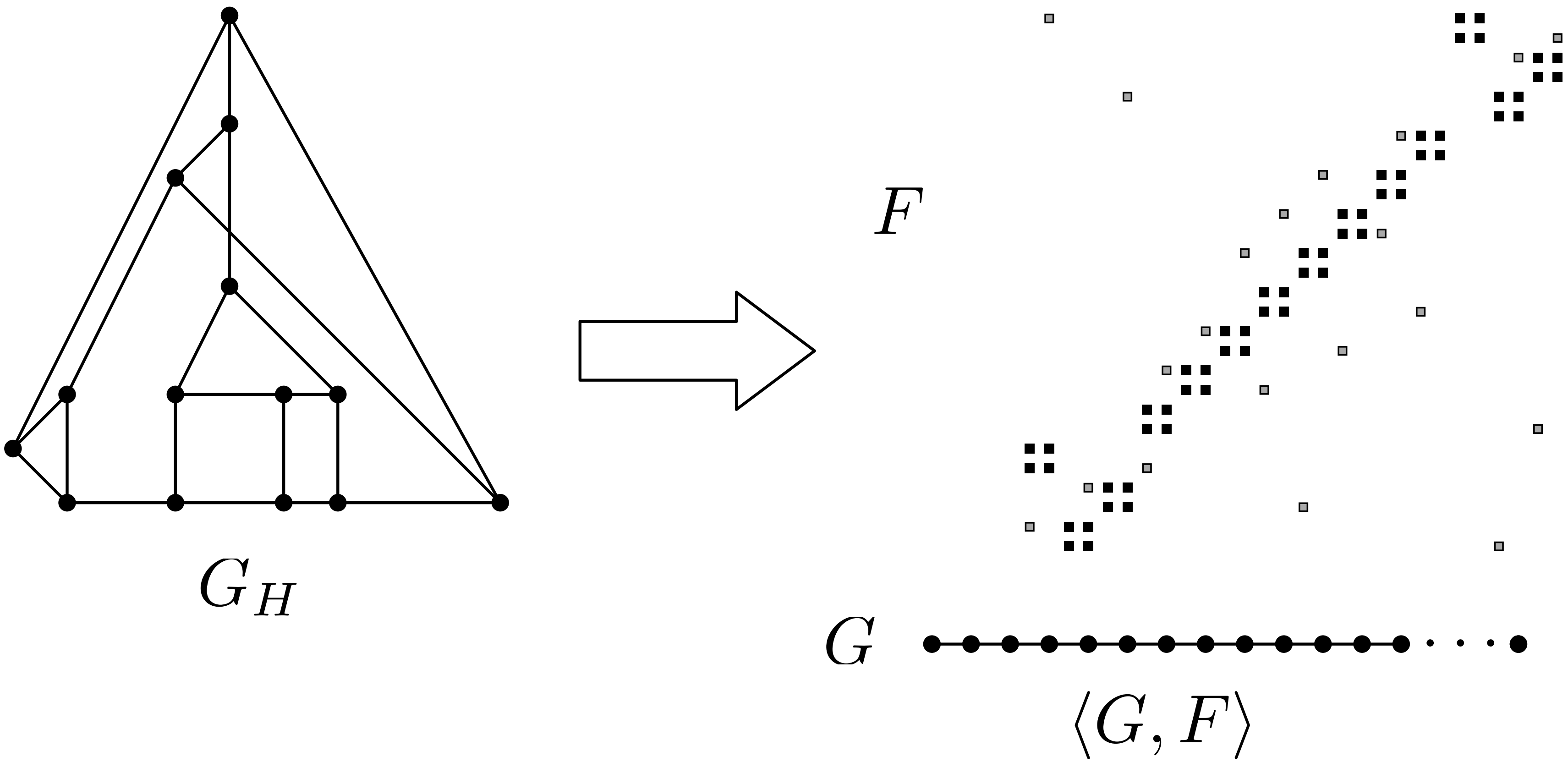
This is the footprint F of the instance $\langle G, F \rangle$

2.5D-GBR: NP-hardness



There are at most three squares in each row/column

2.5D-GBR: NP-hardness



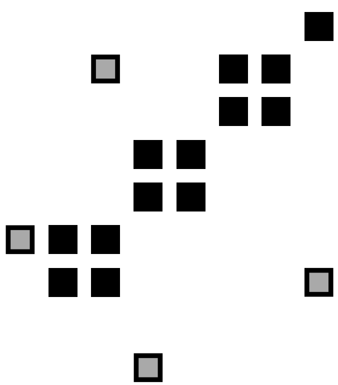
2.5D-GBR: NP-hardness

G_H has a Hamiltonian path $\Leftrightarrow$ G has a 2.5D-GBR with footprint F

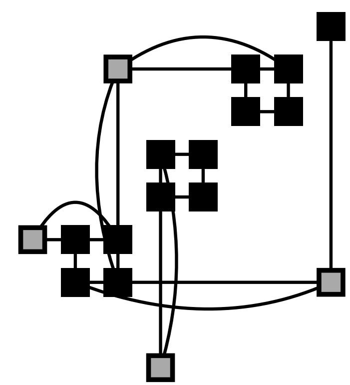
Footprint graph

Starting from F we define a graph F^* that:

- has a vertex for each square of F;
- has an edge between two squares if they are horizontally or vertically aligned



F



F^*

2.5D-GBR: NP-hardness

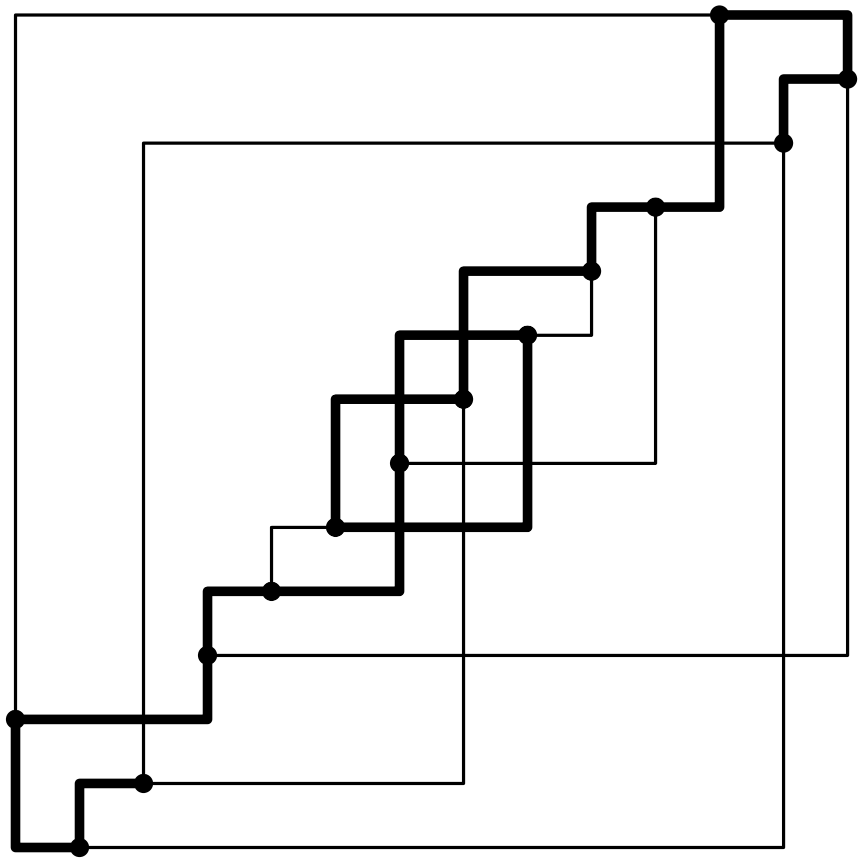
G_H has a Hamiltonian path $\Leftrightarrow$ F^* has a Hamiltonian path

$\Updownarrow$

G has a 2.5D-GBR with footprint F

2.5D-GBR: NP-hardness

G_H has a Hamiltonian path	$\Longrightarrow$	F^* has a Hamiltonian path



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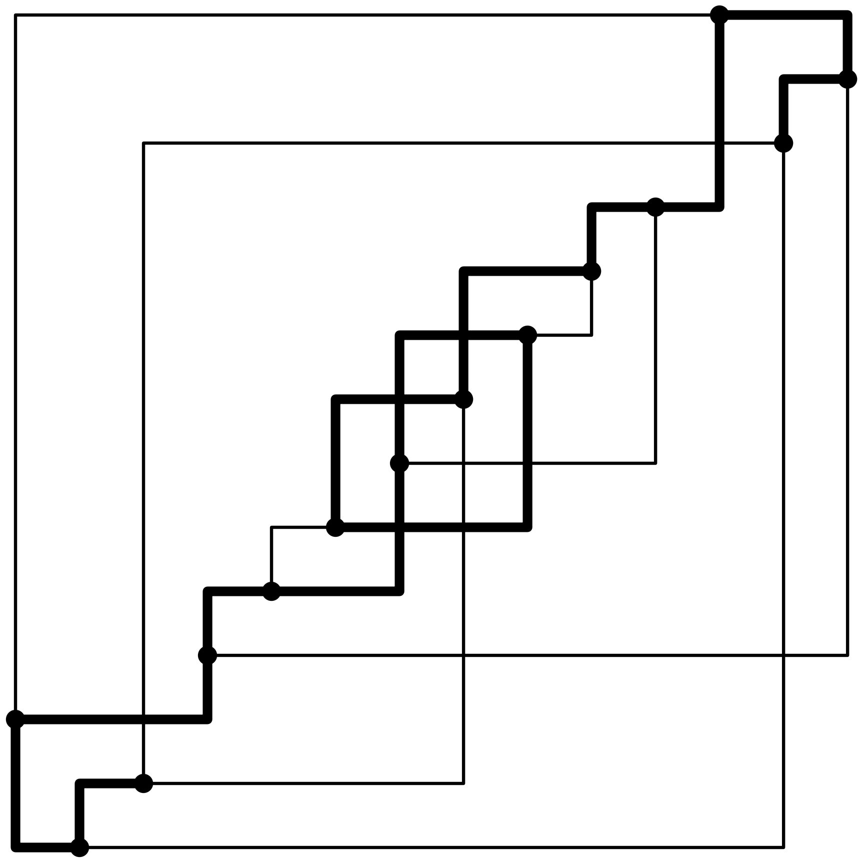

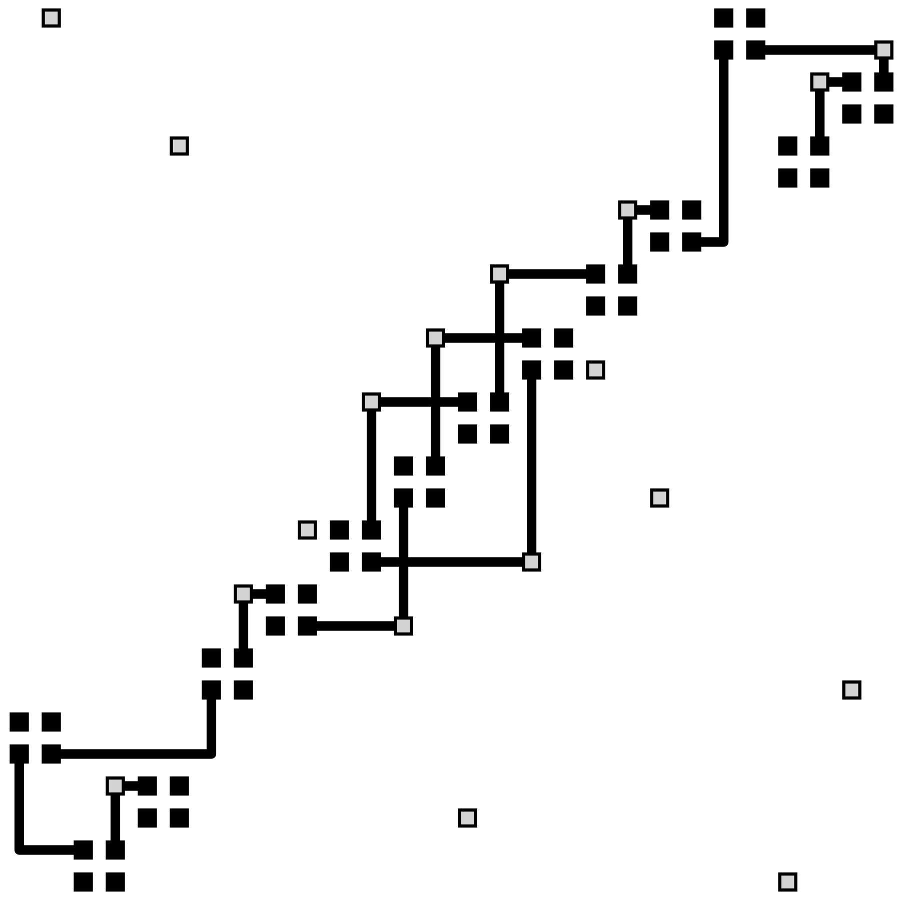

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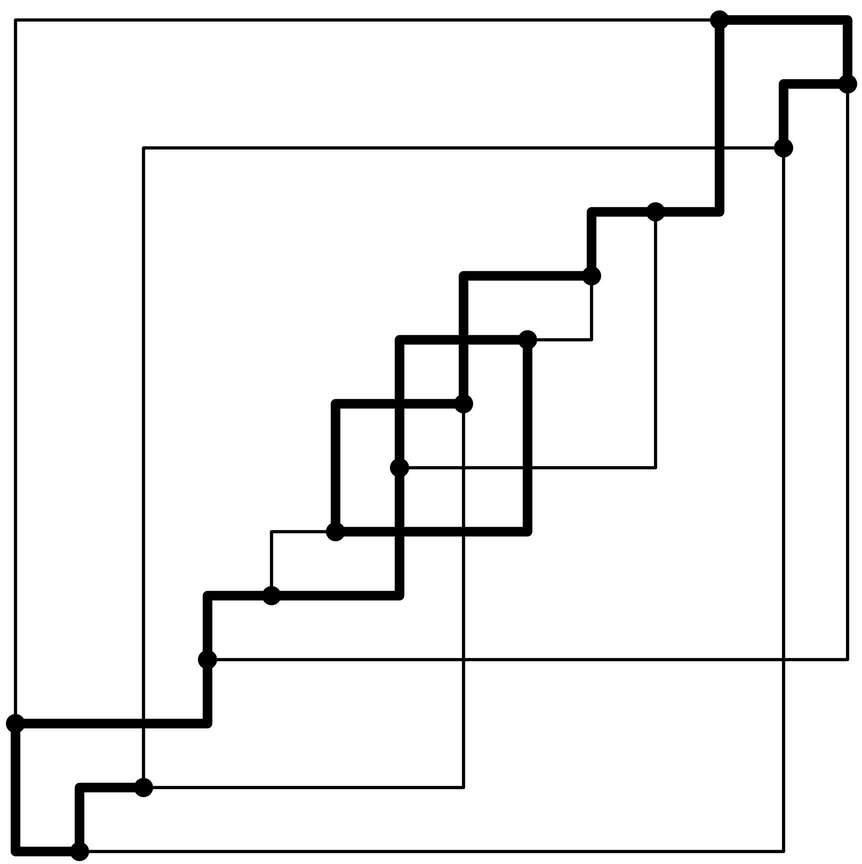

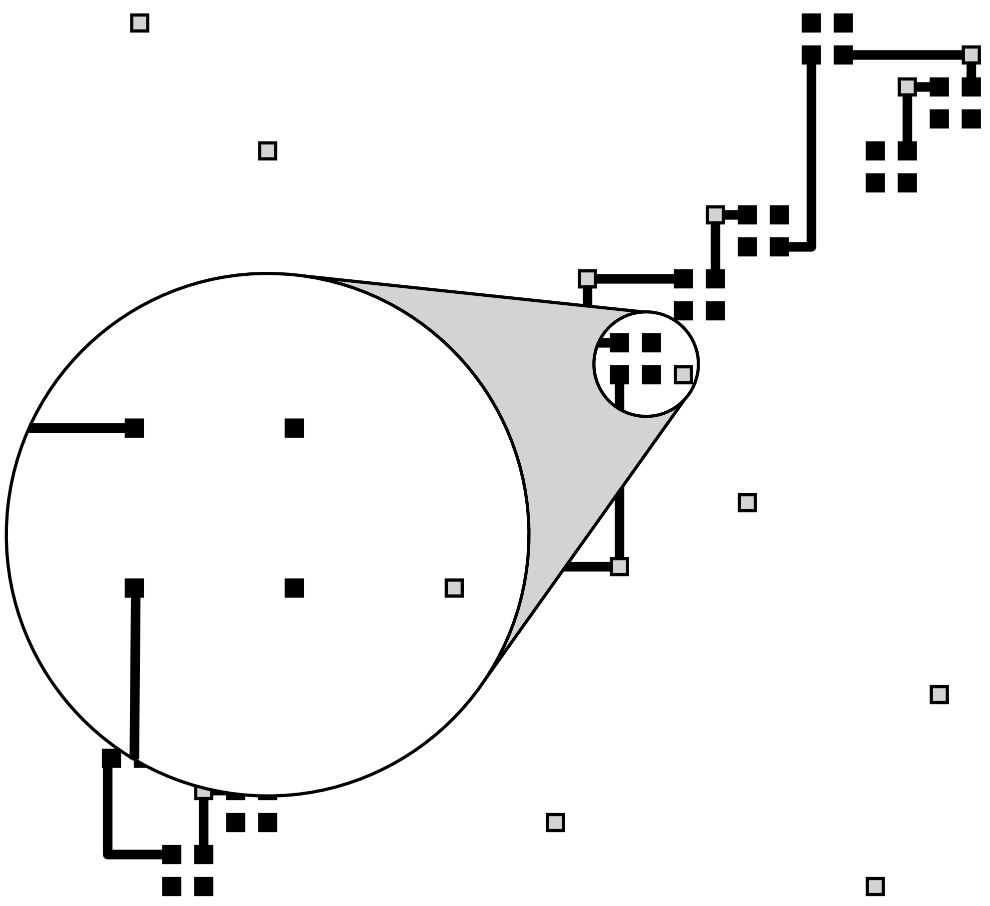

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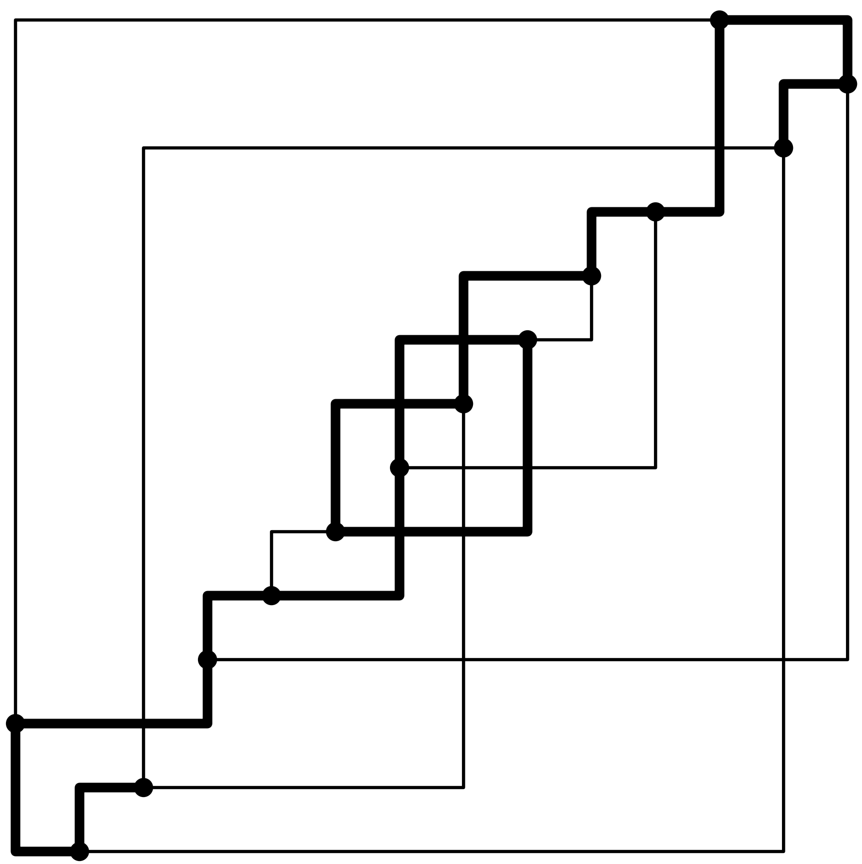

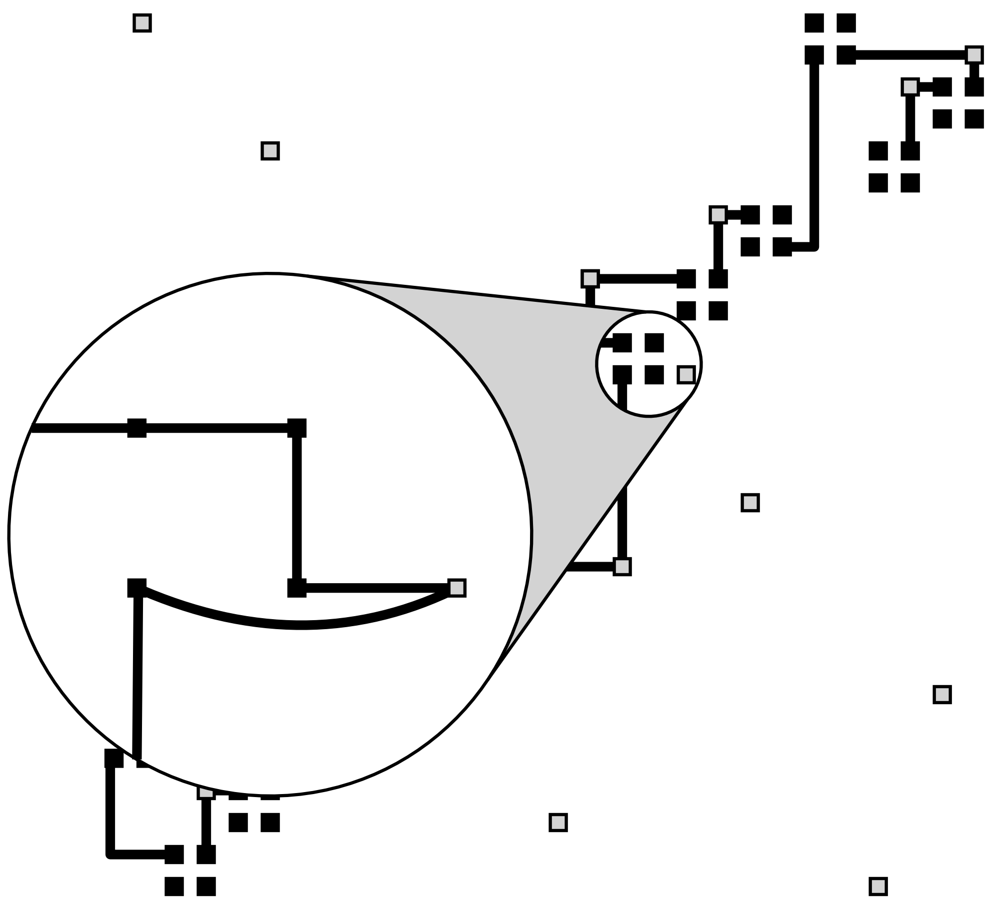

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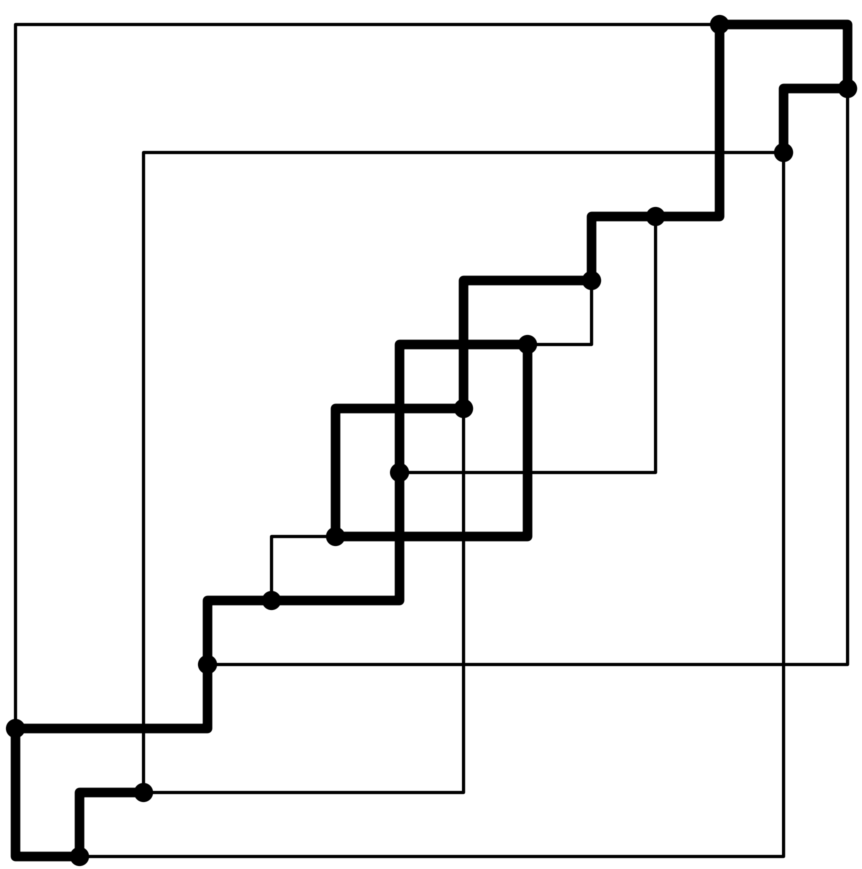

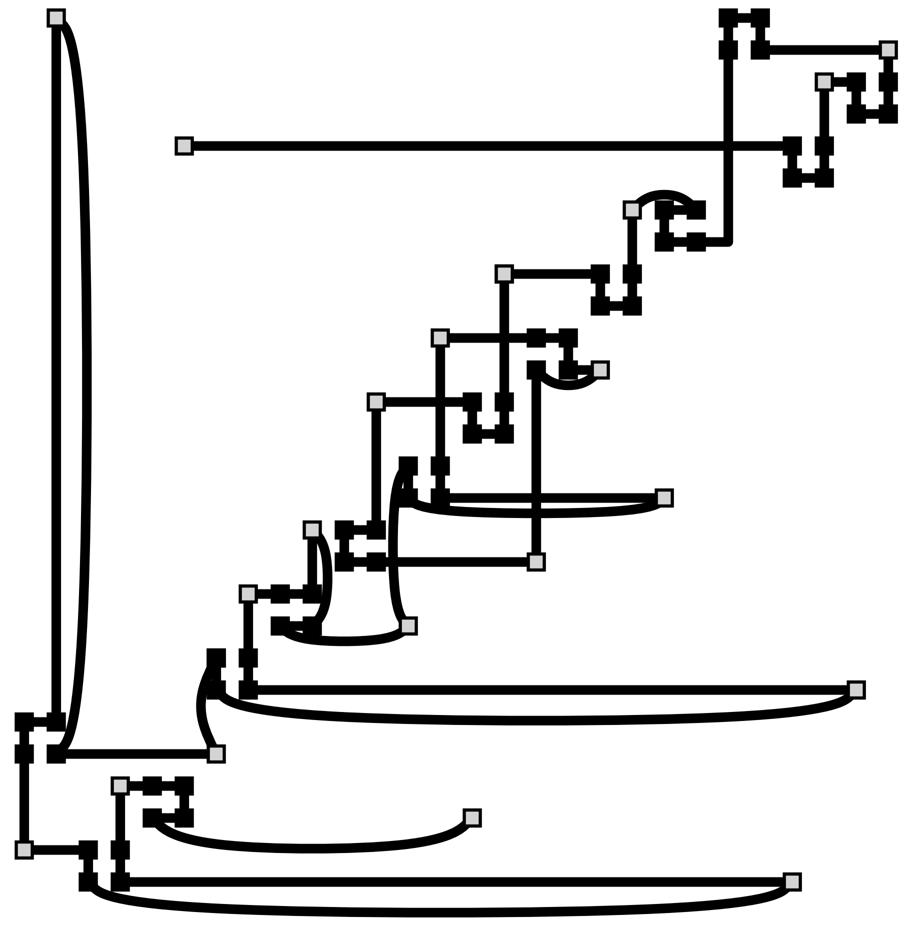

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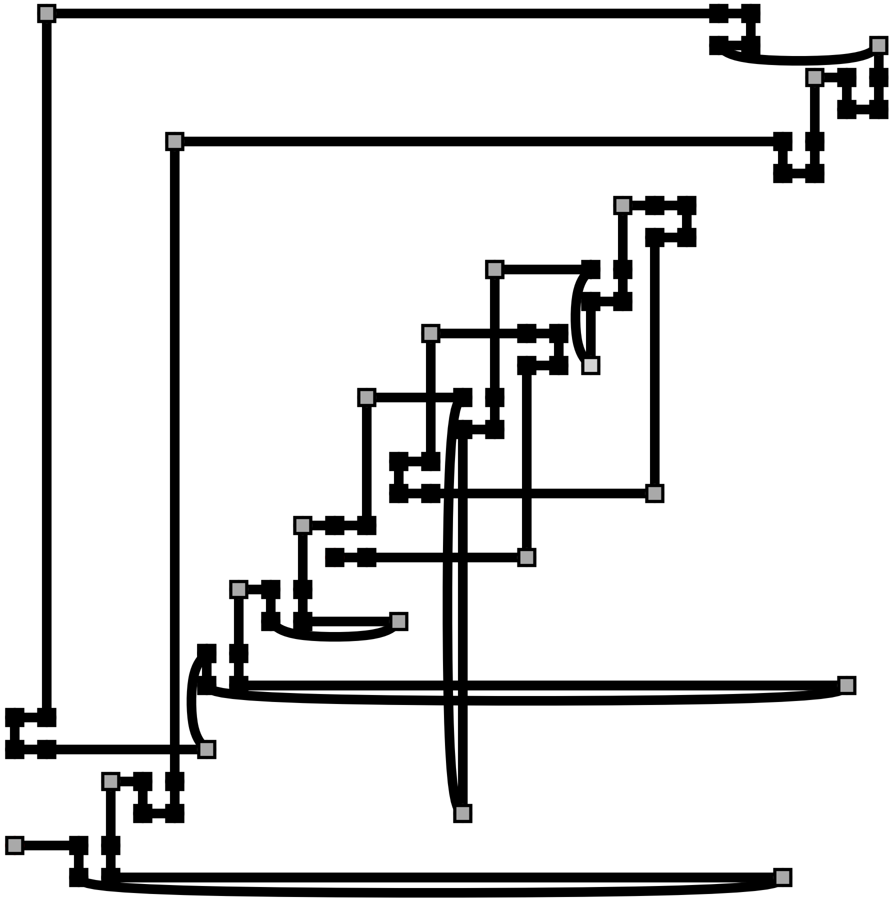

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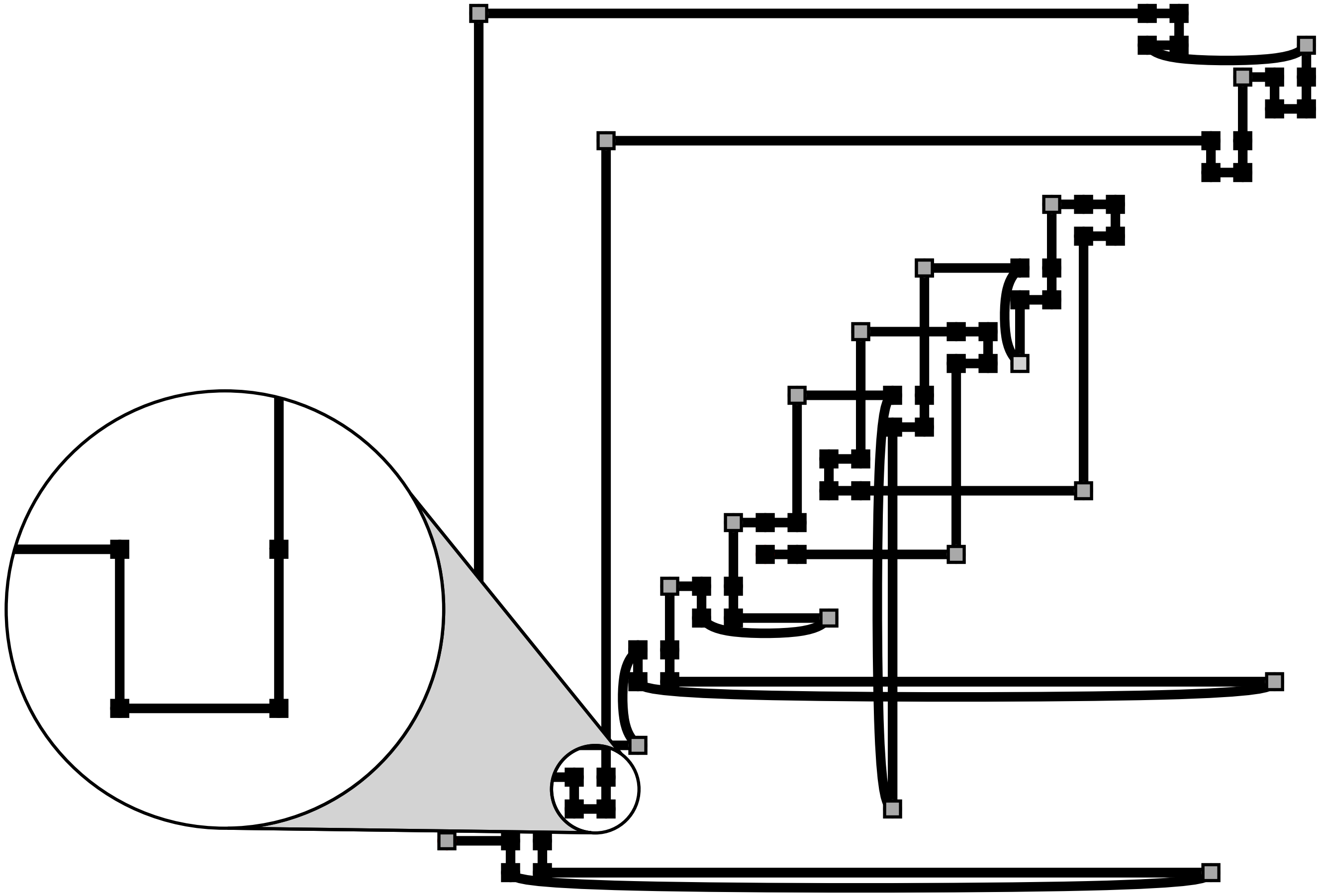

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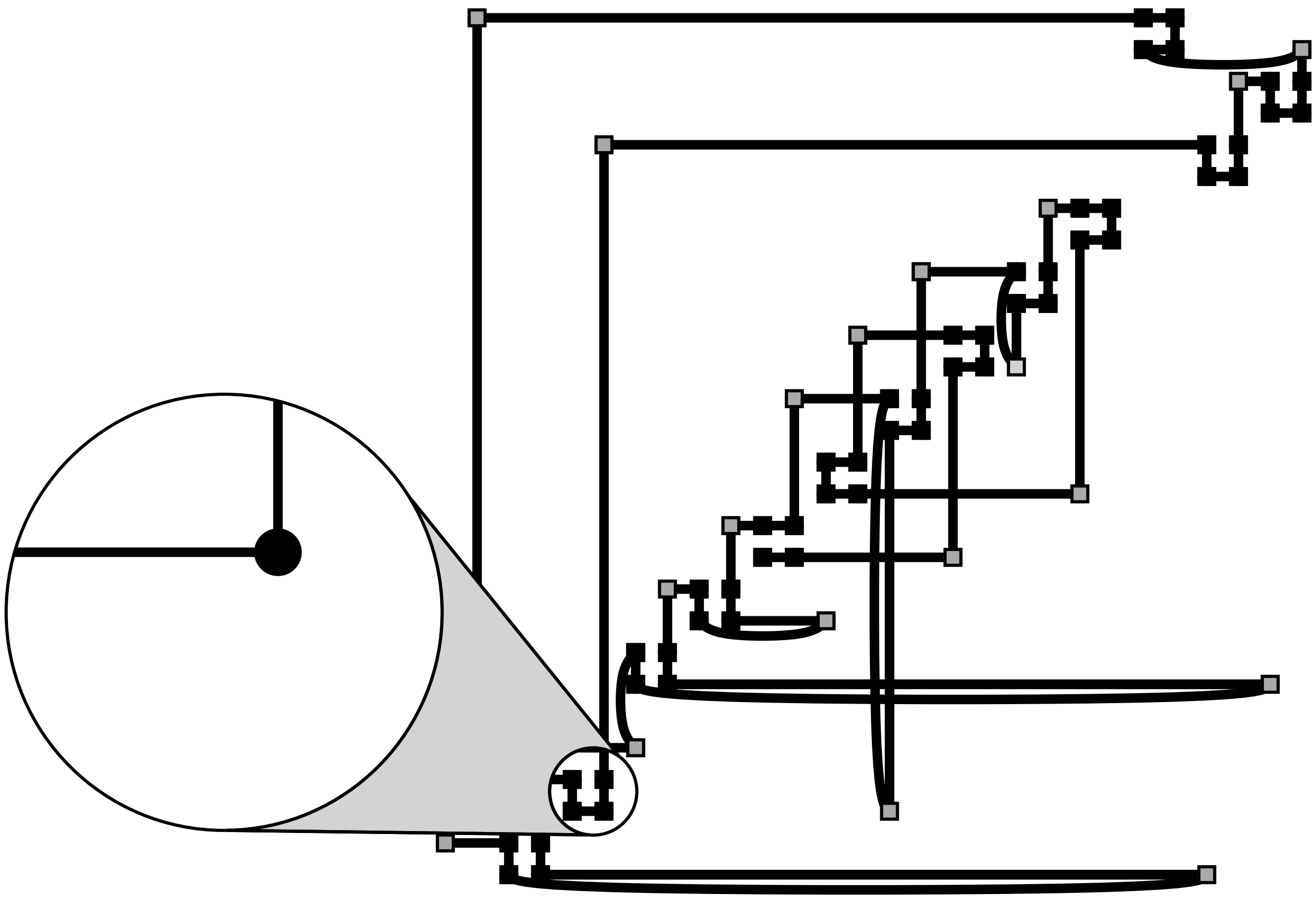

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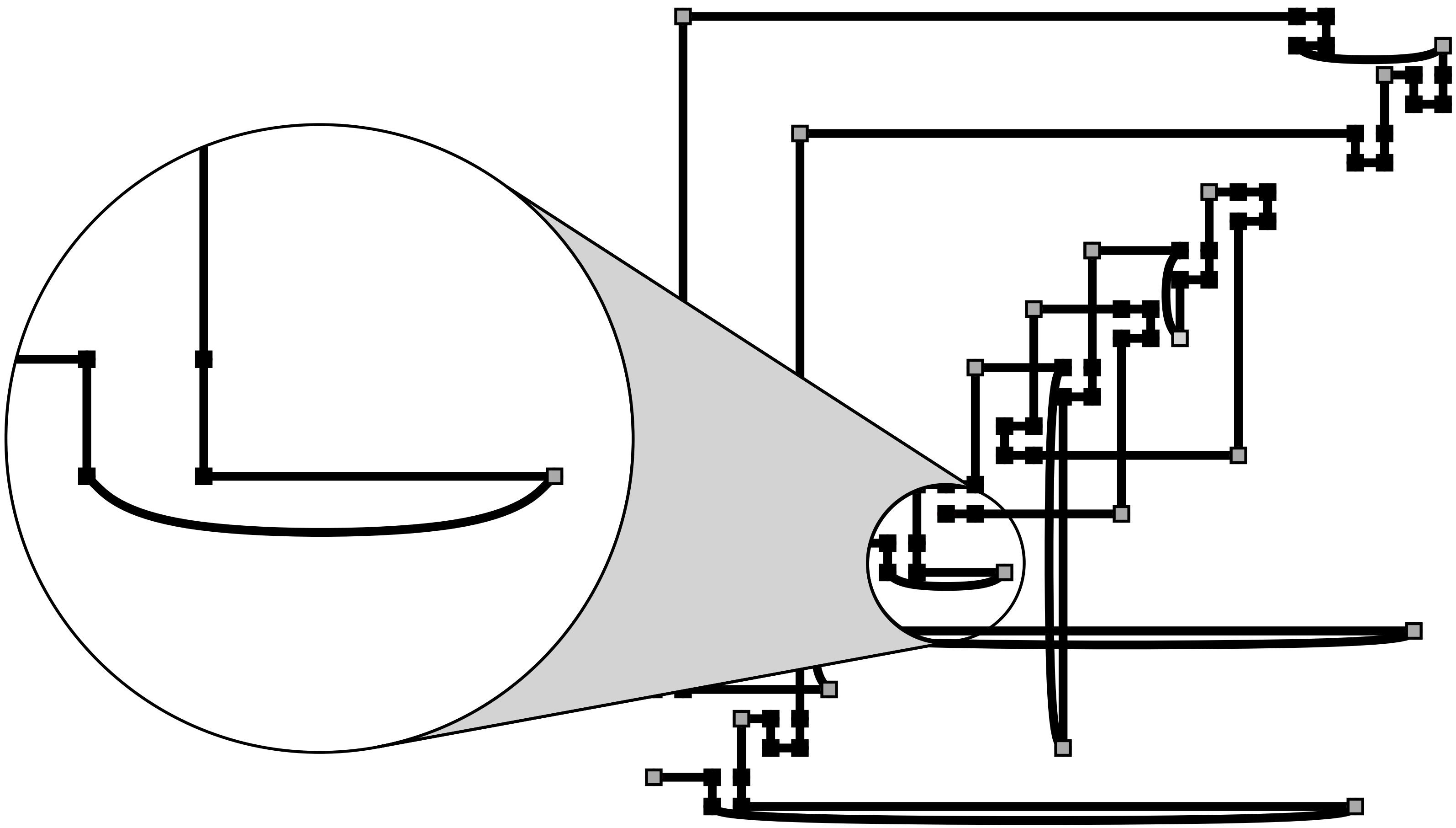

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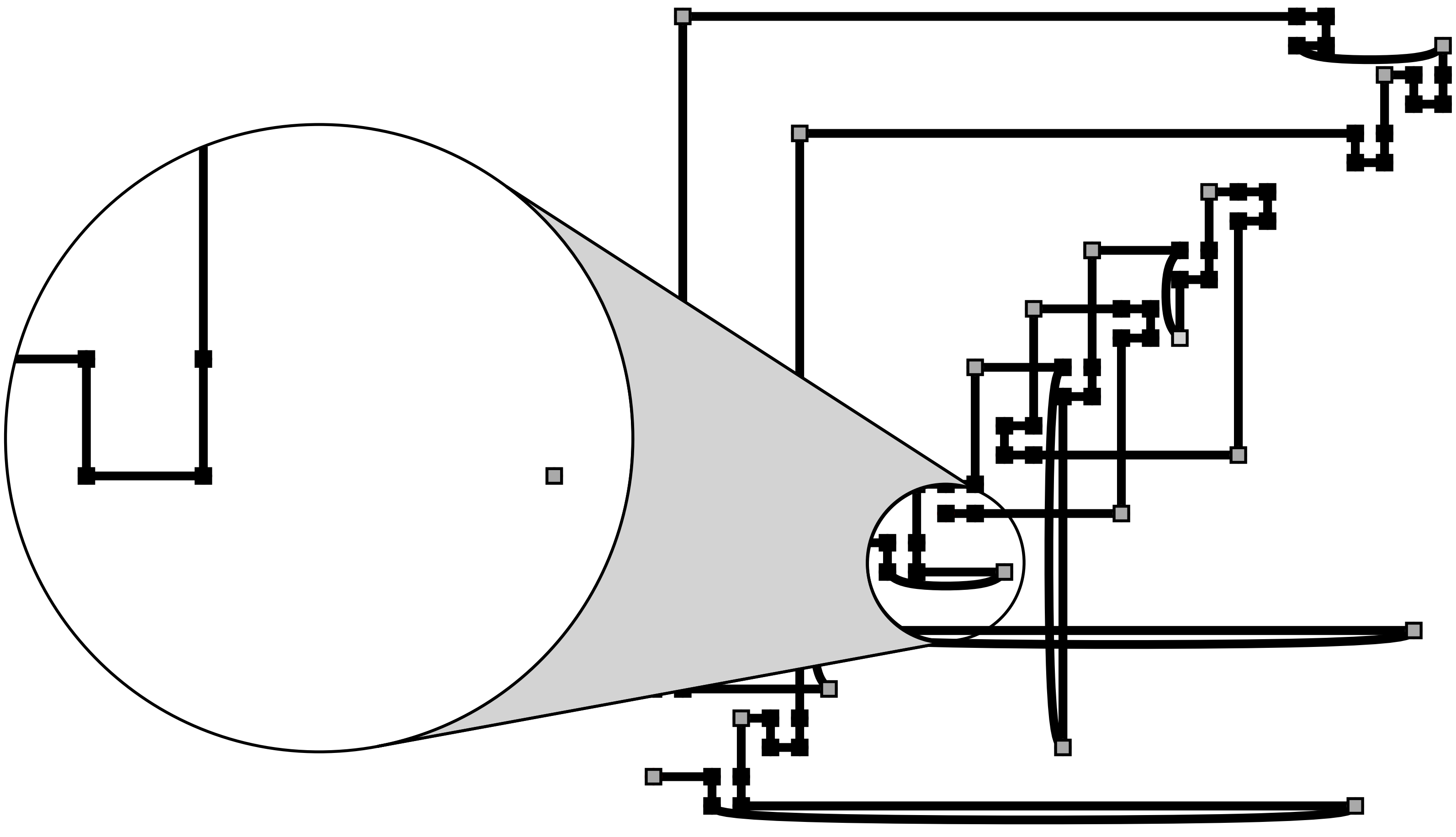

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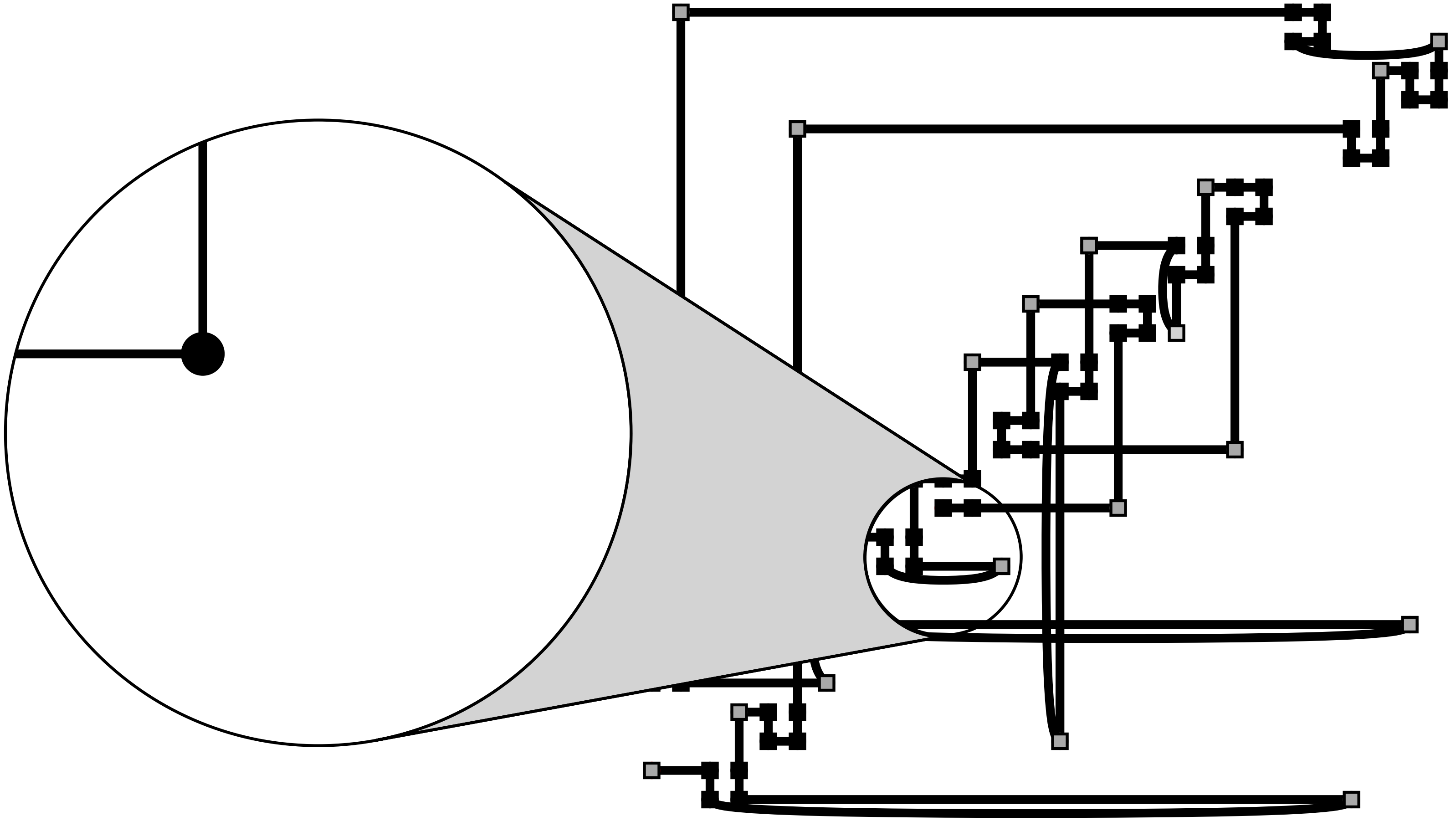

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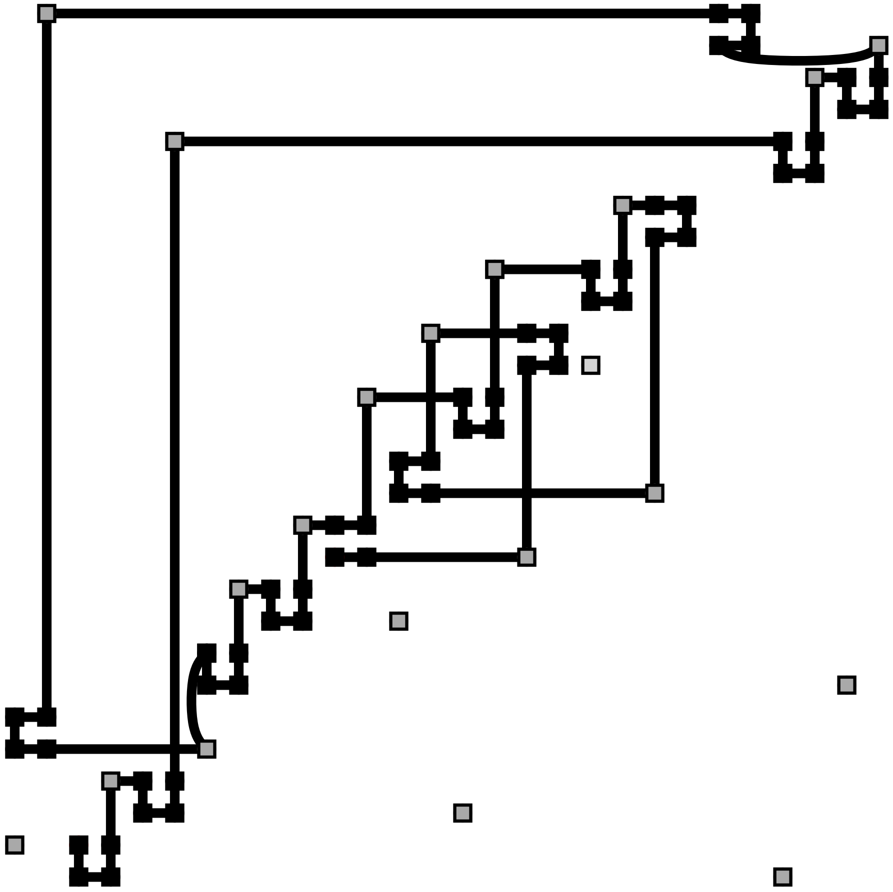

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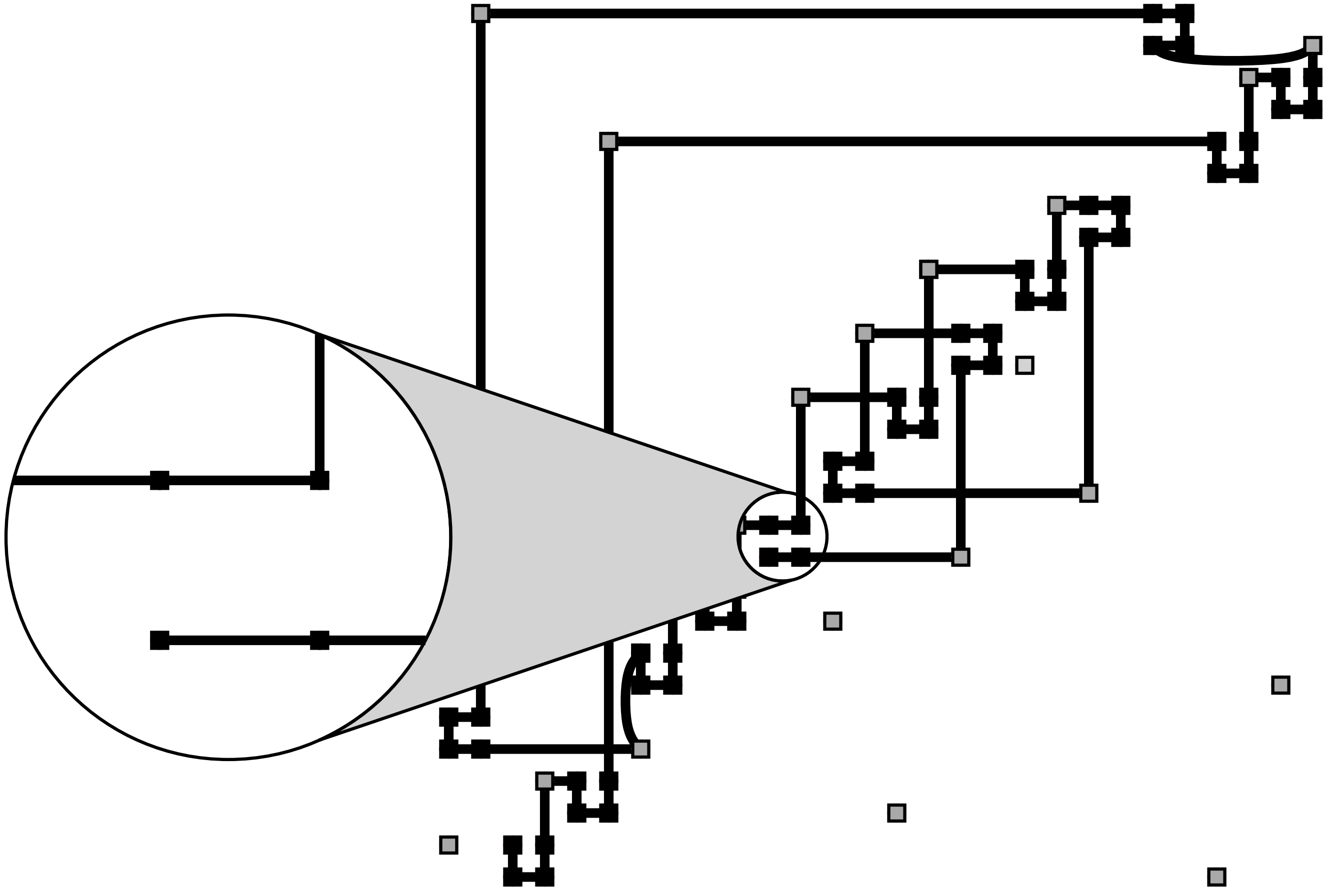

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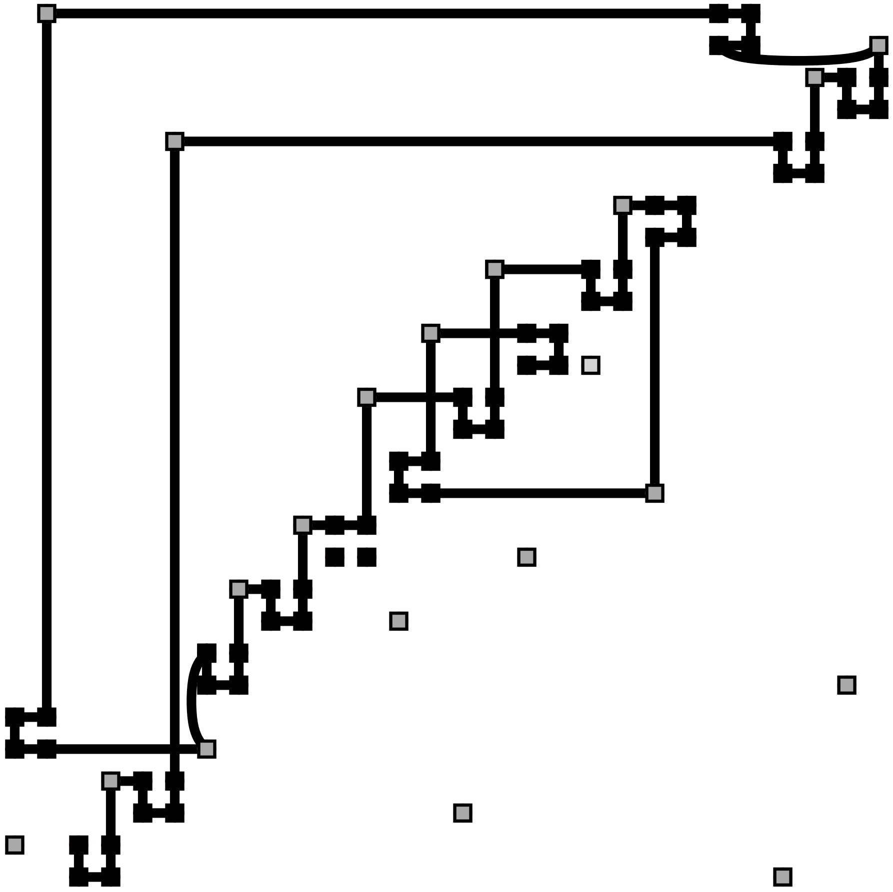

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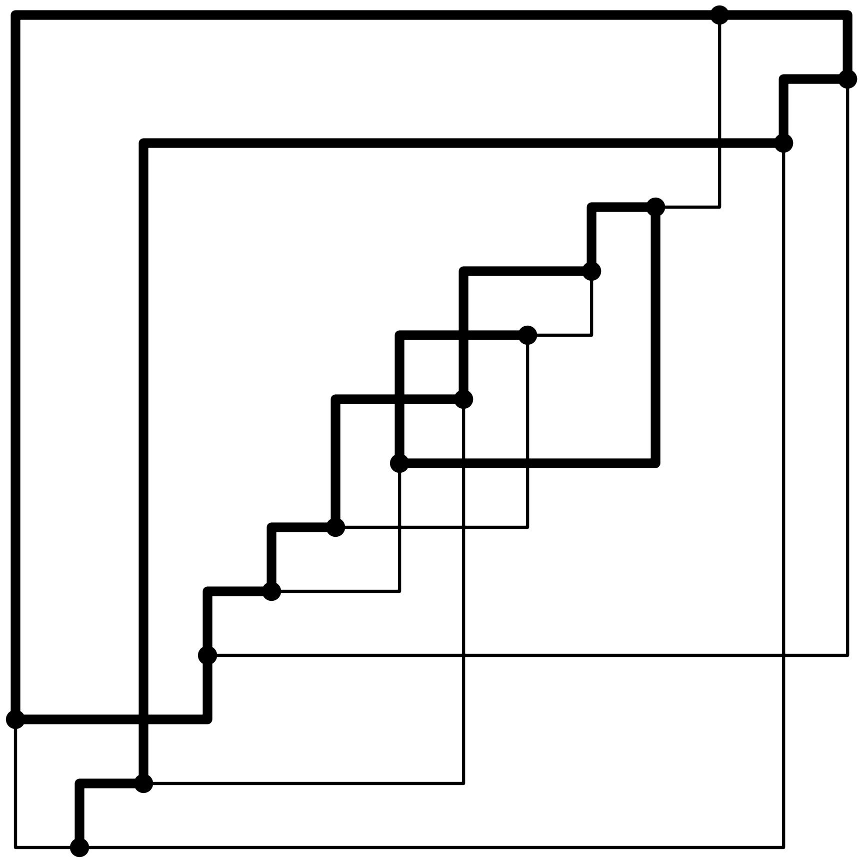
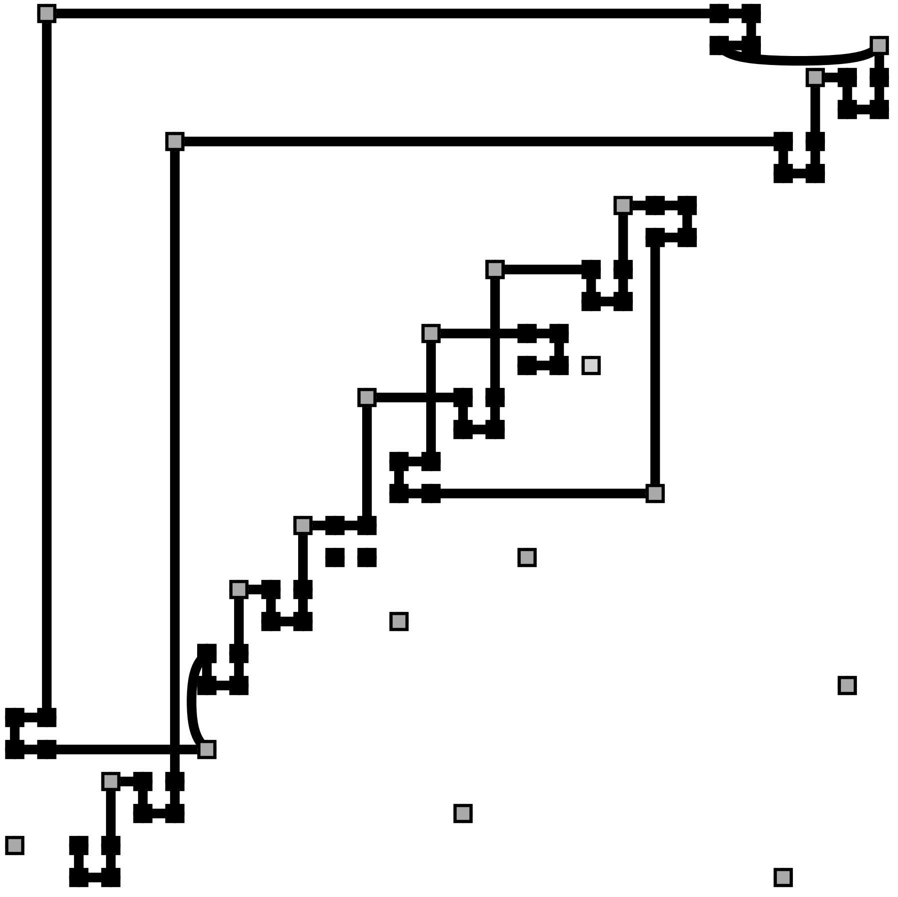
G_H has a Hamiltonian path $\Leftarrow$ F^* has a Hamiltonian path





2.5D-GBR: NP-hardness

G_H has a Hamiltonian path

$\overset{\checkmark}{\Leftrightarrow}$

F^* has a Hamiltonian path

$\Updownarrow$

G has a 2.5D-GBR with footprint F

2.5D-GBR: NP-hardness

F^* has a Hamiltonian path

$\Uparrow$

G has a 2.5D-GBR with footprint F

2.5D-GBR: NP-hardness

Every graphs that has a 2.5D-GBR with footprint F is a spanning subgraph of F^*

$$\boxed{F^* \text{ has a Hamiltonian path}}$$

$$\Uparrow$$

$$\boxed{G \text{ has a 2.5D-GBR with footprint } F}$$

2.5D-GBR: NP-hardness

Every graphs that has a 2.5D-GBR with footprint F is a spanning subgraph of F^*

G is a path

F^* has a Hamiltonian path

$\Uparrow$

G has a 2.5D-GBR with footprint F

2.5D-GBR: NP-hardness

Every graphs that has a 2.5D-GBR with footprint F is a spanning subgraph of F^*

G is a path

$\Rightarrow$ G is a Hamiltonian path of F^*

F^* has a Hamiltonian path

$\Uparrow$

G has a 2.5D-GBR with footprint F

2.5D-GBR: NP-hardness

Suppose that F^* has a Hamiltonian path H^*

$\boxed{F^* \text{ has a Hamiltonian path}}$

$$\Downarrow$$

$\boxed{G \text{ has a 2.5D-GBR with footprint } F}$

2.5D-GBR: NP-hardness

Suppose that F^* has a Hamiltonian path H^*

We have to choose the heights of the squares of F so that all the edges of H^* are realized as visibilities between the resulting boxes

$\boxed{F^* \text{ has a Hamiltonian path}}$

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2.5D-GBR: NP-hardness

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F^* has a Hamiltonian path

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G has a 2.5D-GBR with footprint F

If an edge of H^* connects consecutive squares on a row/column, any choice is fine

2.5D-GBR: NP-hardness

Suppose that F^* has a Hamiltonian path H^*

F^* has a Hamiltonian path

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We have to choose the heights of the squares of F so that all the edges of H^* are realized as visibilities between the resulting boxes

If an edge of H^* connects consecutive squares on a row/column, any choice is fine

If an edge of H^* connects non-consecutive squares on a row/column, their heights must be larger than the heights of the square in the middle

2.5D-GBR: NP-hardness

F^* has a Hamiltonian path

$$\Downarrow$$

G has a 2.5D-GBR with footprint F

2.5D-GBR: NP-hardness

We assigne the heights to one square per step following the path H^*

$\boxed{F^* \text{ has a Hamiltonian path}}$

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2.5D-GBR: NP-hardness

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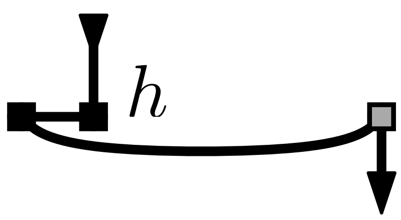

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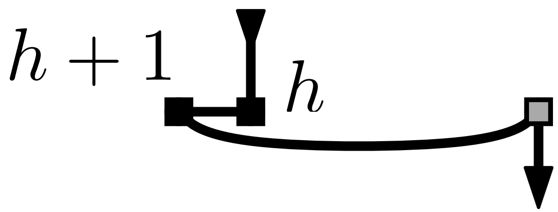

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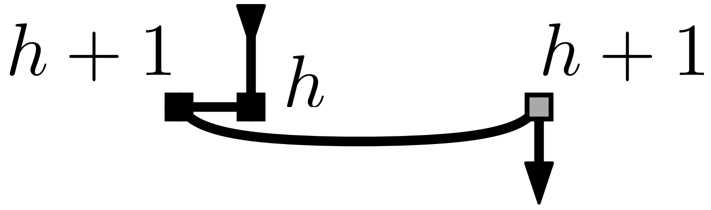

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2.5D-GBR: NP-hardness

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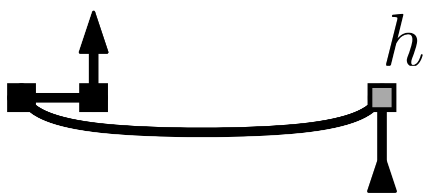

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2.5D-GBR: NP-hardness

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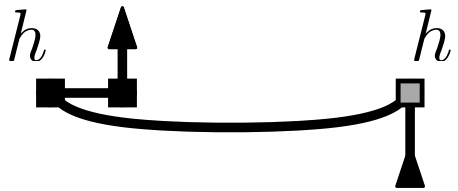

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2.5D-GBR: NP-hardness

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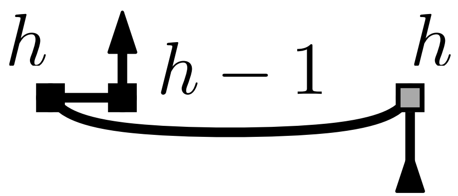


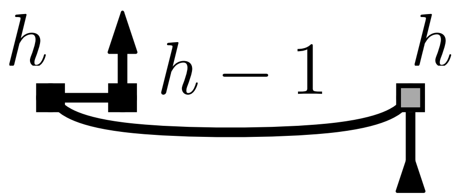
F^* has a Hamiltonian path

$$\Downarrow$$

G has a 2.5D-GBR with footprint F

2.5D-GBR: NP-hardness

We assigne the heights to one square per step following the path H^*



Assigning height n to the first square, all heights are guaranteed to be positive

$\boxed{F^* \text{ has a Hamiltonian path}}$

$$\Downarrow$$

$\boxed{G \text{ has a 2.5D-GBR with footprint } F}$

2.5D-GBR: NP-hardness

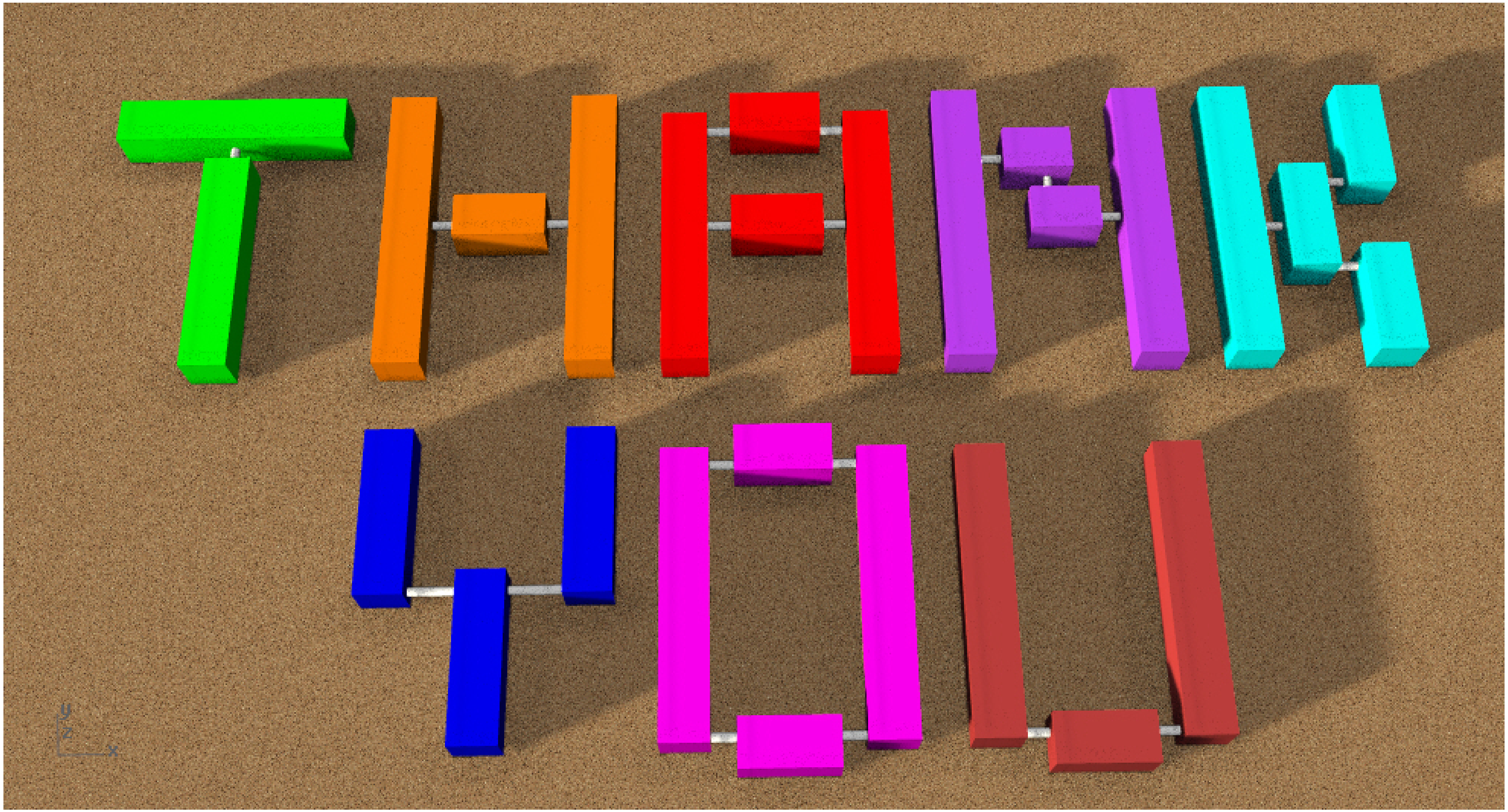
$\boxed{G_H \text{ has a Hamiltonian path}}$ $\overset{\checkmark}{\Longleftrightarrow}$ $\boxed{F^* \text{ has a Hamiltonian path}}$

$\Updownarrow \checkmark$

$\boxed{G \text{ has a 2.5D-GBR with footprint } F}$

Open Problems

- Study the complexity of deciding if a given graph admits a 2.5D-BR.
  - *Deciding if a graph admits an RVR is NP-hard*
- Investigate other classes of graphs that admit a 2.5D-BR. For example, 1-planar graphs or partial 5-trees?
  - *There are both 1-planar graphs and partial 5-trees not admitting an RVR*
- Study the 2.5D-BRs under the strong visibility model
  - *All our results assume the weak visibility model*


THANK
YOU