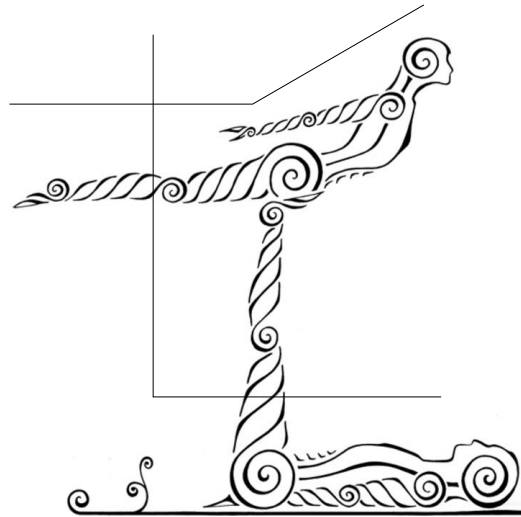
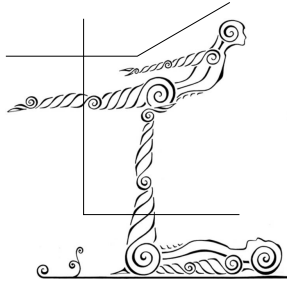


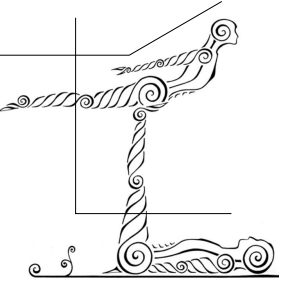
1-bend RAC Drawings of 1-Planar Graphs



Walter Didimo, Giuseppe Liotta,
Saeed Mehrabi, Fabrizio Montecchiani

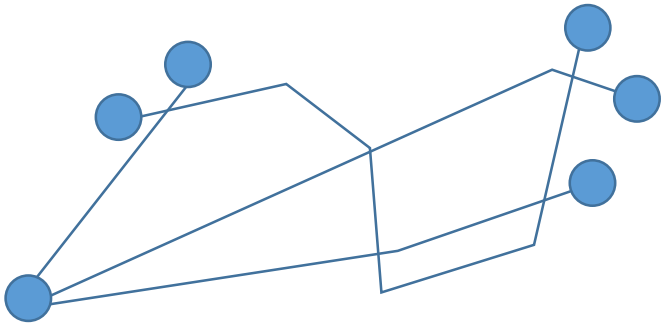


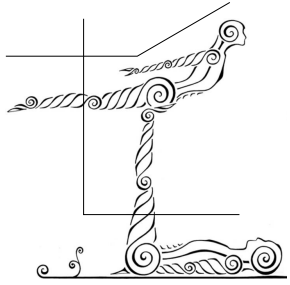
Things to avoid in graph drawing



Things to avoid in graph drawing

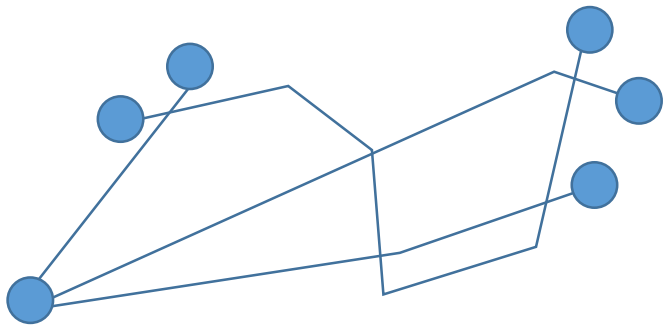
- Too many crossings



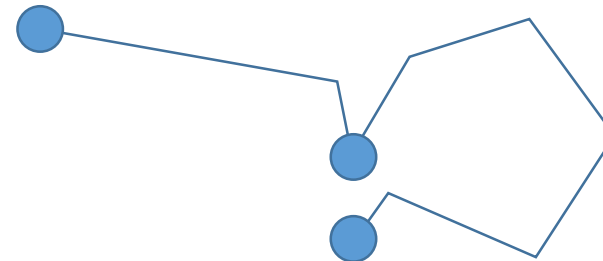


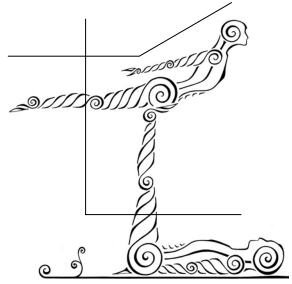
Things to avoid in graph drawing

- Too many crossings

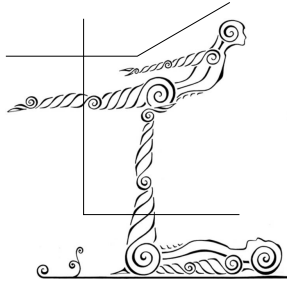


- Too many bends





A good property

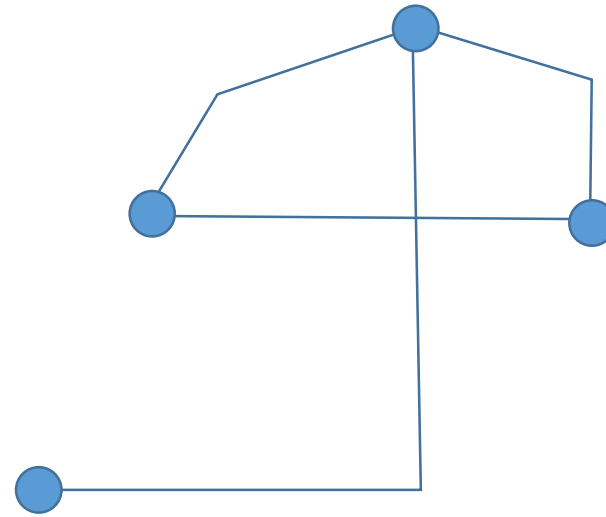


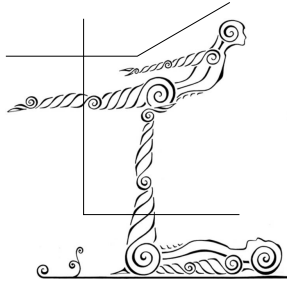
A good property

- Right angle crossings (RAC)!



[Huang, Hong, Eades – 2008]





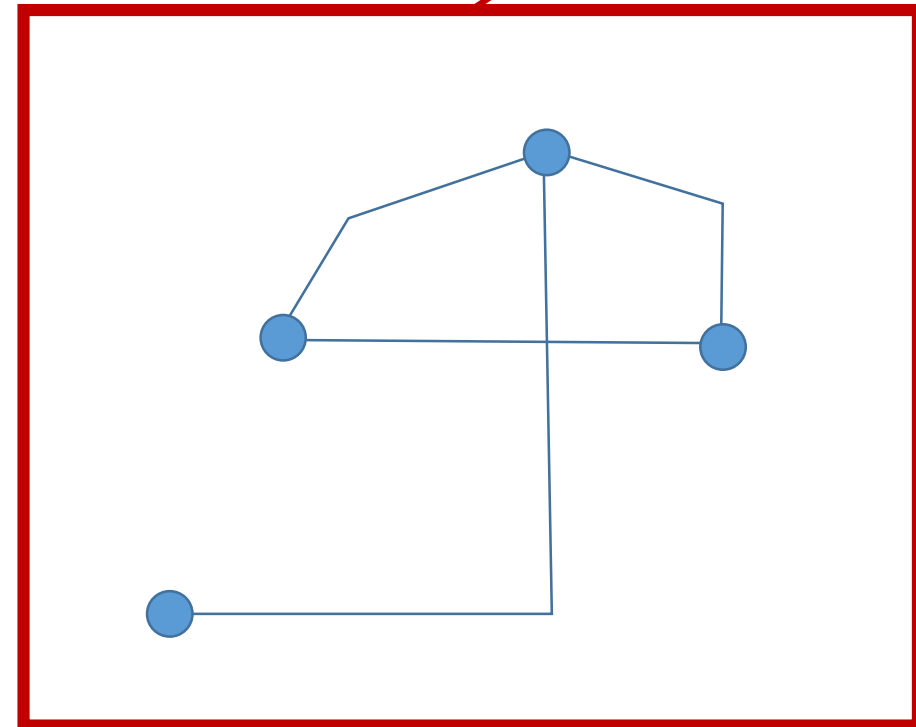
A good property

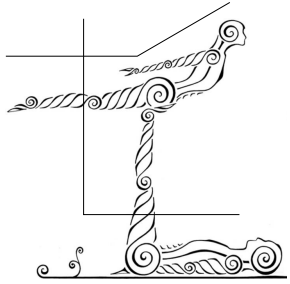
- Right angle crossings (RAC)!



[Huang, Hong, Eades – 2008]

1-bend 1-planar RAC drawing

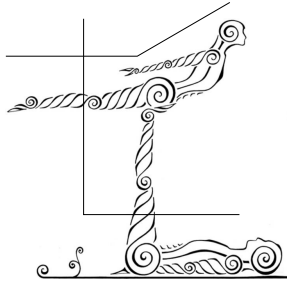




Questions

- **General:** Which kind of graphs can be drawn with:
 - a few crossings per edge,
 - a few bends per edge,
 - right angle crossings (RAC)?





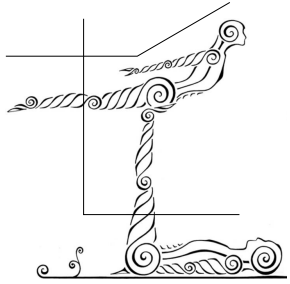
Questions

- **General:** Which kind of graphs can be drawn with:

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- a few bends per edge,
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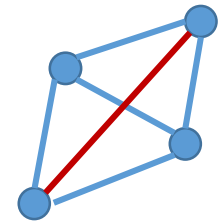


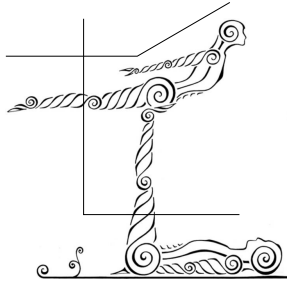
- **Specific:** Does every 1-planar graph admit a 1-bend RAC drawing?



1-planar RAC drawings

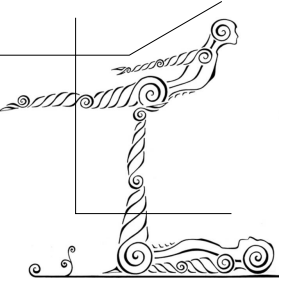
- Not all 1-planar graphs have a straight-line RAC drawing
[consequence of edge density results]
- Not all straight-line RAC drawable graphs are 1-planar
[Eades and Liotta - 2013]
- Every 1-plane *kite-triangulation* has a 1-bend RAC drawing
[Angelini et al. - 2009]
- Every 1-plane graph with *independent crossings* (IC-planar) has a straight-line RAC drawing [Brandenburg et al. - 2013]





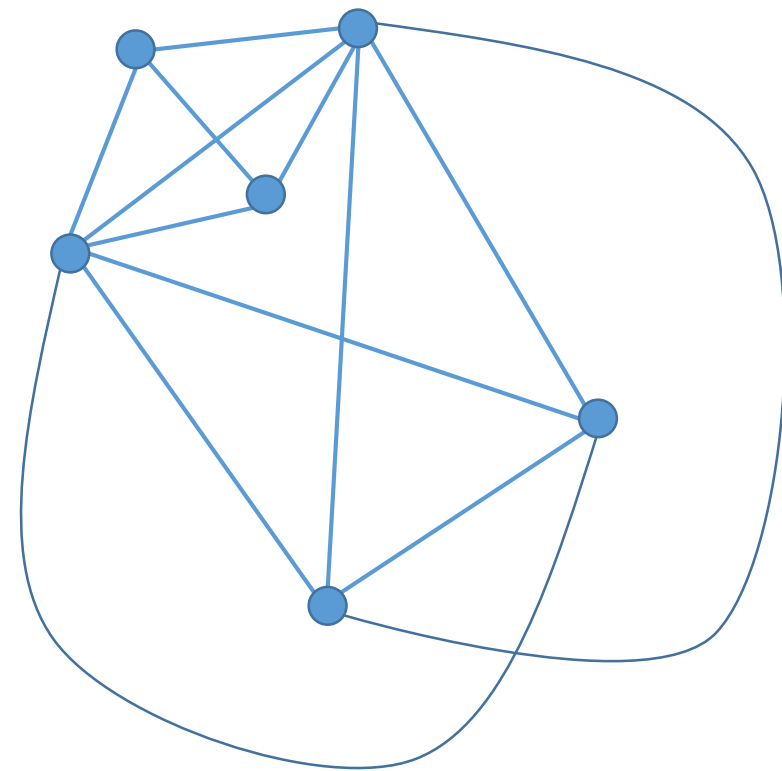
Our result

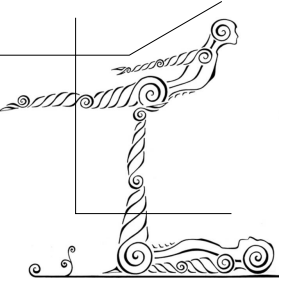
- **Theorem.** Every simple 1-planar graph admits a 1-bend RAC drawing, which can be computed in linear time if an initial 1-planar embedding is given



Some definitions

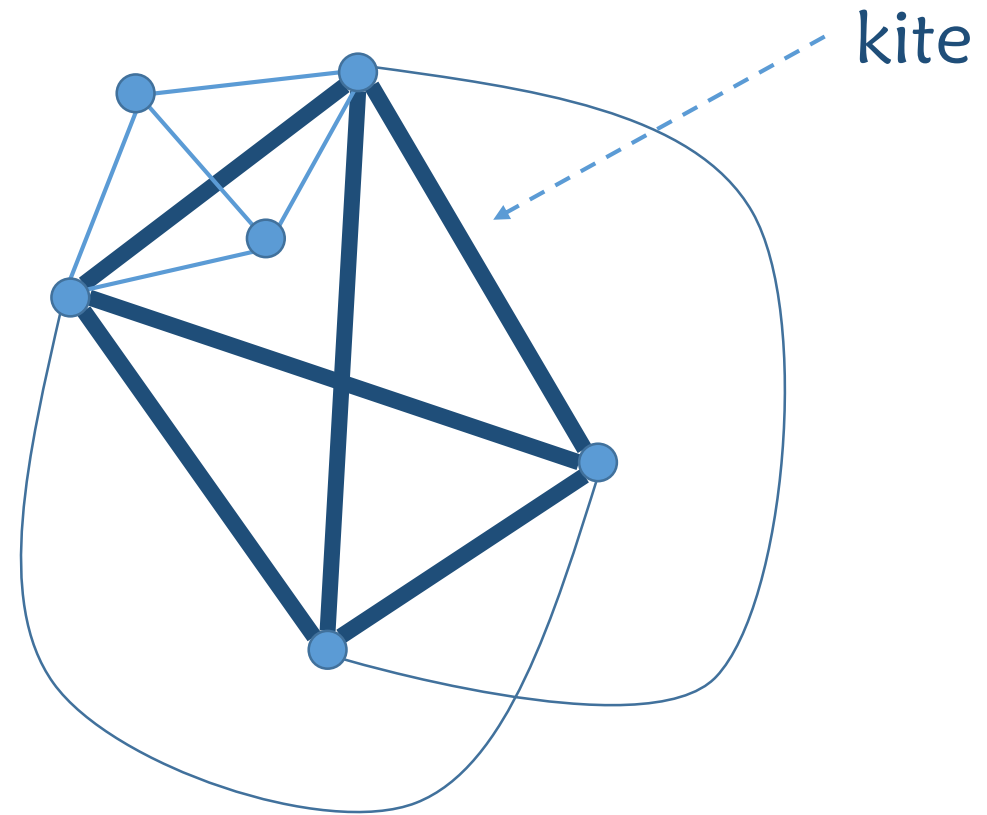
1-plane graph
(not necessarily simple)

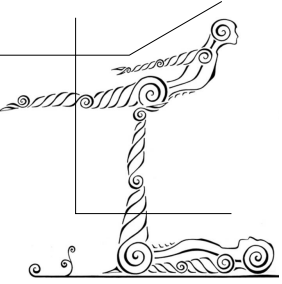




Some definitions

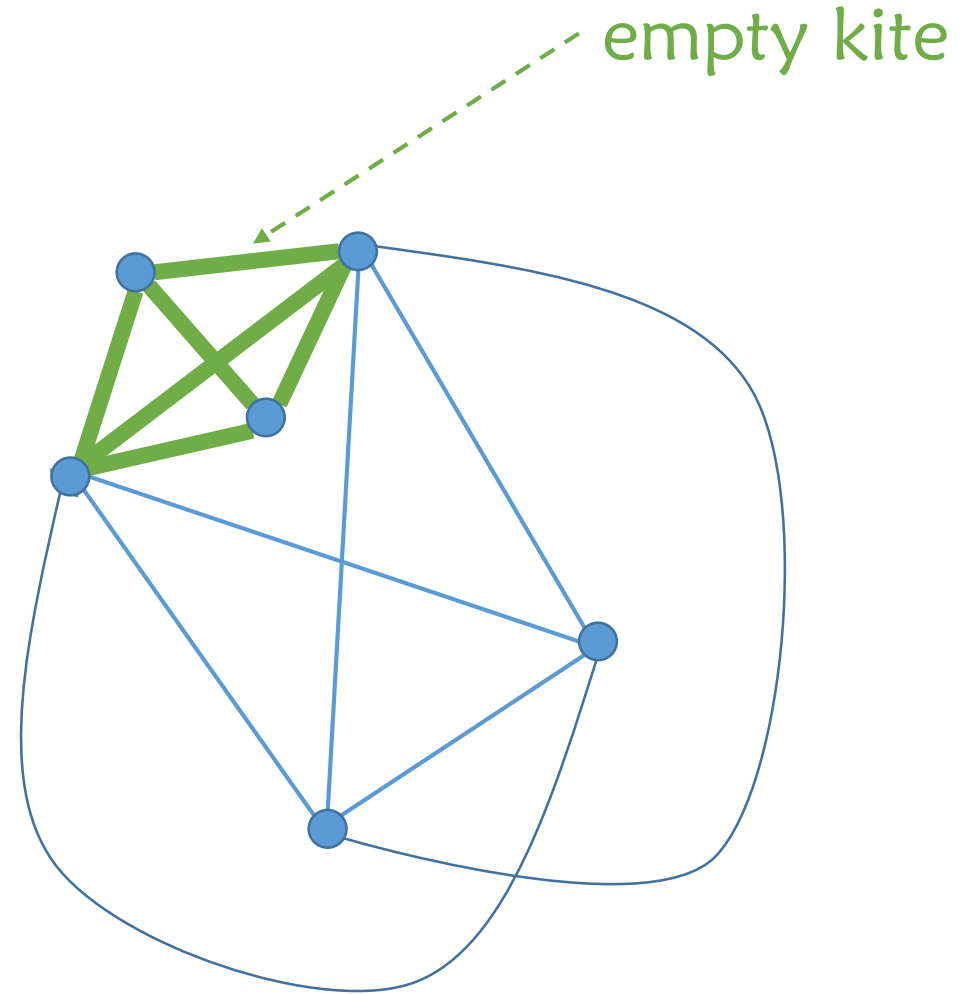
1-plane graph
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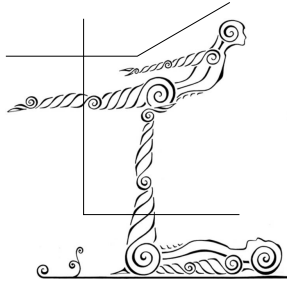




Some definitions

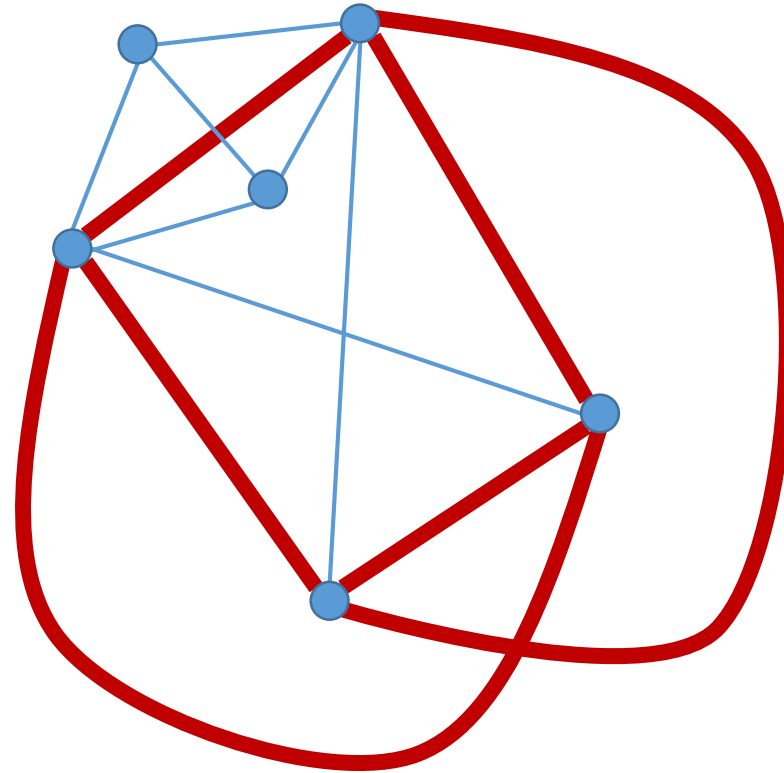
1-plane graph
(not necessarily simple)



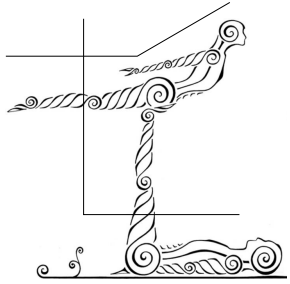


Some definitions

1-plane graph
(not necessarily simple)

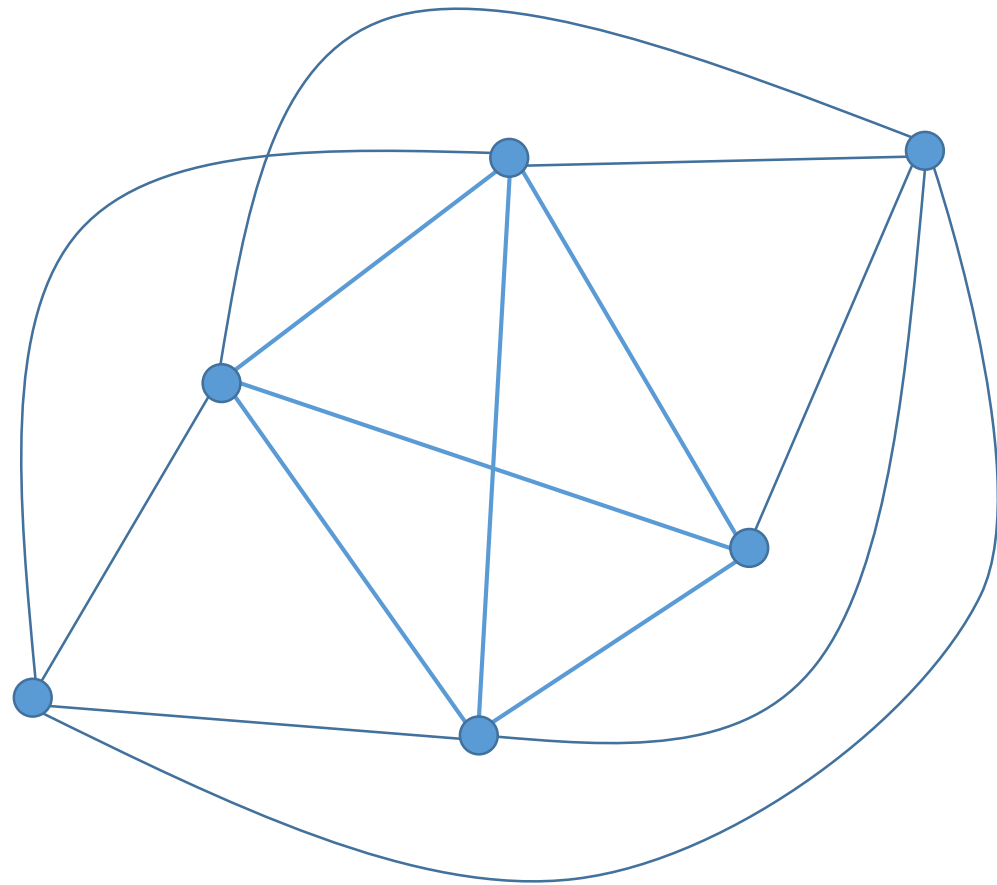


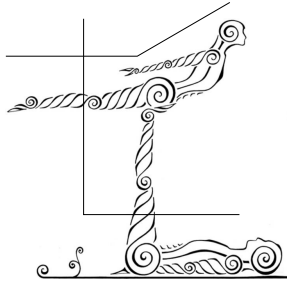
not a kite!



Observation

triangulated 1-plane graph
(not necessarily simple)



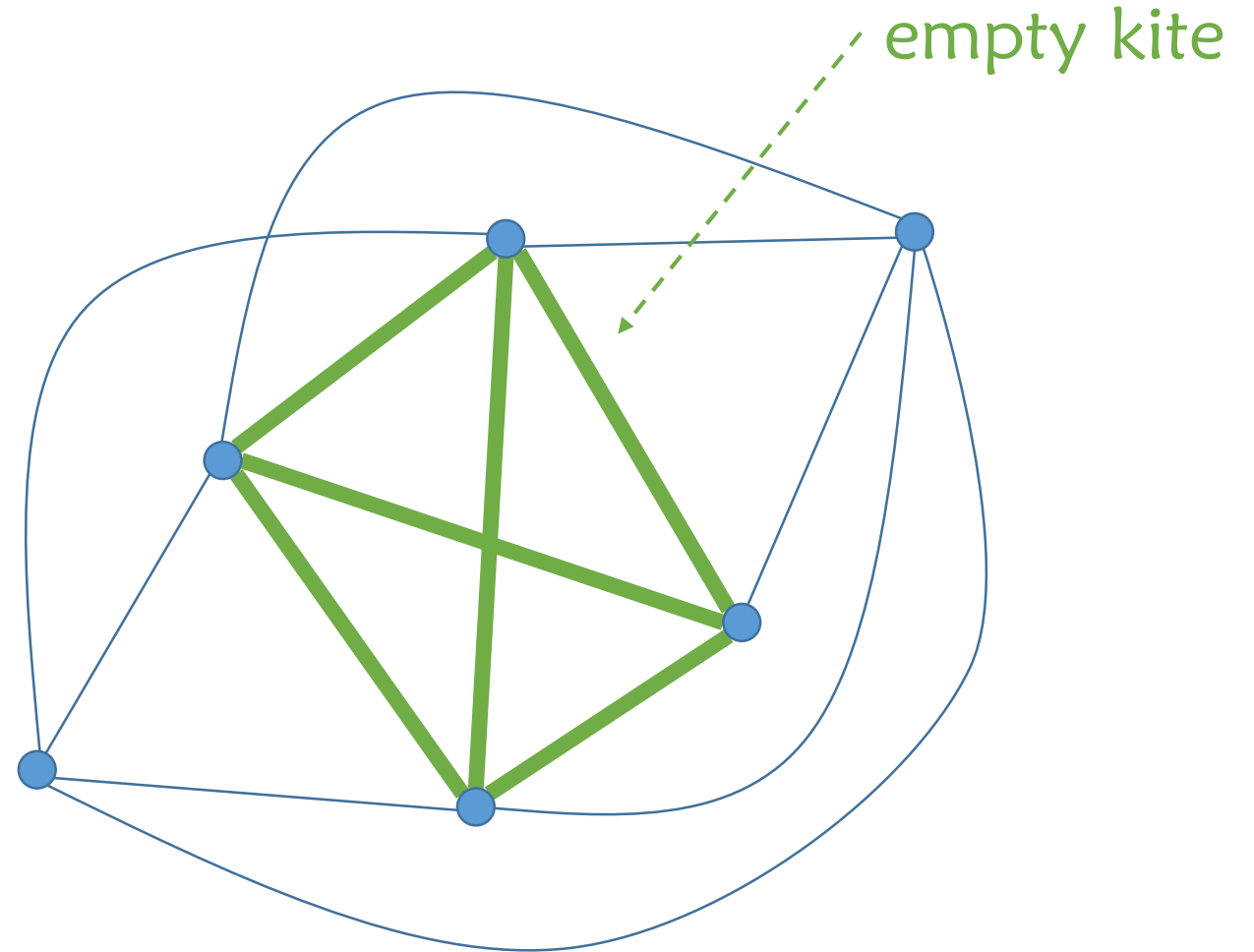


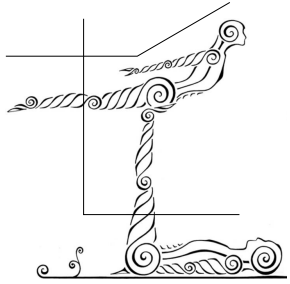
Observation

triangulated 1-plane graph
(not necessarily simple)



Every pair of crossing edges
forms an empty kite except
possibly for a pair of crossing
edges on the outer face



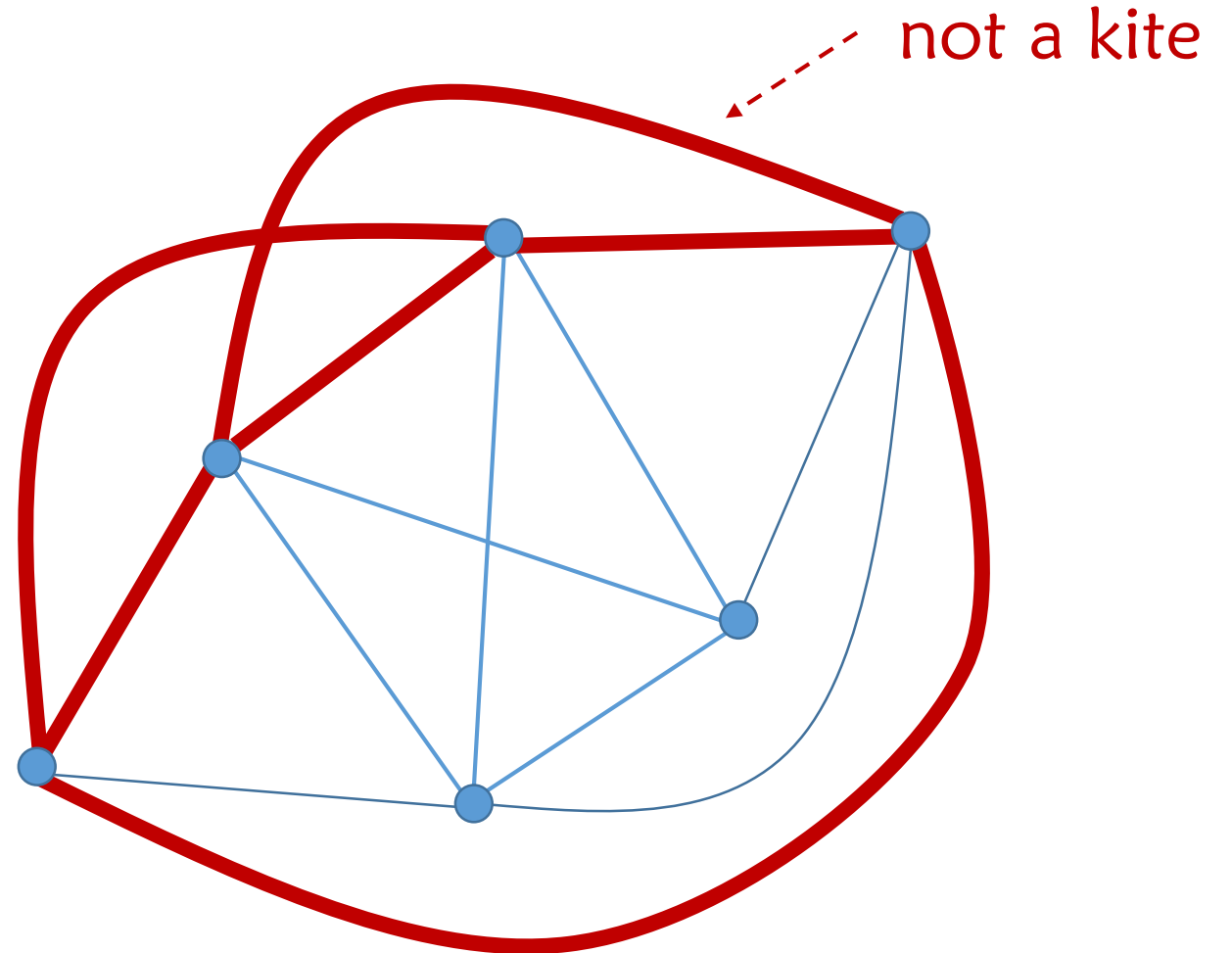


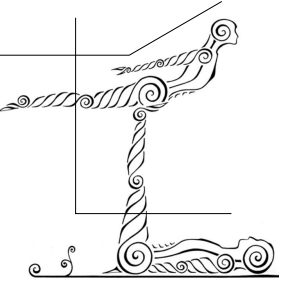
Observation

triangulated 1-plane graph
(not necessarily simple)

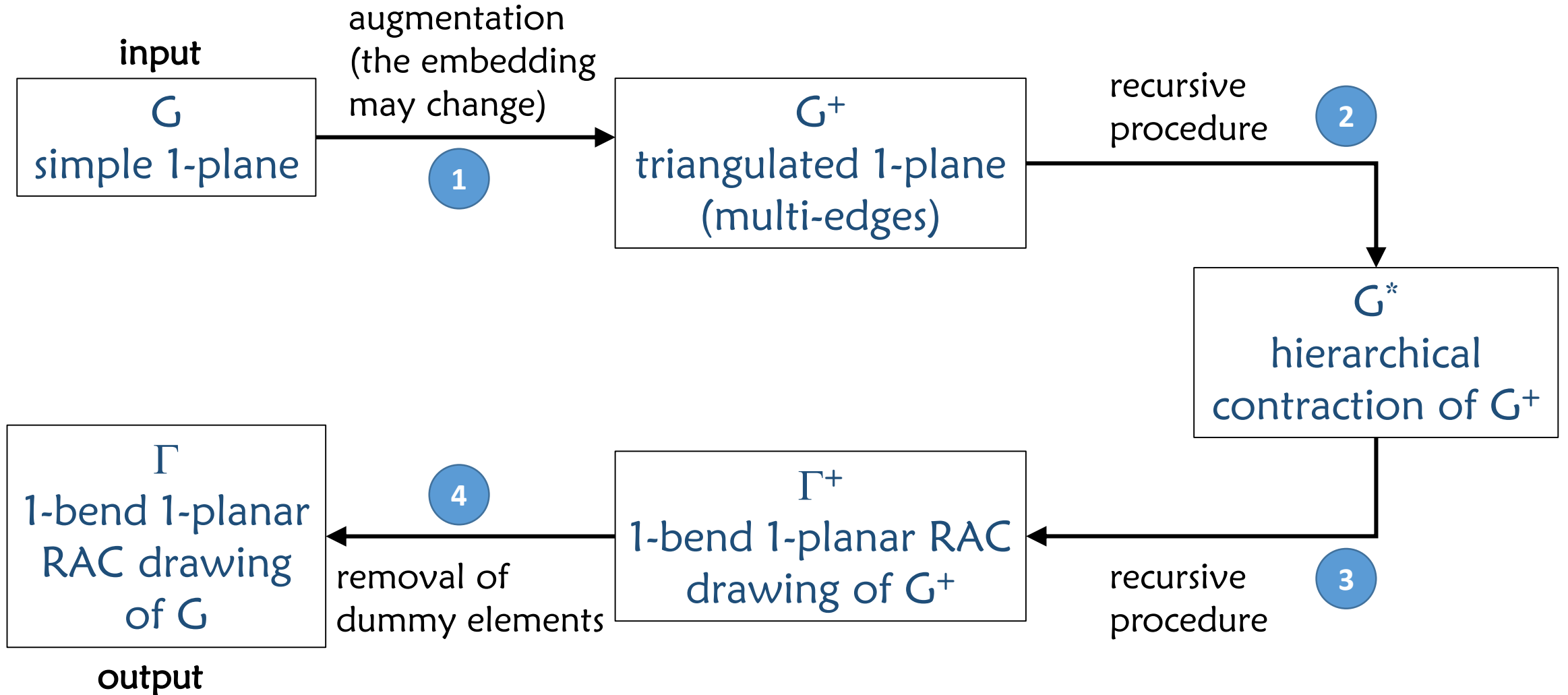


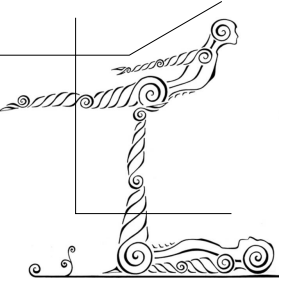
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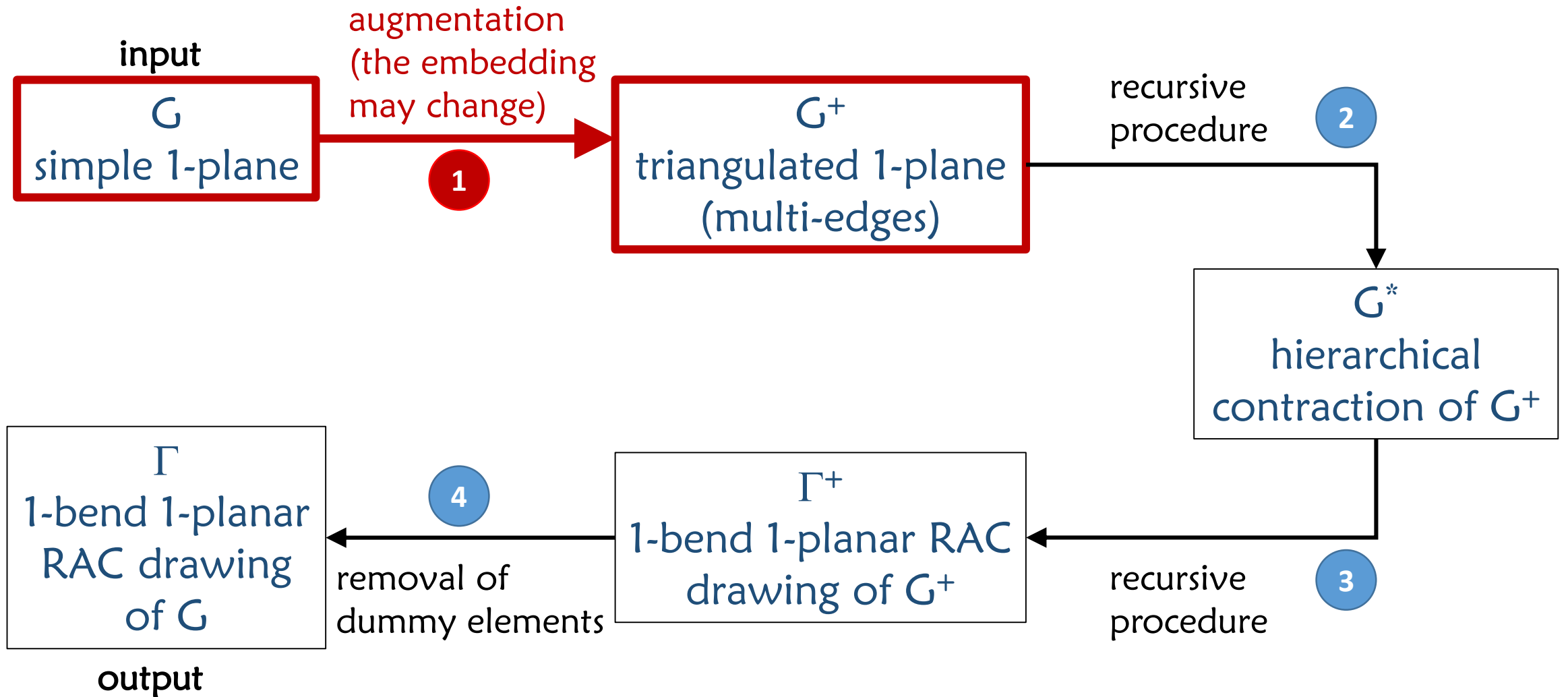


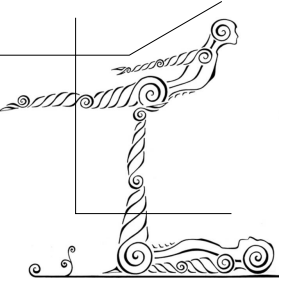
Algorithm Outline





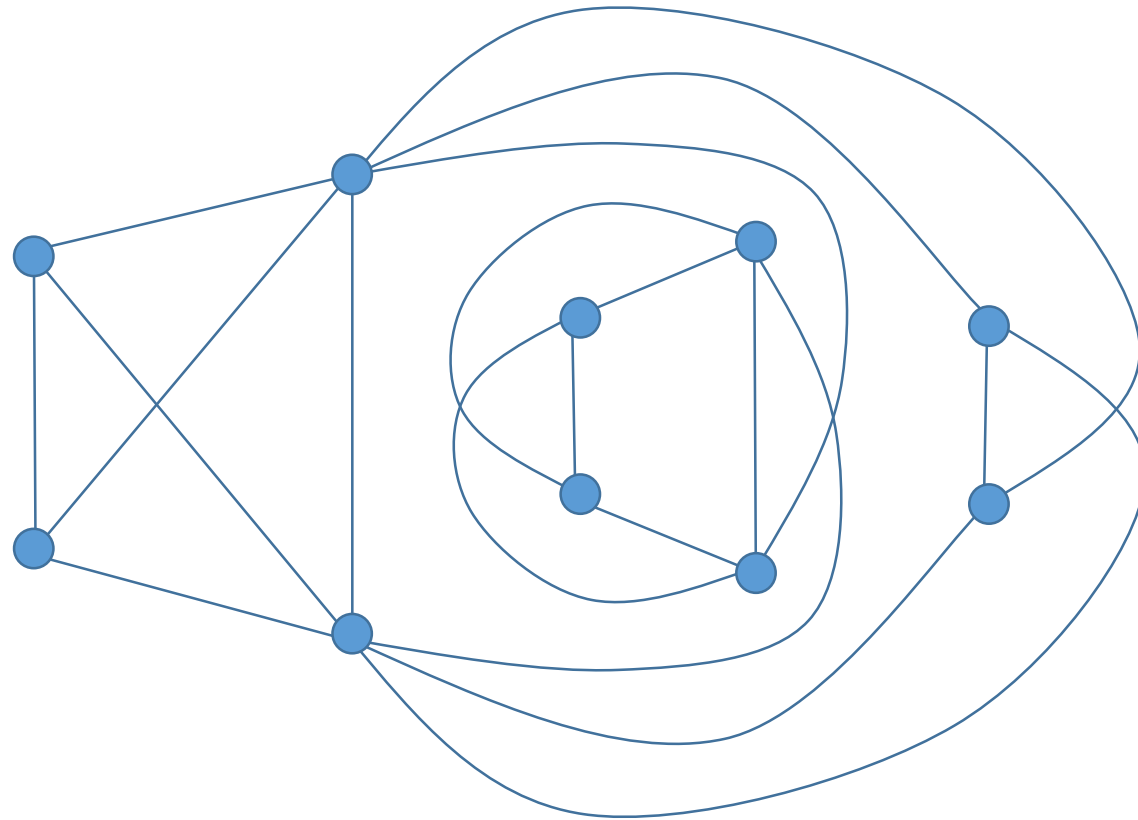
Algorithm Outline

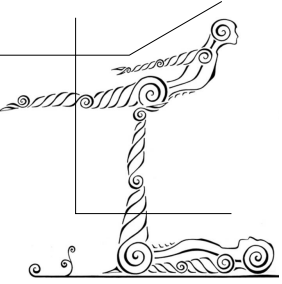




Augmentation

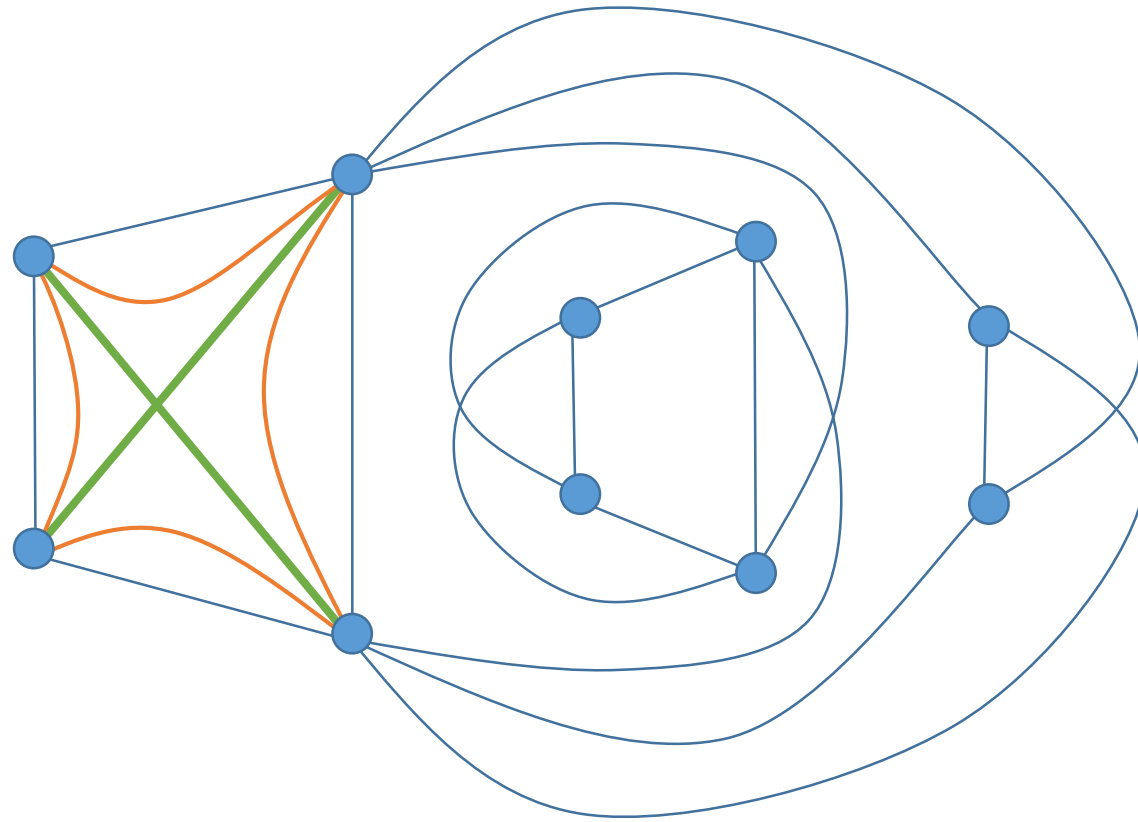
G
simple 1-plane



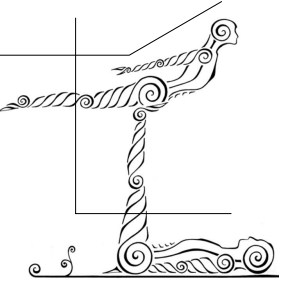


Augmentation

G
simple 1-plane

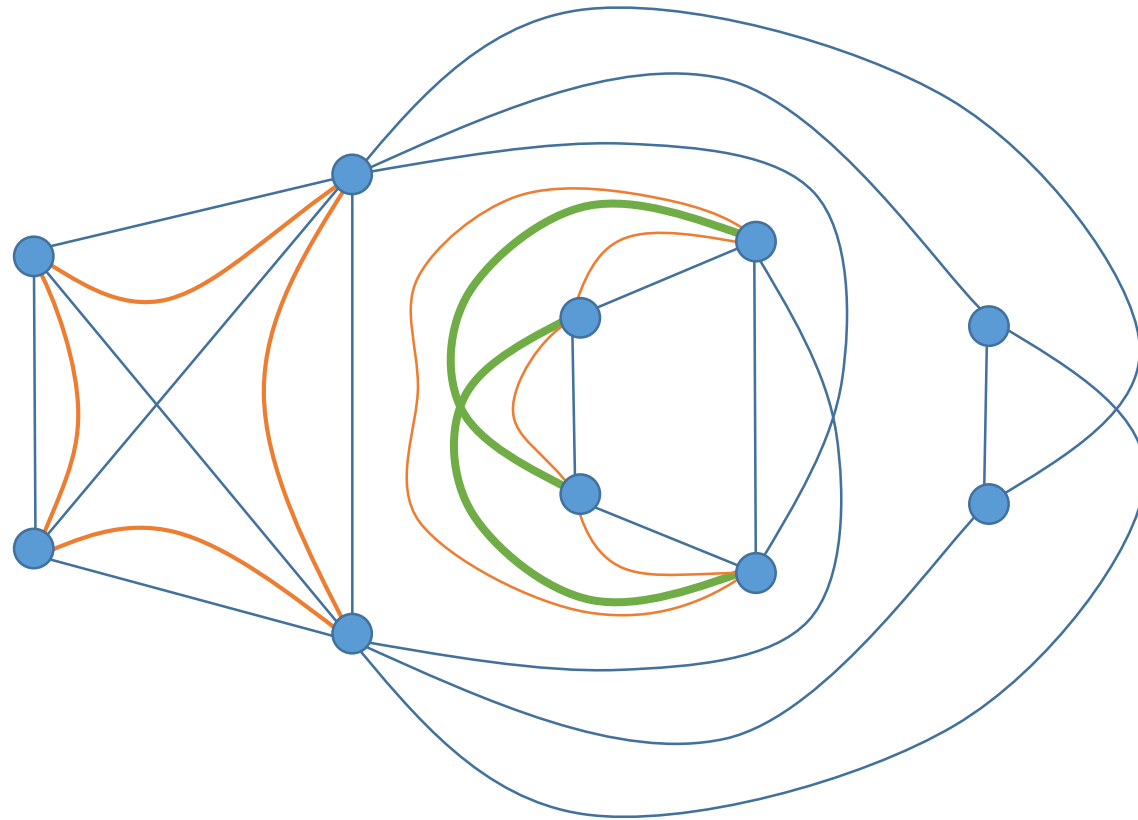


for each
pair of crossing edges
add an enclosing
4-cycle

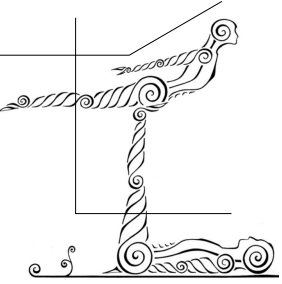


Augmentation

G
simple 1-plane

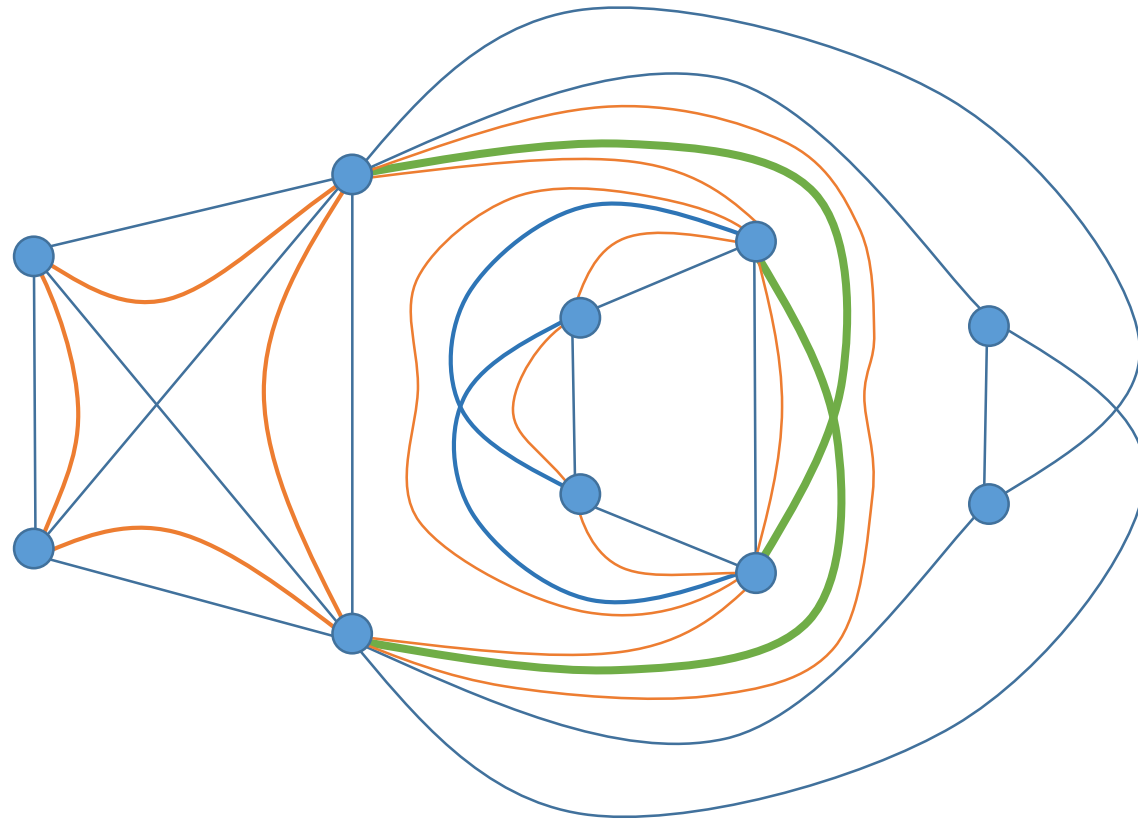


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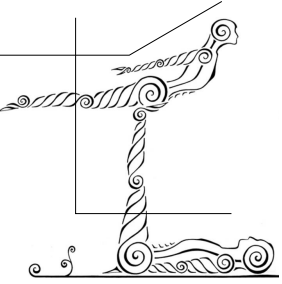


Augmentation

G
simple 1-plane

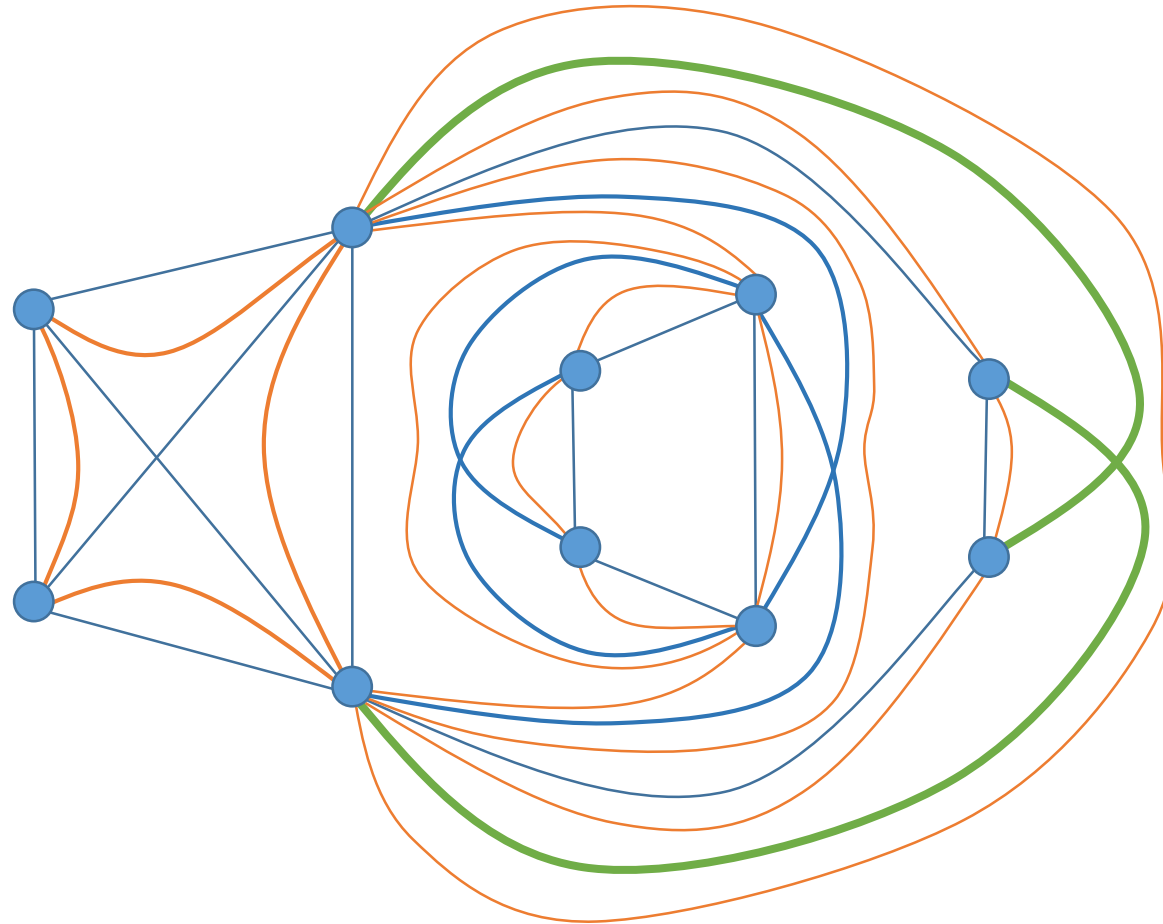


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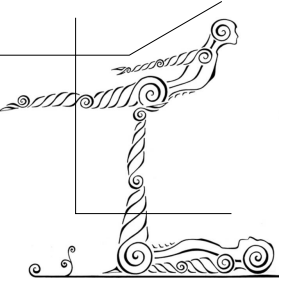


Augmentation

G
simple 1-plane

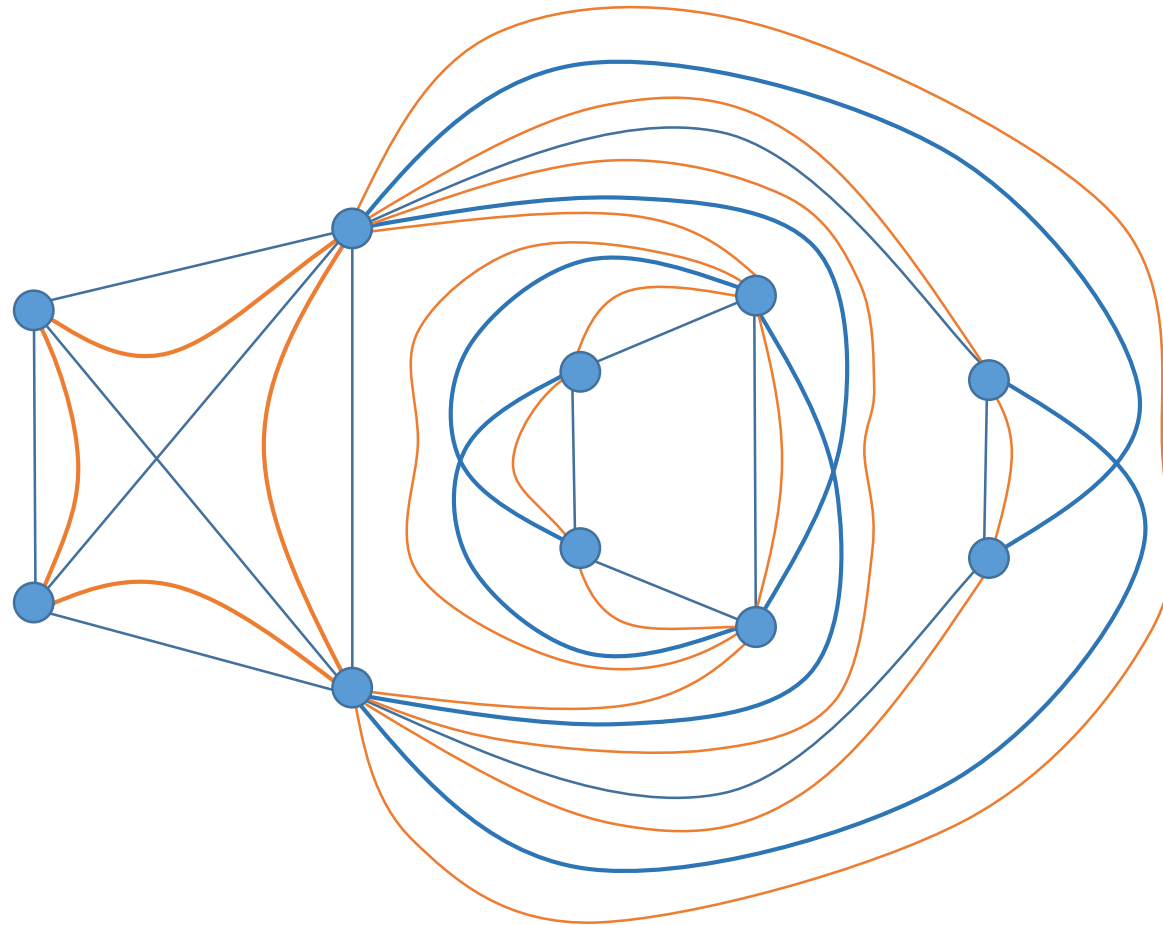


for each
pair of crossing edges
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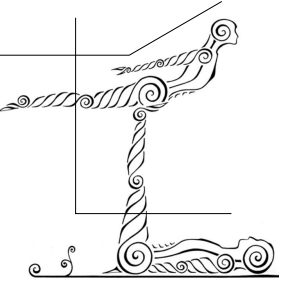


Augmentation

G
simple 1-plane

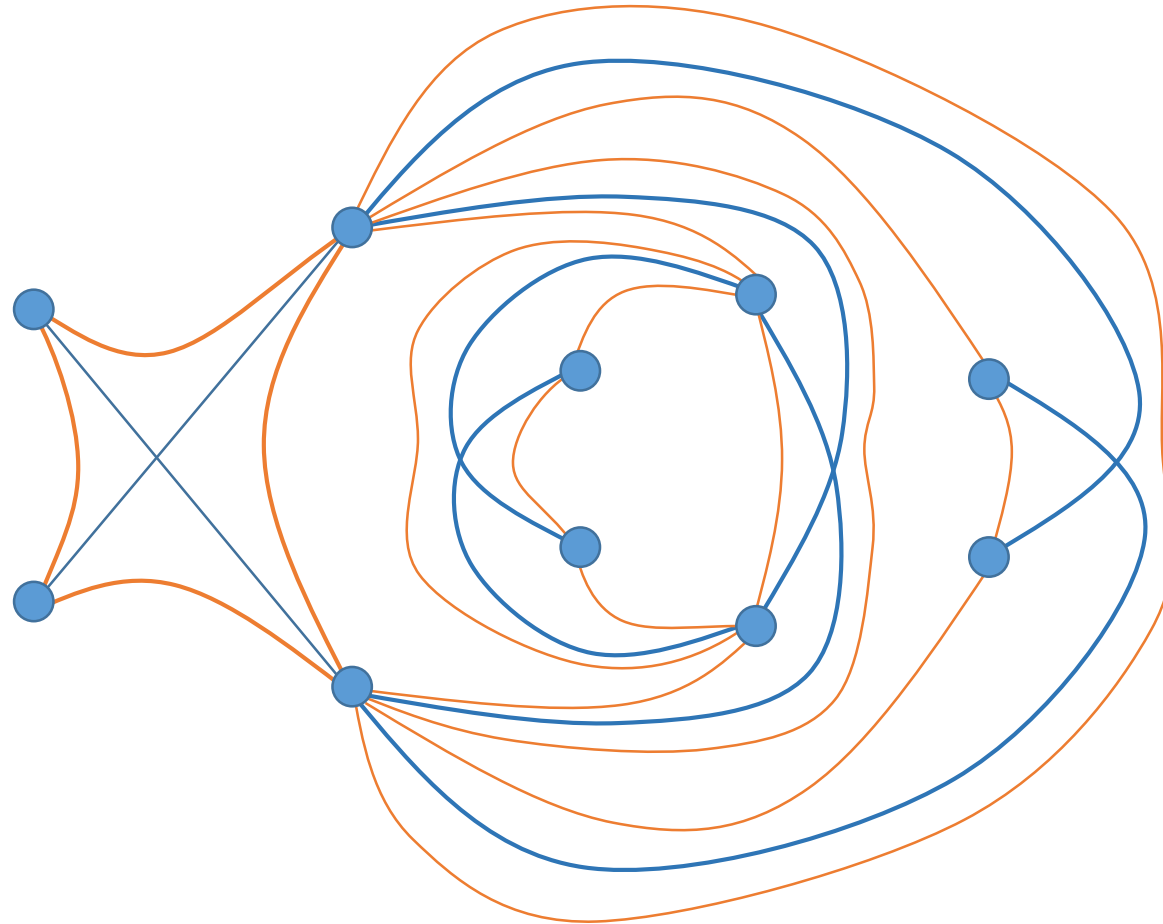


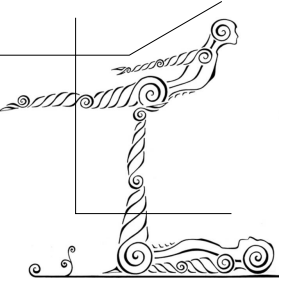
remove those
multiple edges that
belong to the
input graph



Augmentation

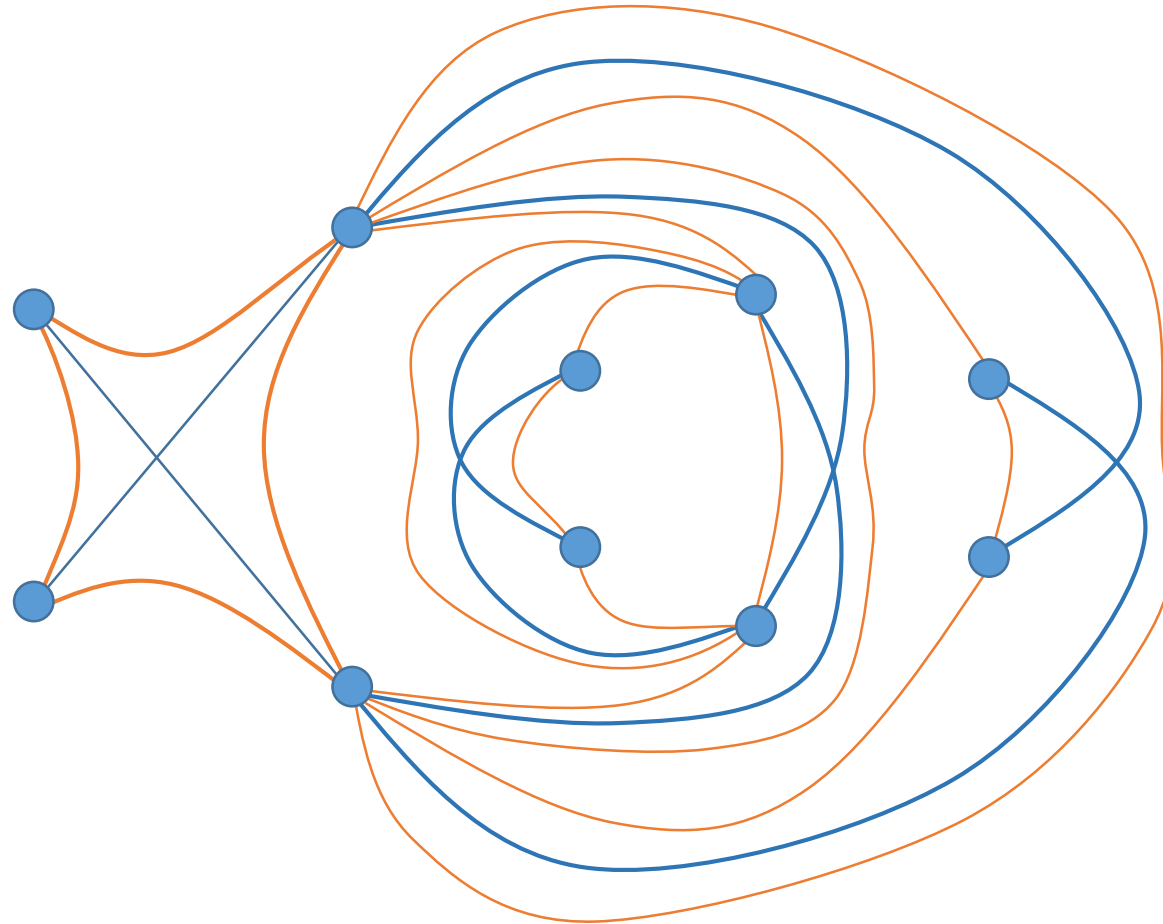
G
simple 1-plane



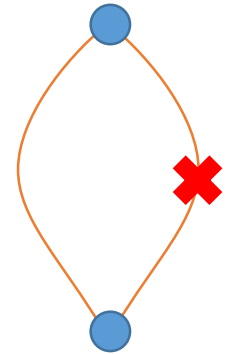


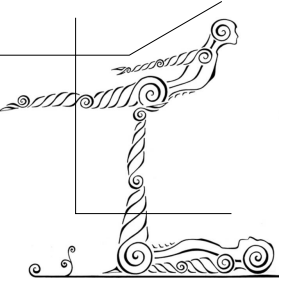
Augmentation

G
simple 1-plane



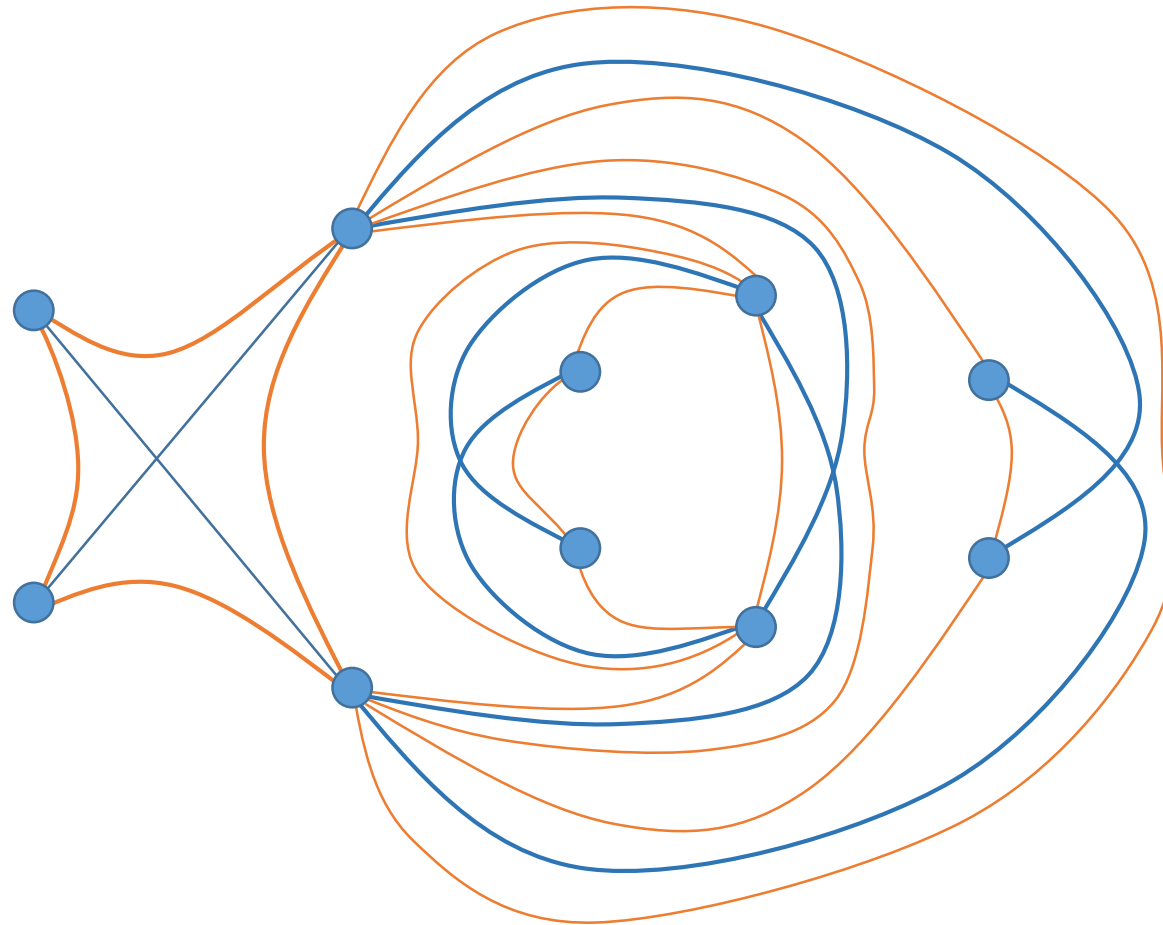
remove one
(multiple) edge
from each face of
degree two, if any



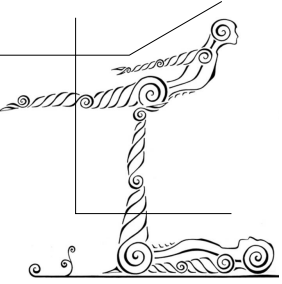


Augmentation

G
simple 1-plane

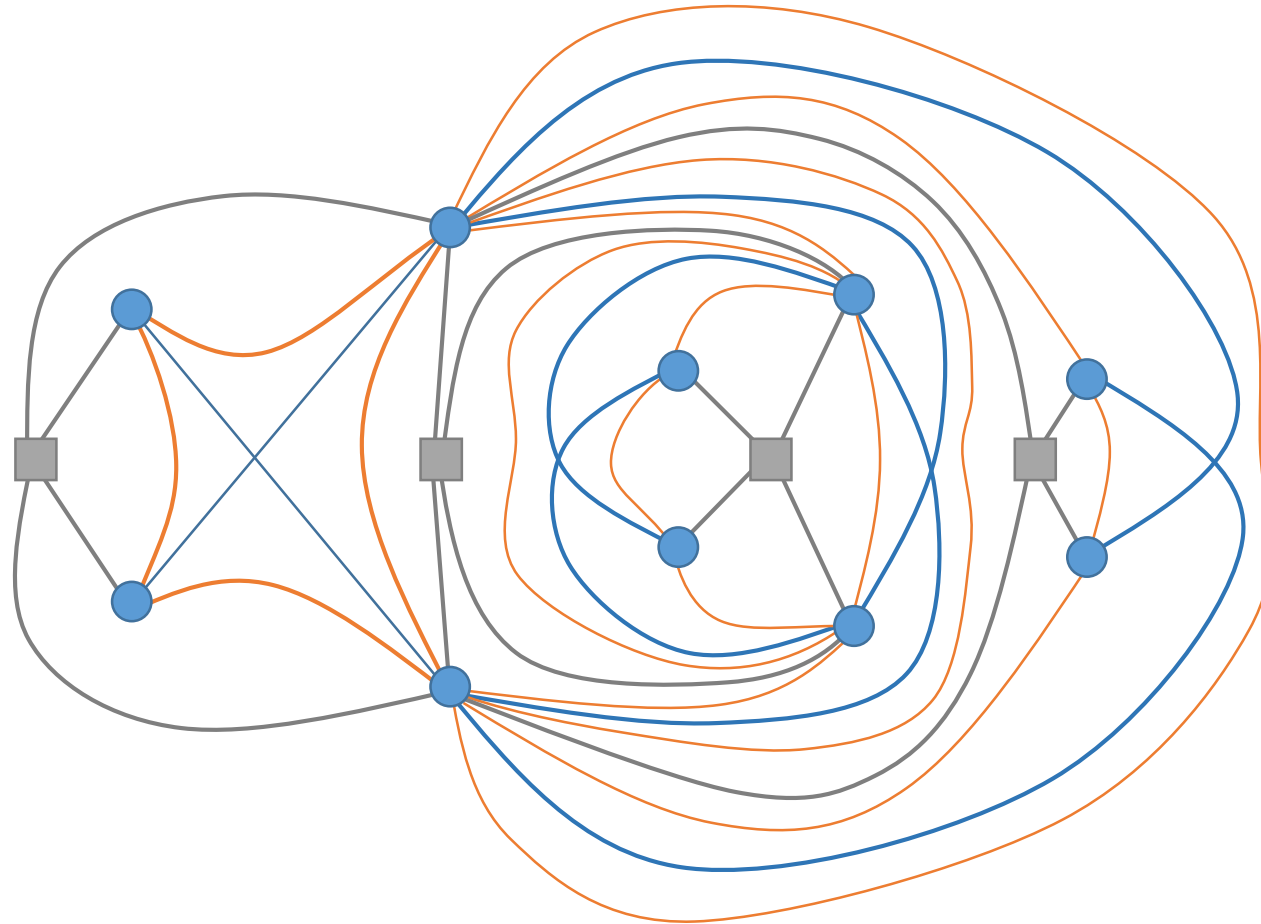


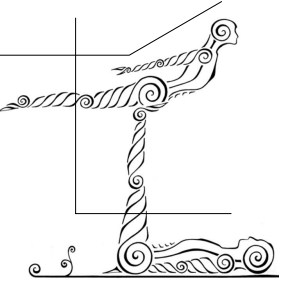
triangulate faces of
degree > 3 by
inserting a star
inside them



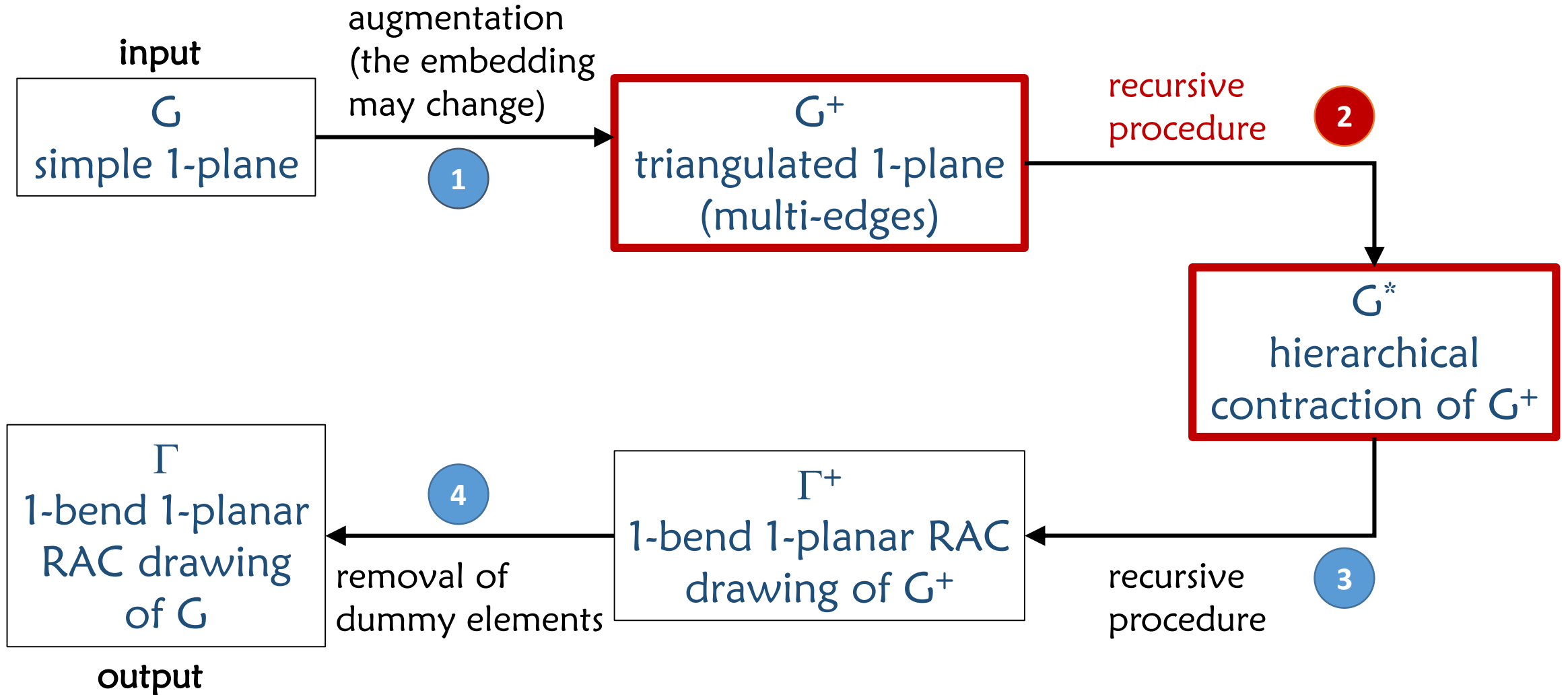
Augmentation

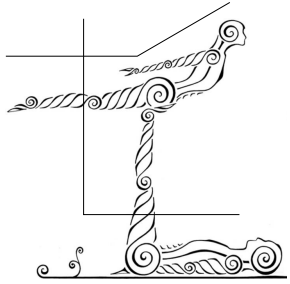
G^+
triangulated
1-plane





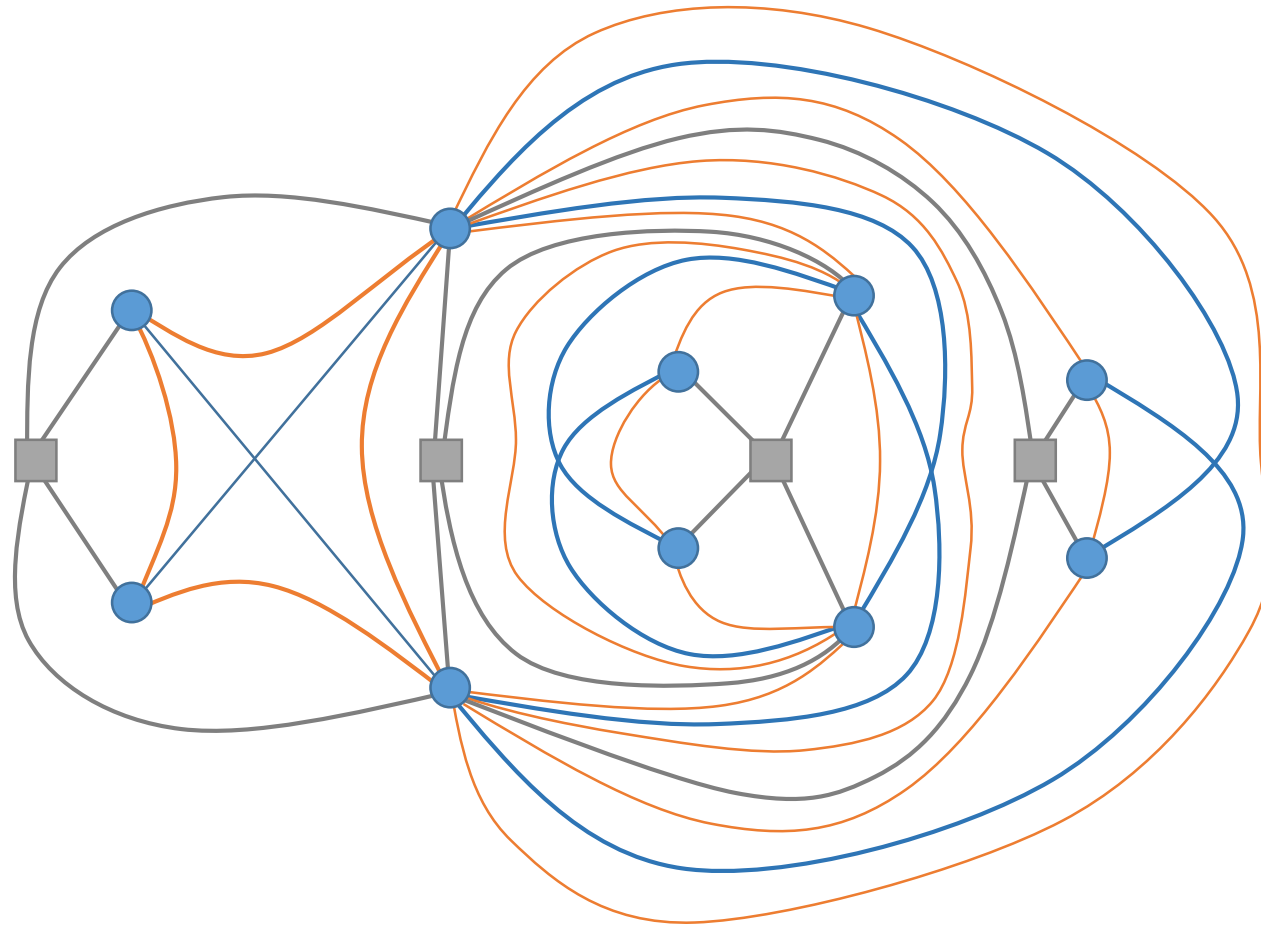
Algorithm Outline



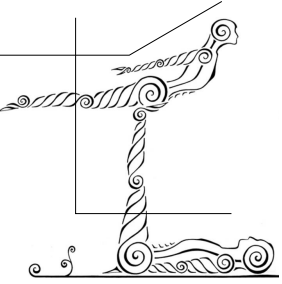


Property of G^+

G^+
triangulated
1-plane

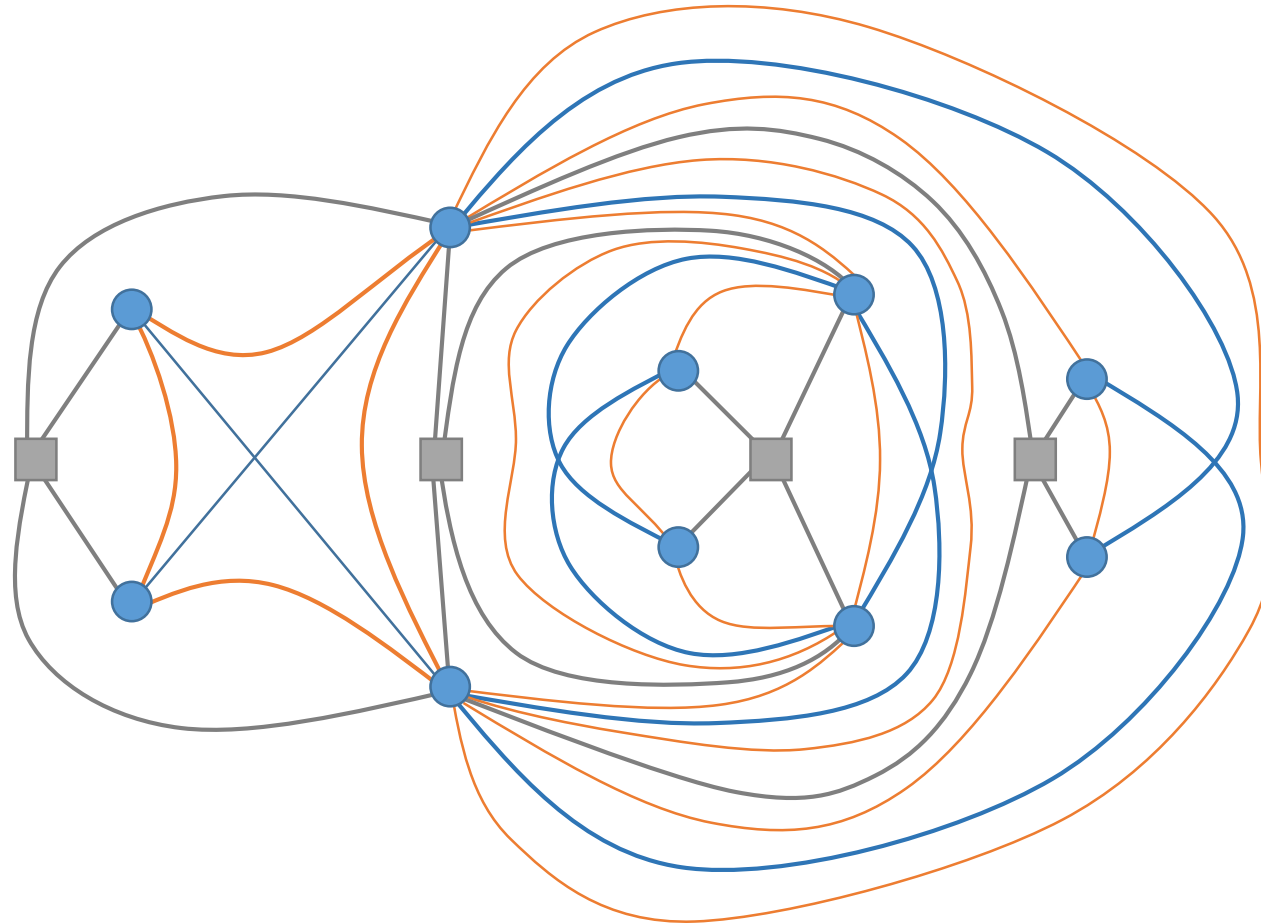


- triangular faces
- multiple edges
- never crossed
- only empty kites

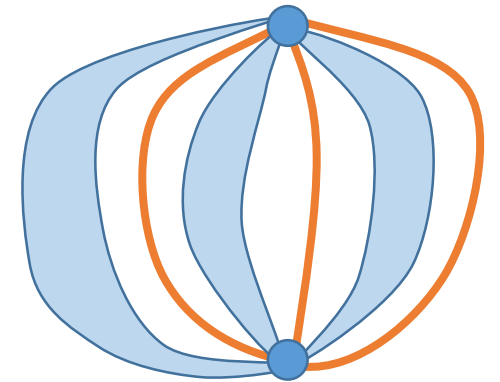


Property of G^+

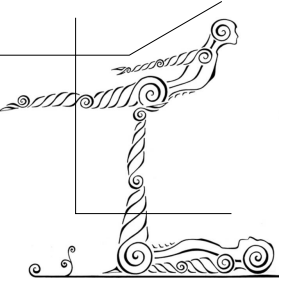
G^+
triangulated
1-plane



- triangular faces
- multiple edges
never crossed
- only empty kites

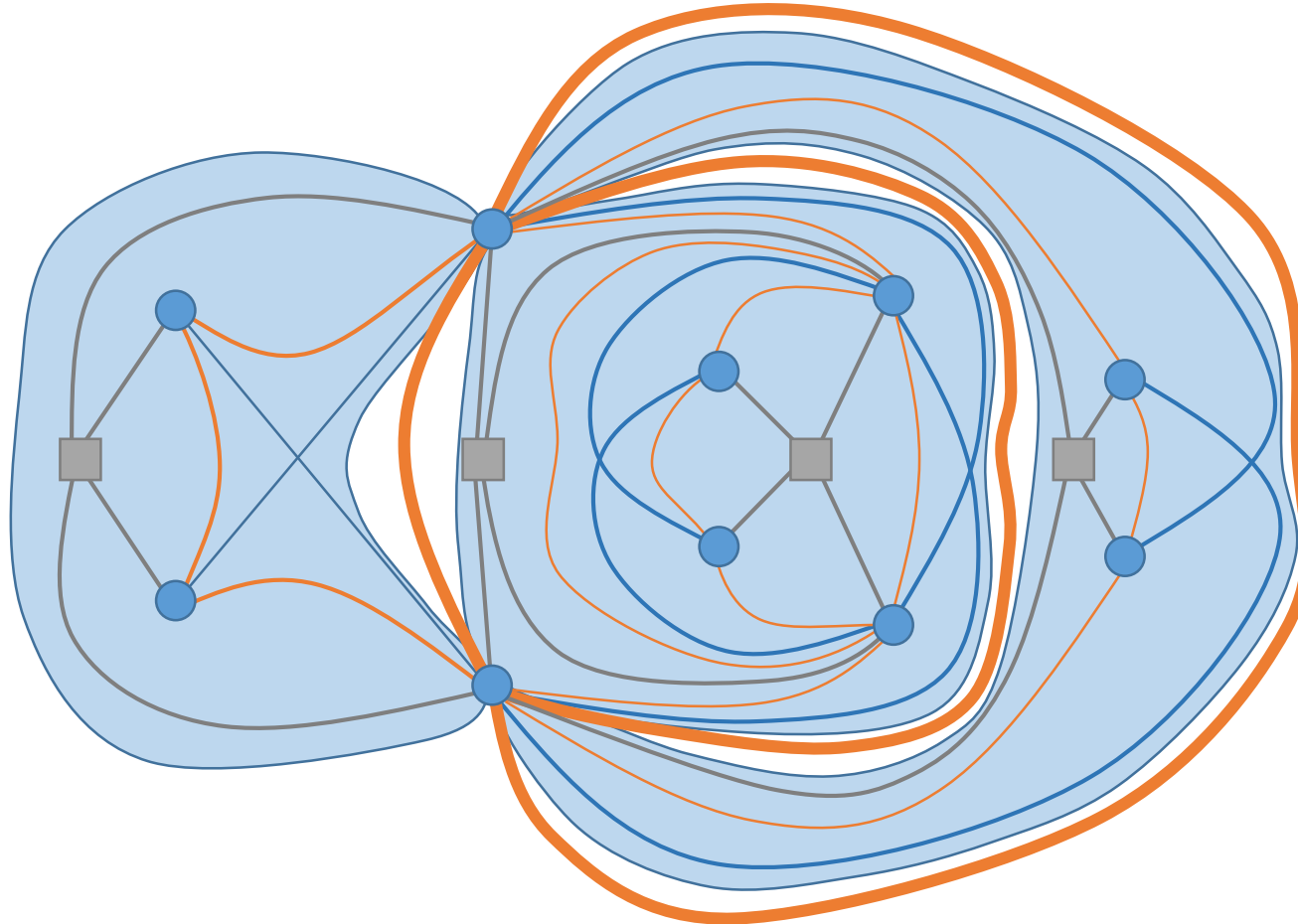


structure of each
separation pair

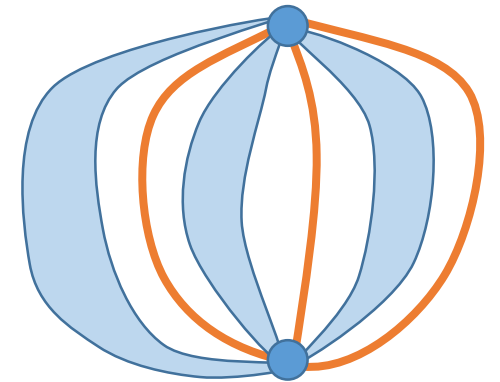


Property of G^+

G^+
triangulated
1-plane

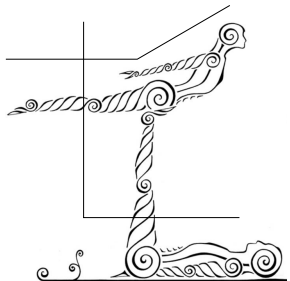


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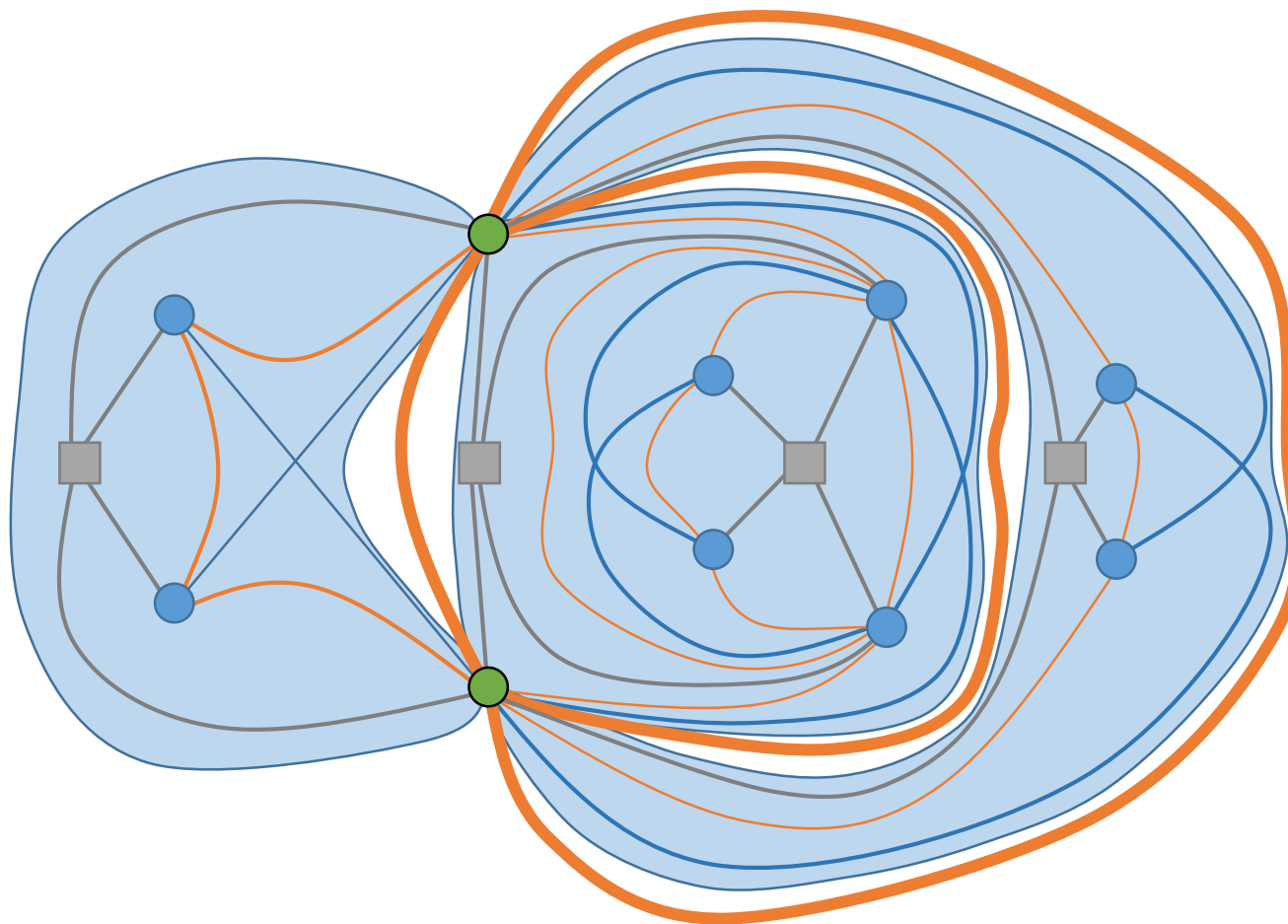


structure of each
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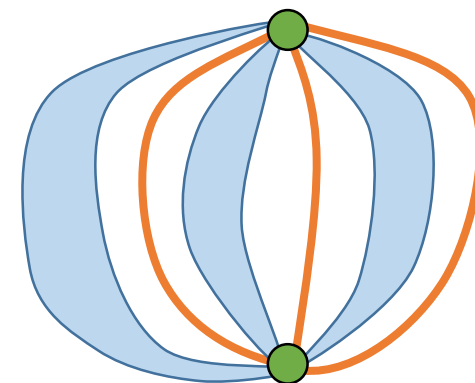
Hierarchical contraction



G^+
triangulated
1-plane

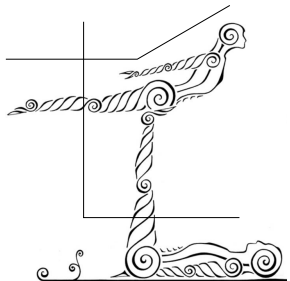


contract all inner
components of
each separation
pair into a
thick edge

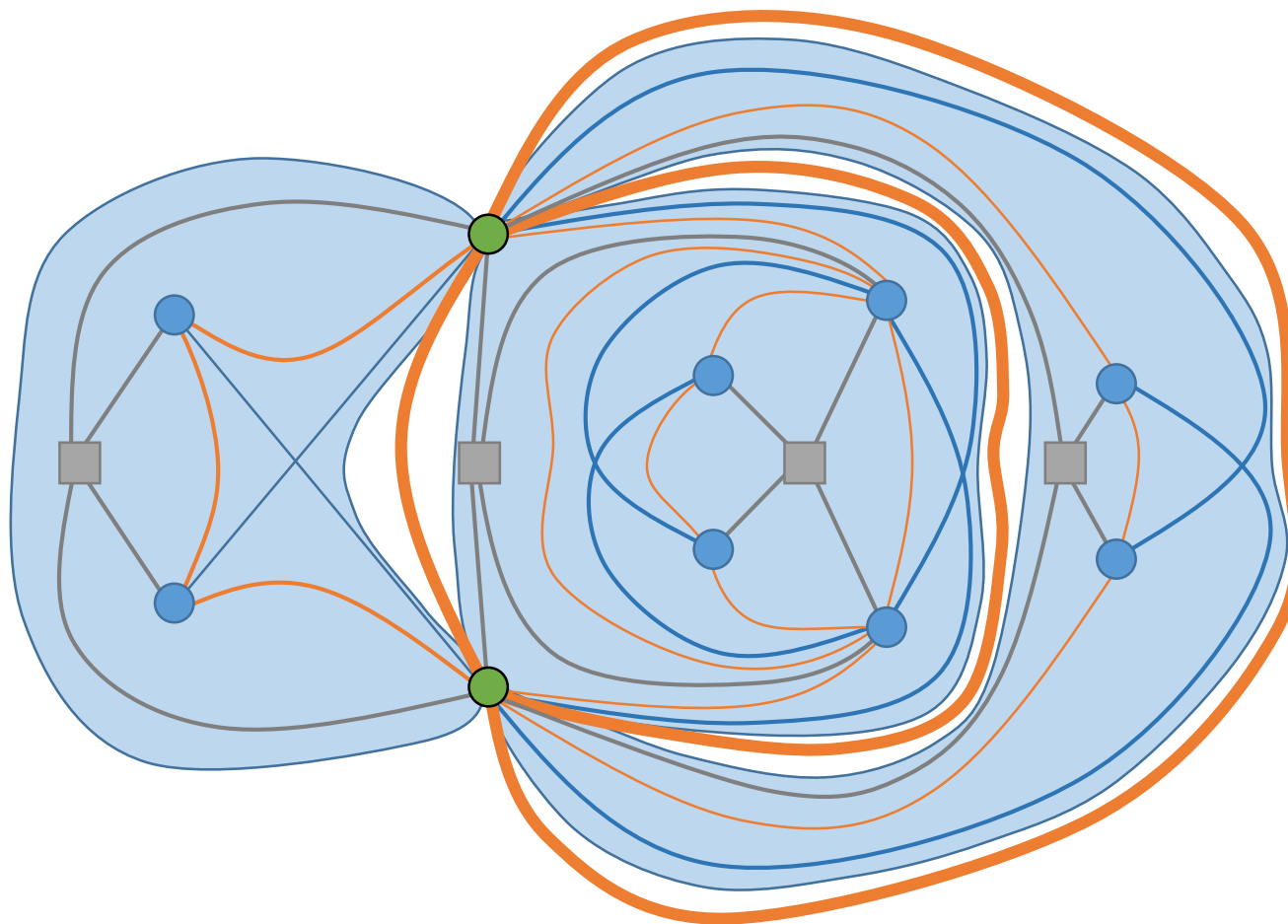


structure of each
separation pair

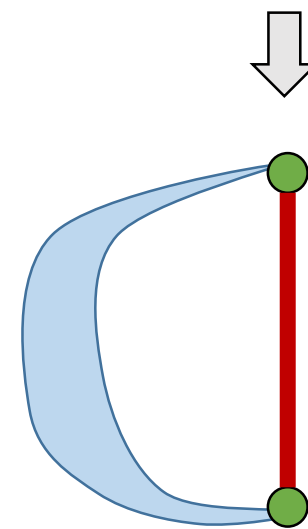
Hierarchical contraction



G^+
triangulated
1-plane

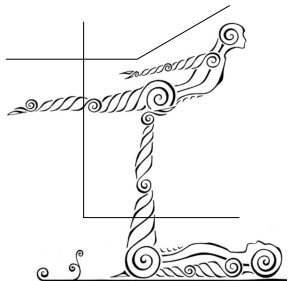


contract all inner
components of
each separation
pair into a
thick edge

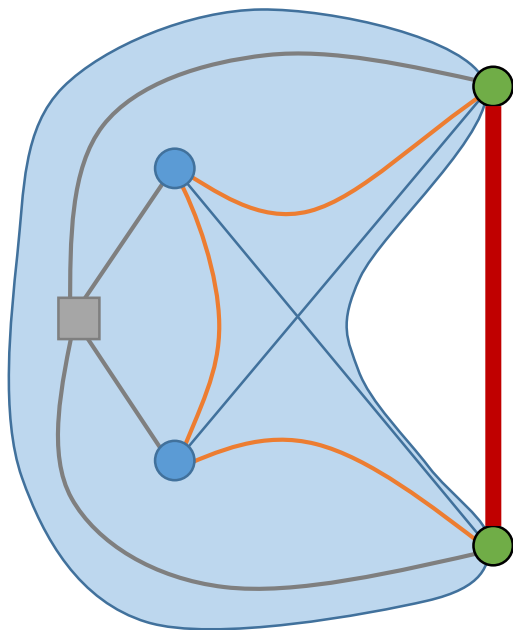


contraction

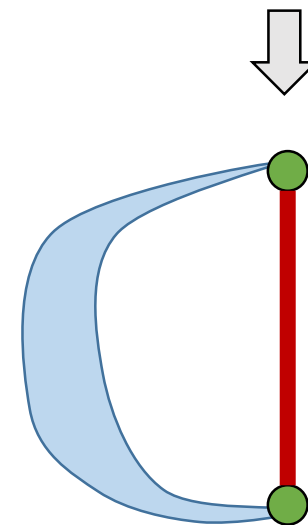
Hierarchical contraction



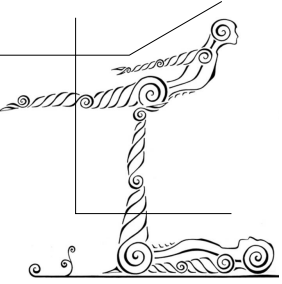
G^+
triangulated
1-plane



contract all inner
components of
each separation
pair into a
thick edge

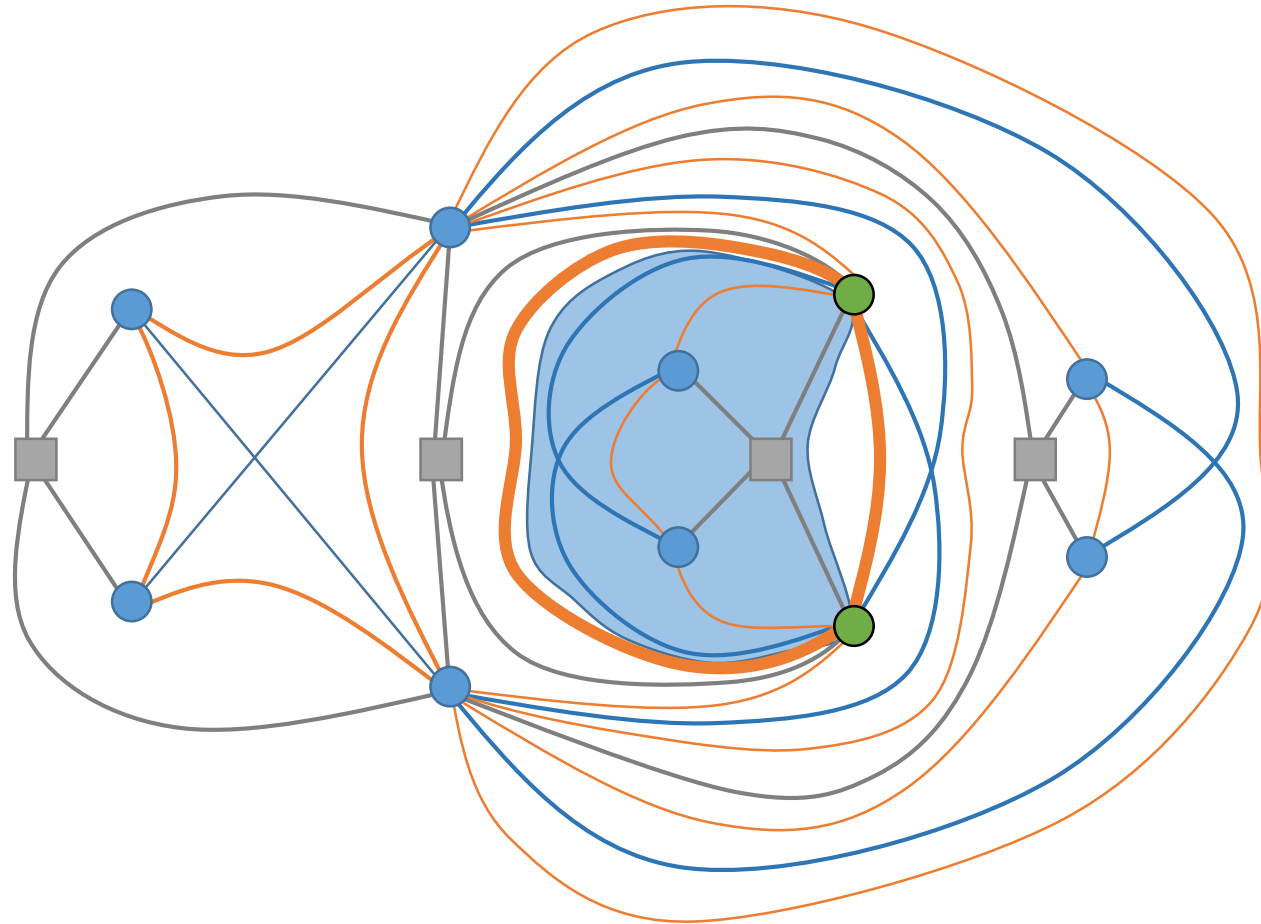
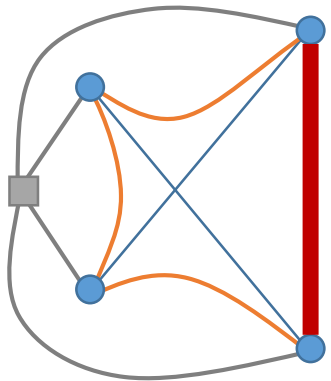


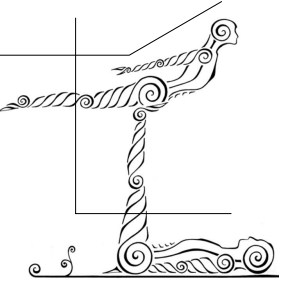
contraction



Hierarchical contraction

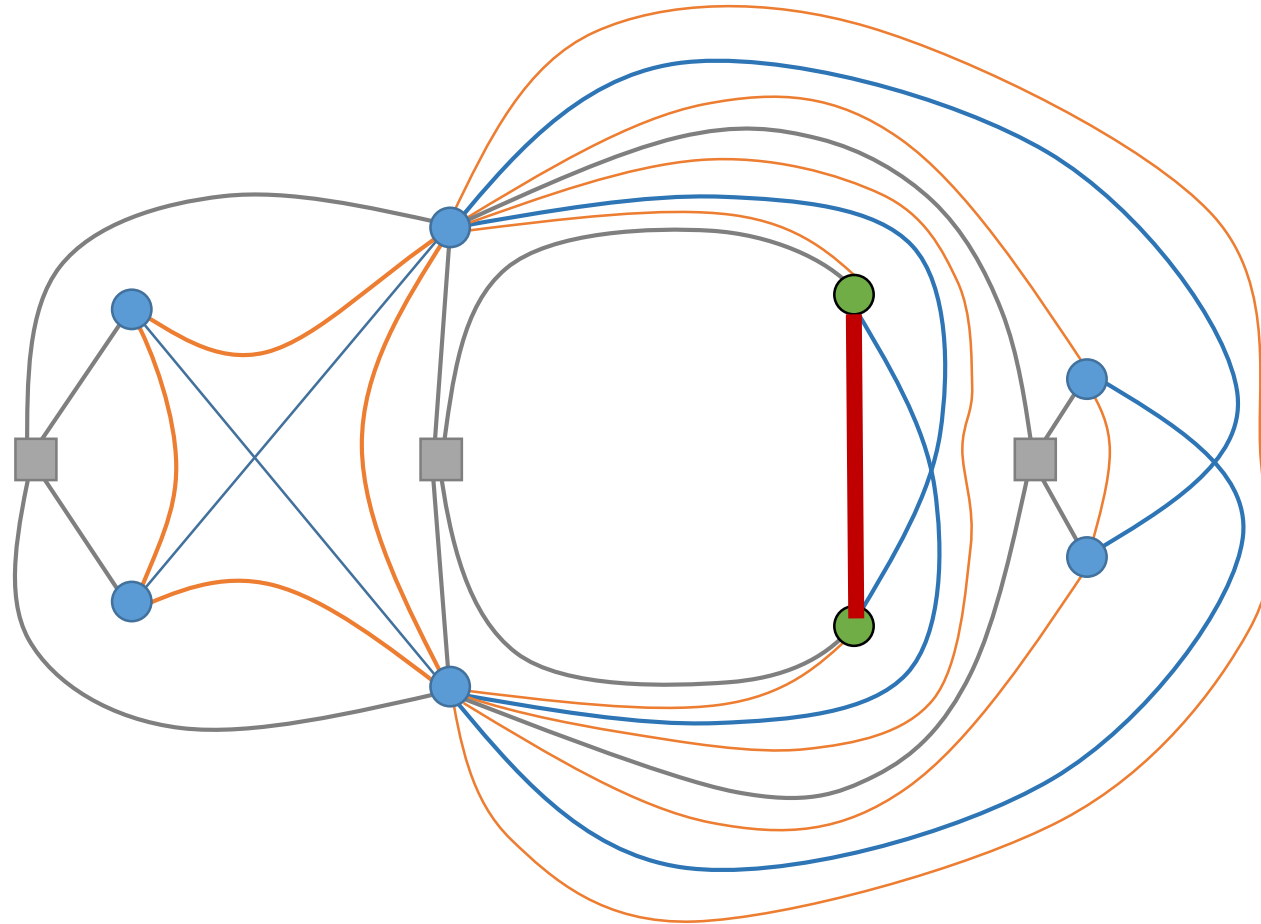
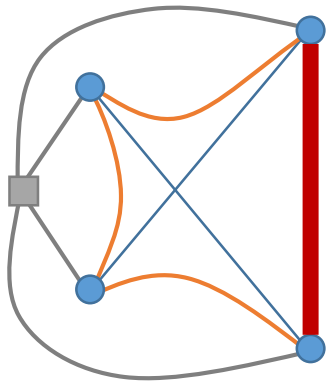
G^+
triangulated
1-plane



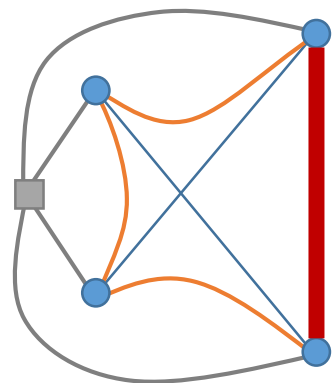
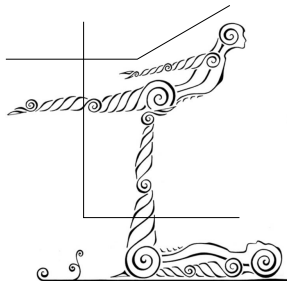


Hierarchical contraction

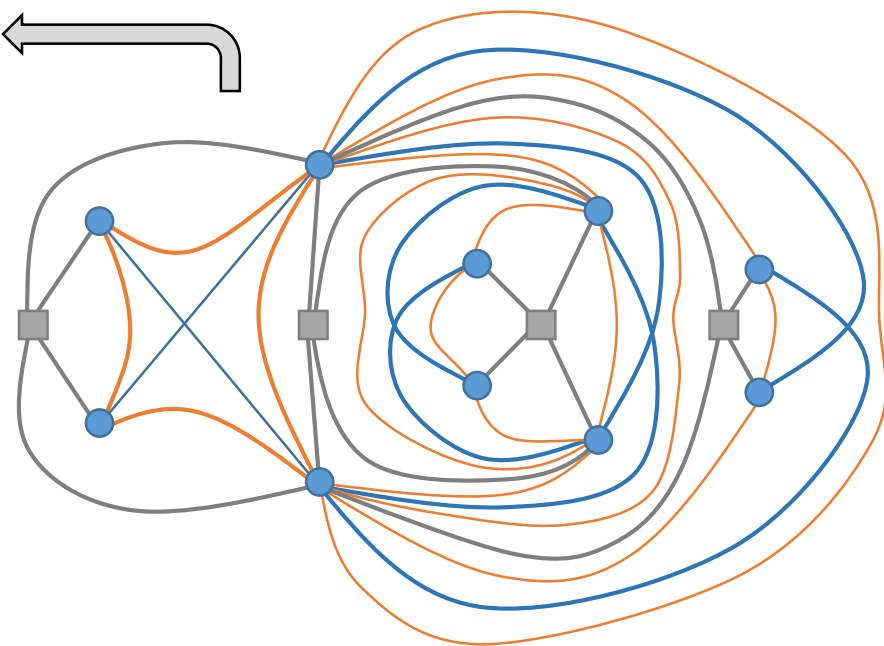
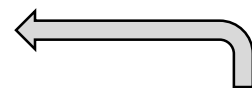
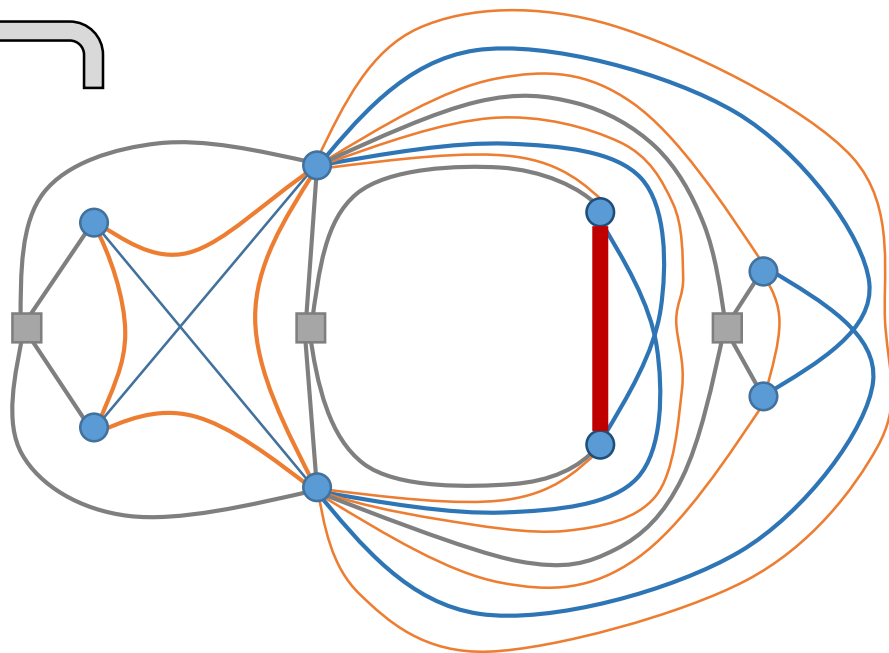
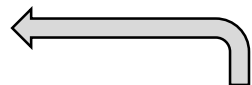
G^+
triangulated
1-plane



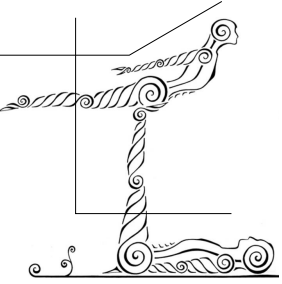
Hierarchical contraction



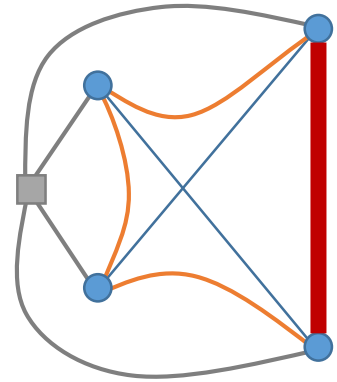
G^*
hierarchical
contraction



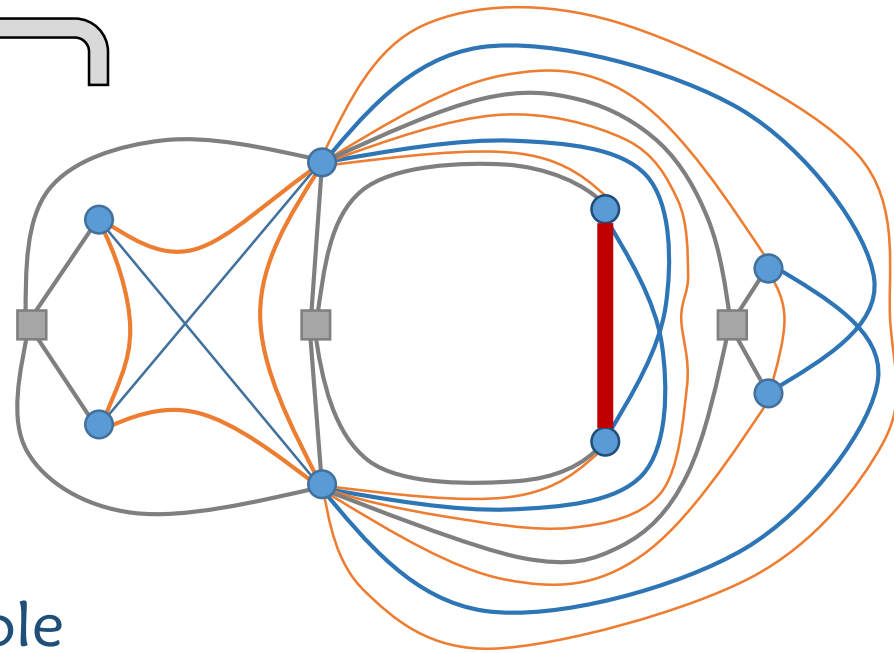
G^+
triangulated
1-plane



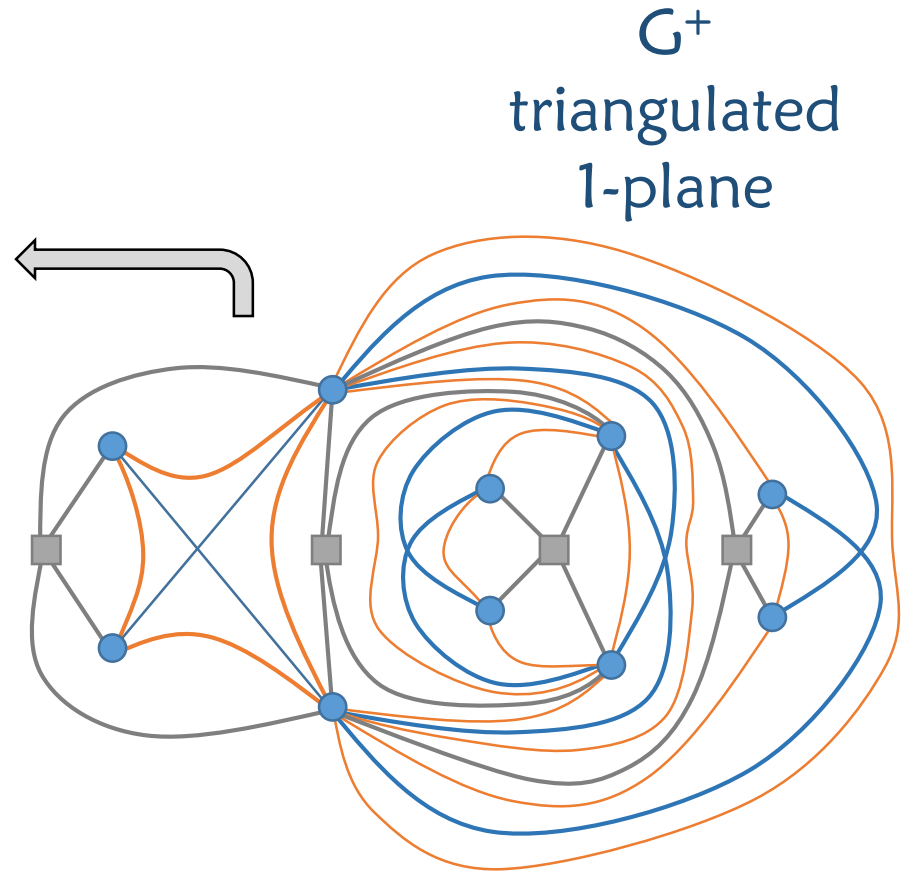
Hierarchical contraction



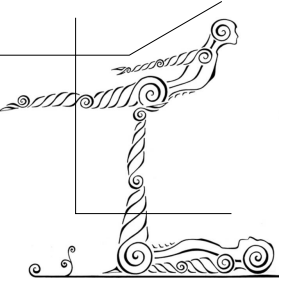
G^*
hierarchical
contraction



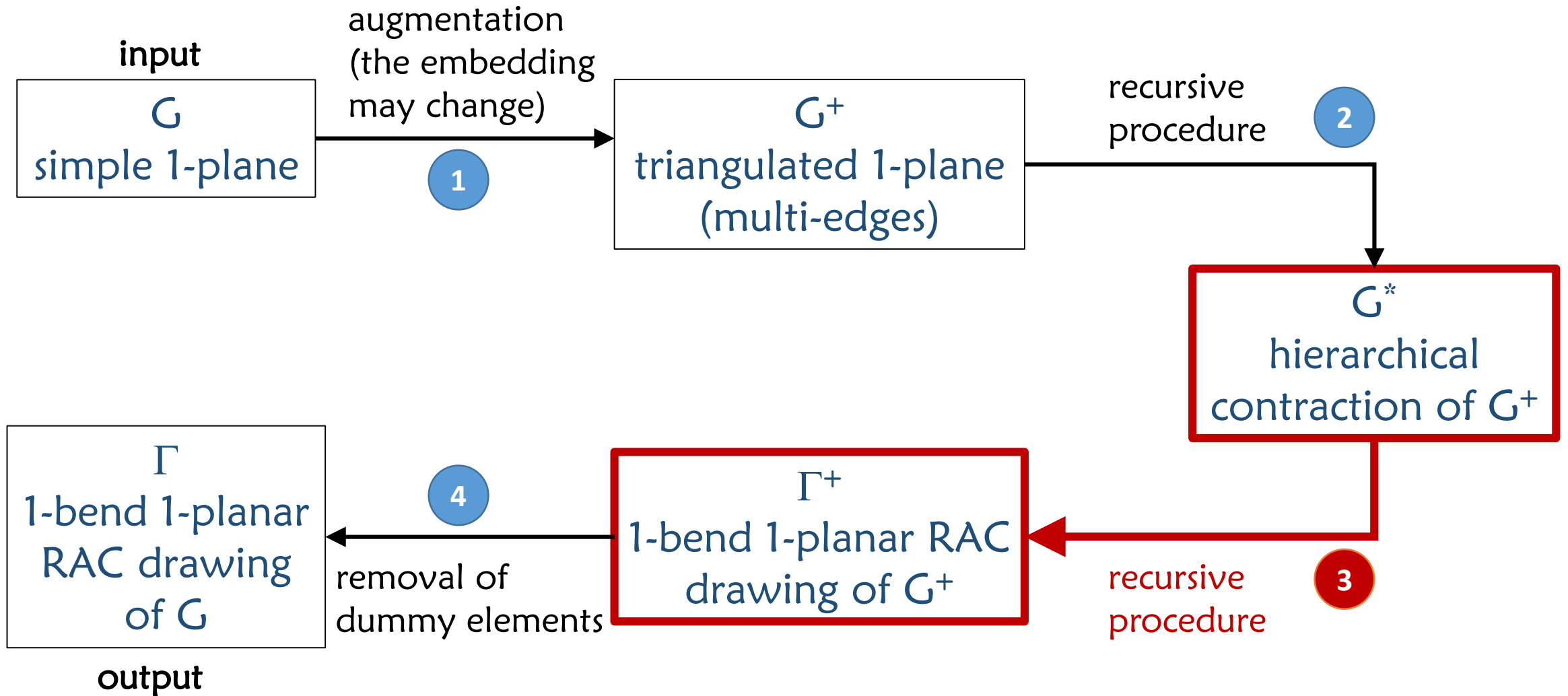
simple
3-connected
triangulated
1-plane graph

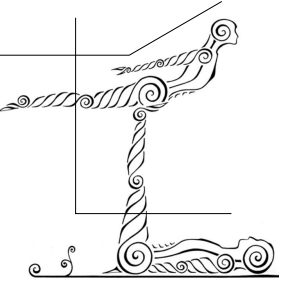


G^+
triangulated
1-plane

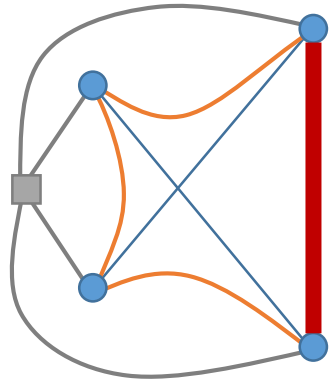


Algorithm Outline

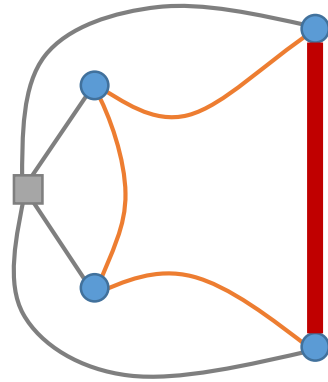
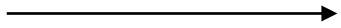




Drawing procedure

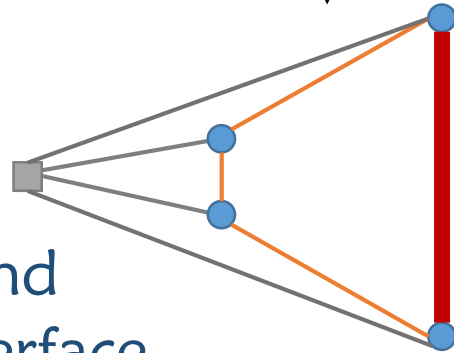


remove
crossing edges



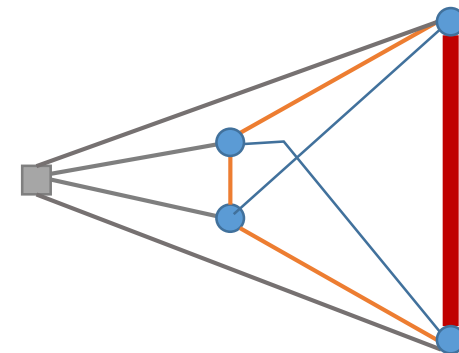
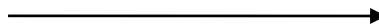
3-connected
plane graph

apply Chiba et
al. 1984

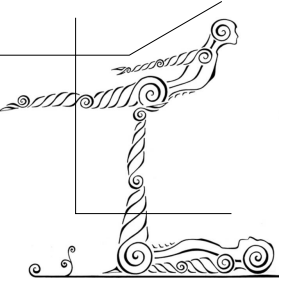


convex faces and
prescribed outerface

reinsert
crossing edges

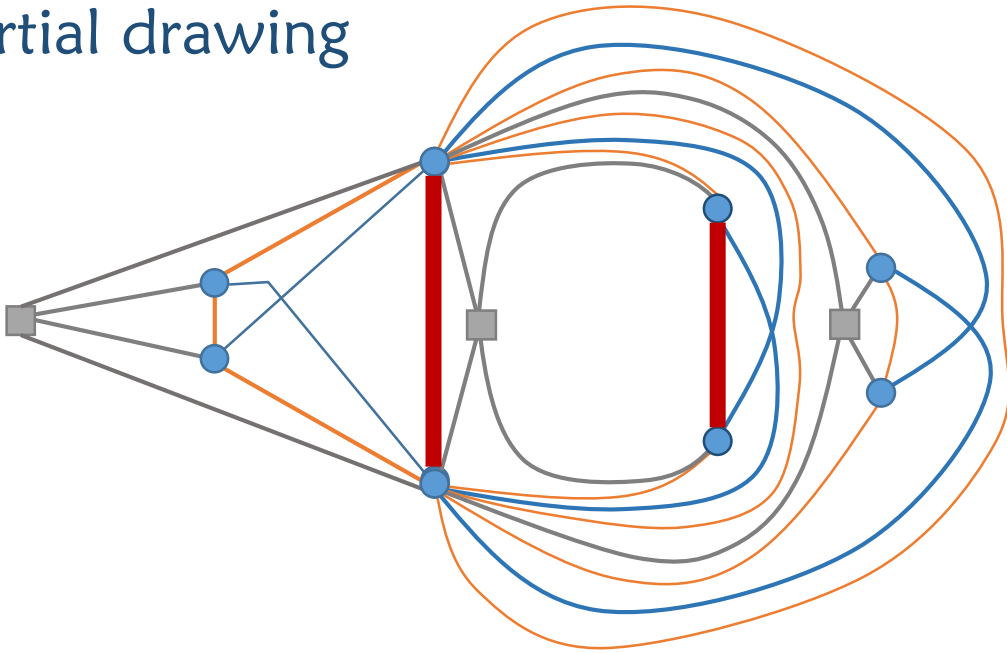


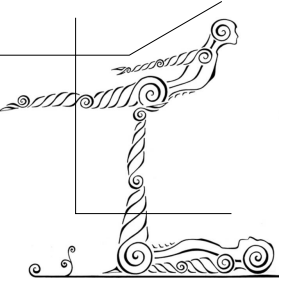
partial drawing



Drawing procedure

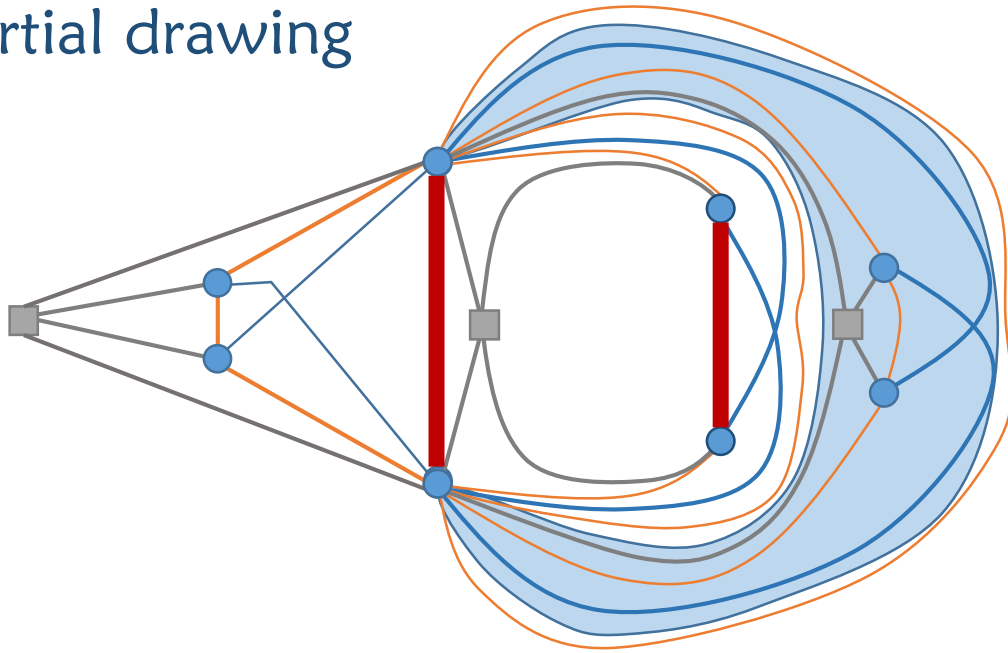
partial drawing

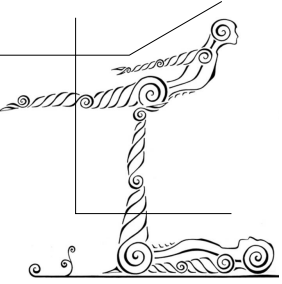




Drawing procedure

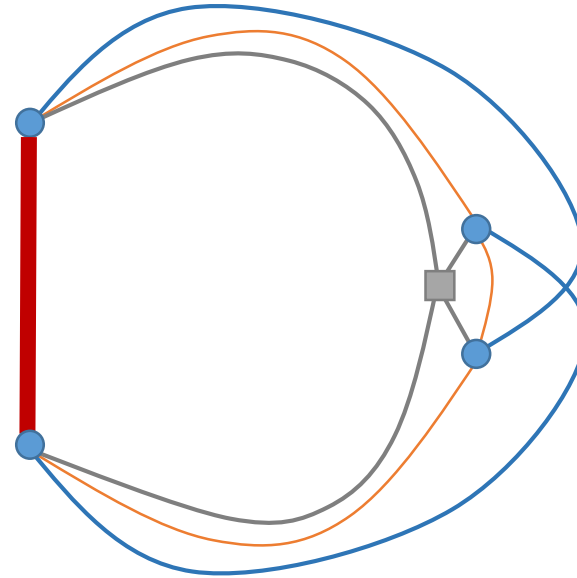
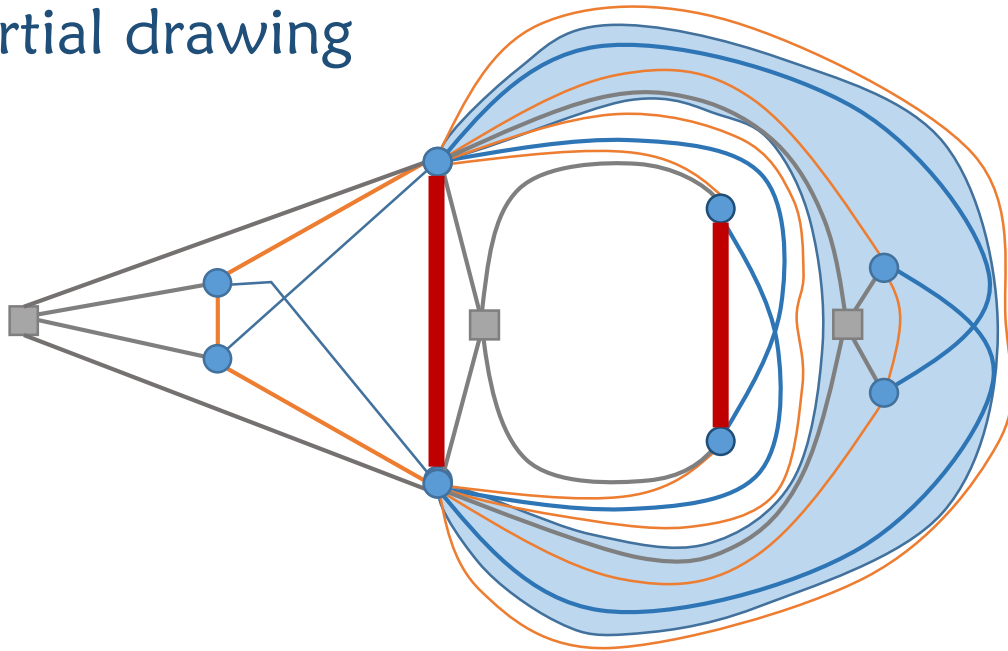
partial drawing

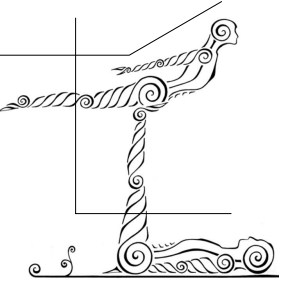




Drawing procedure

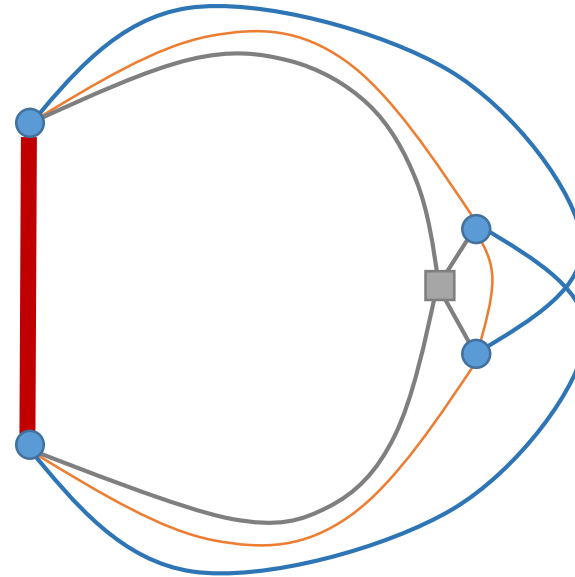
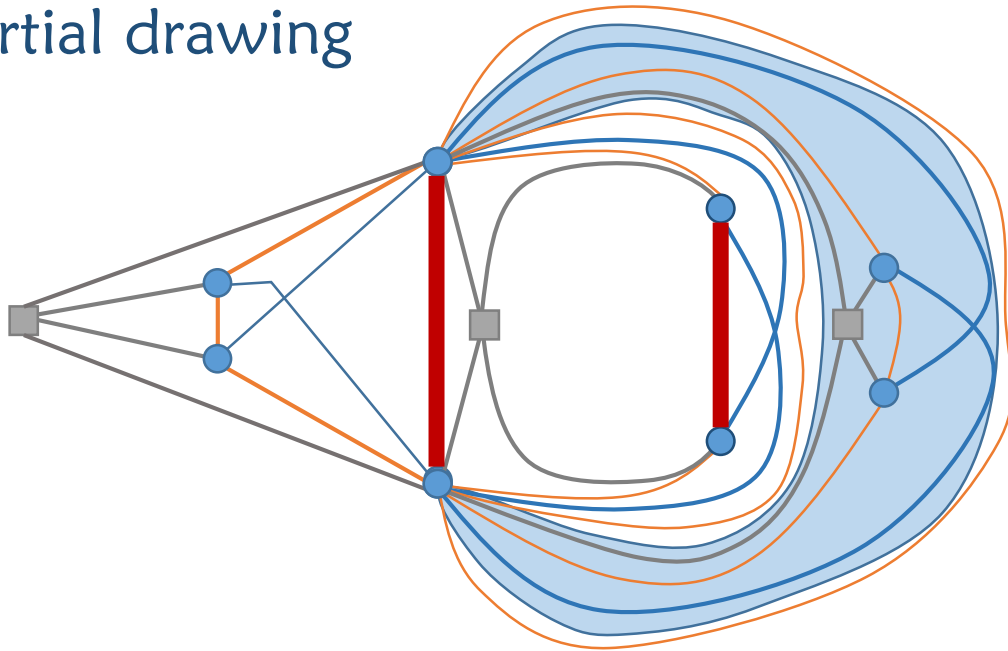
partial drawing



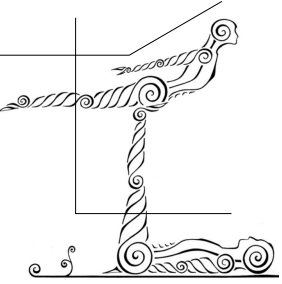


Drawing procedure

partial drawing

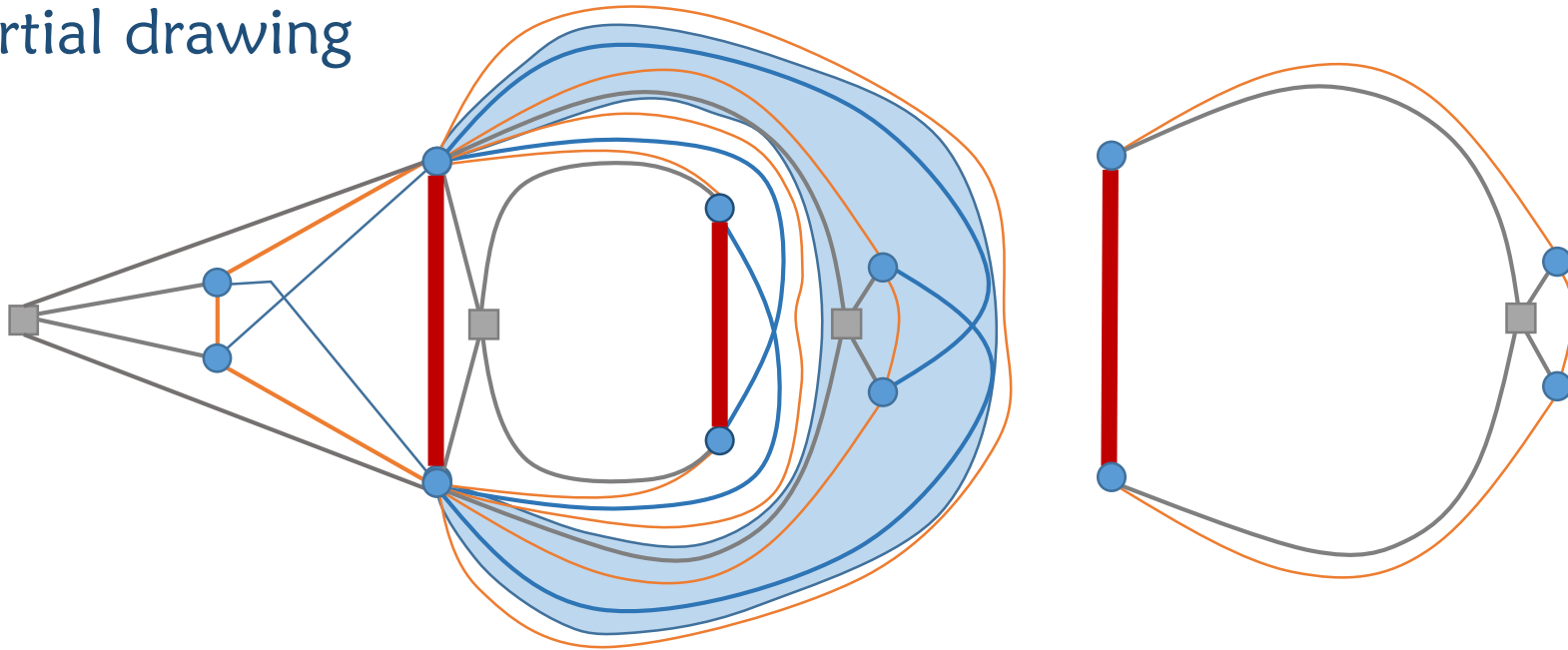


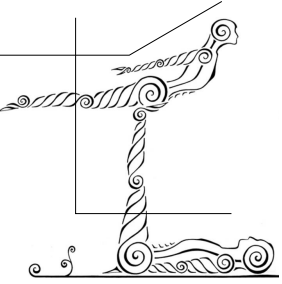
remove
crossing edges



Drawing procedure

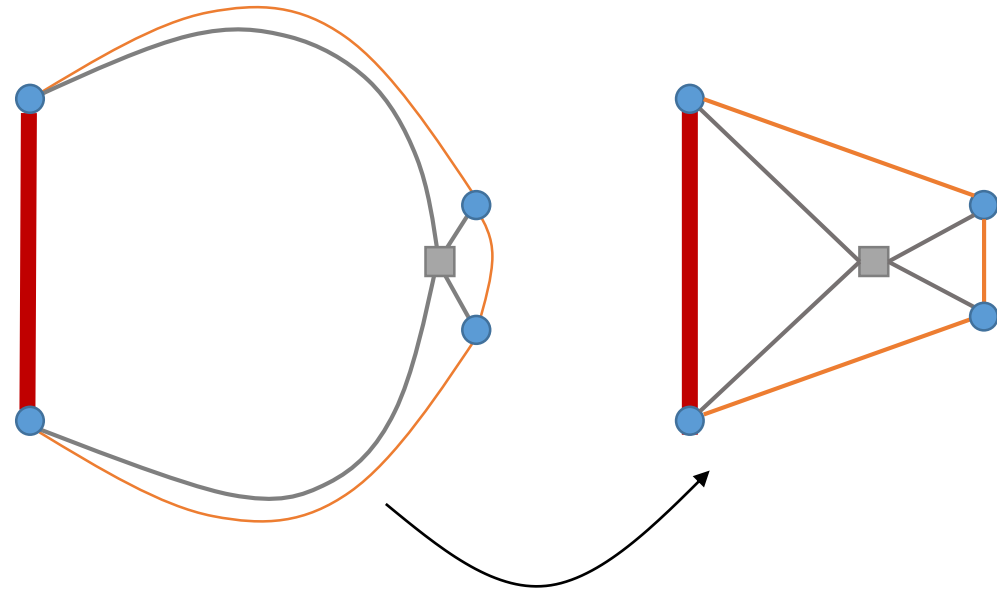
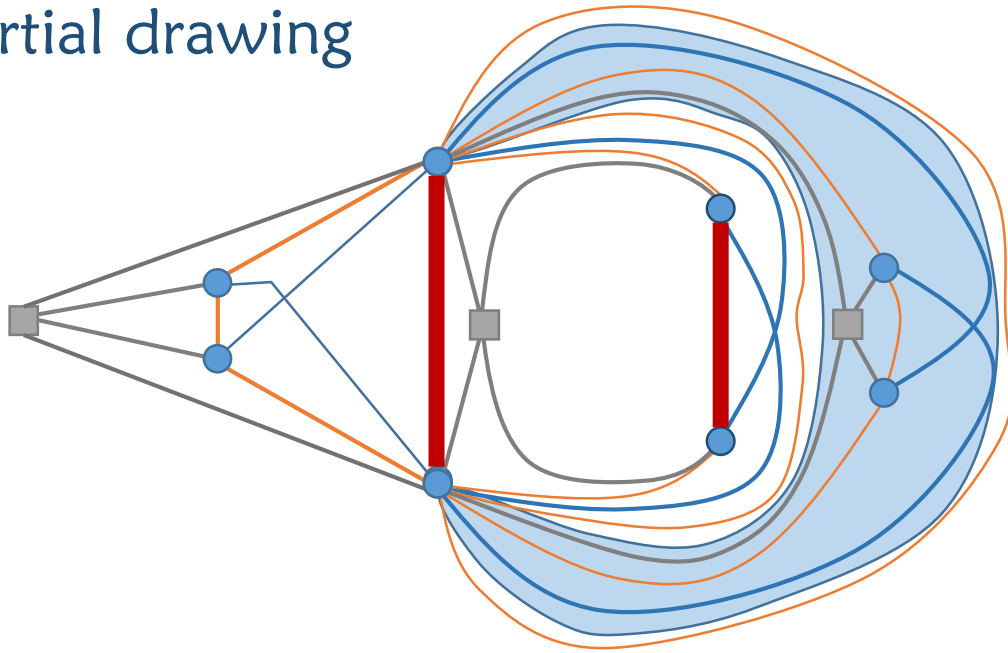
partial drawing



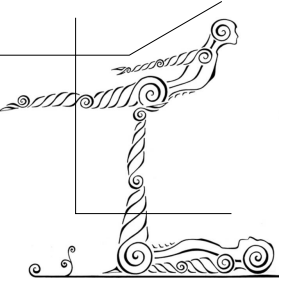


Drawing procedure

partial drawing

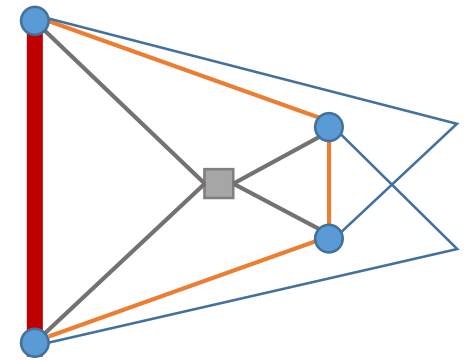
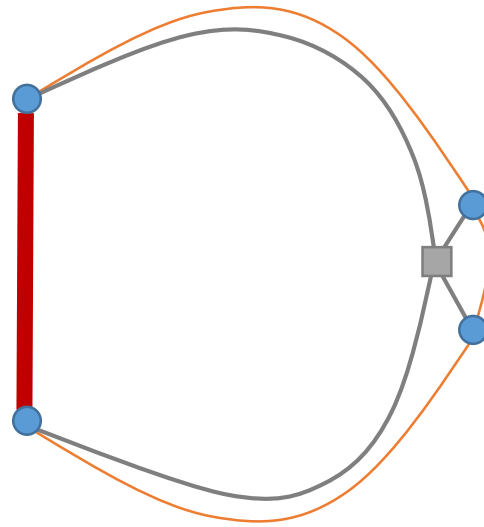
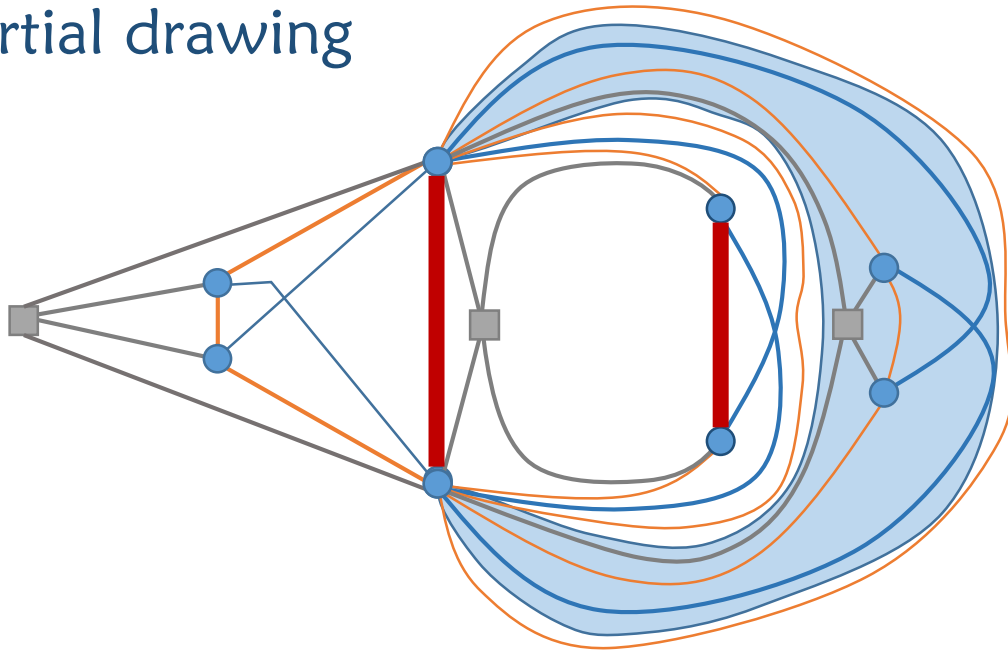


apply Chiba et al. 1984

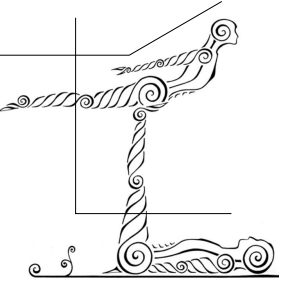


Drawing procedure

partial drawing

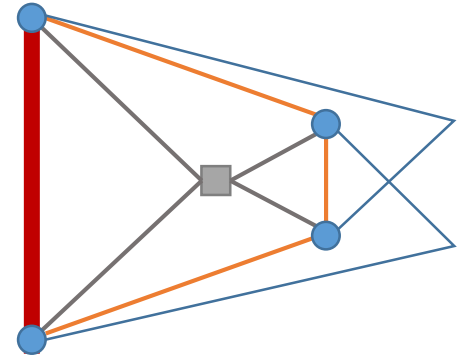
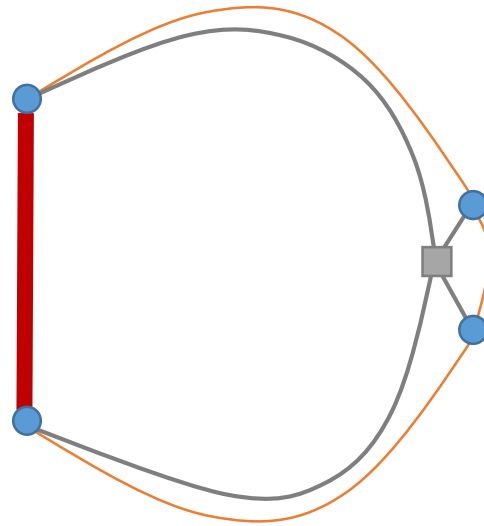
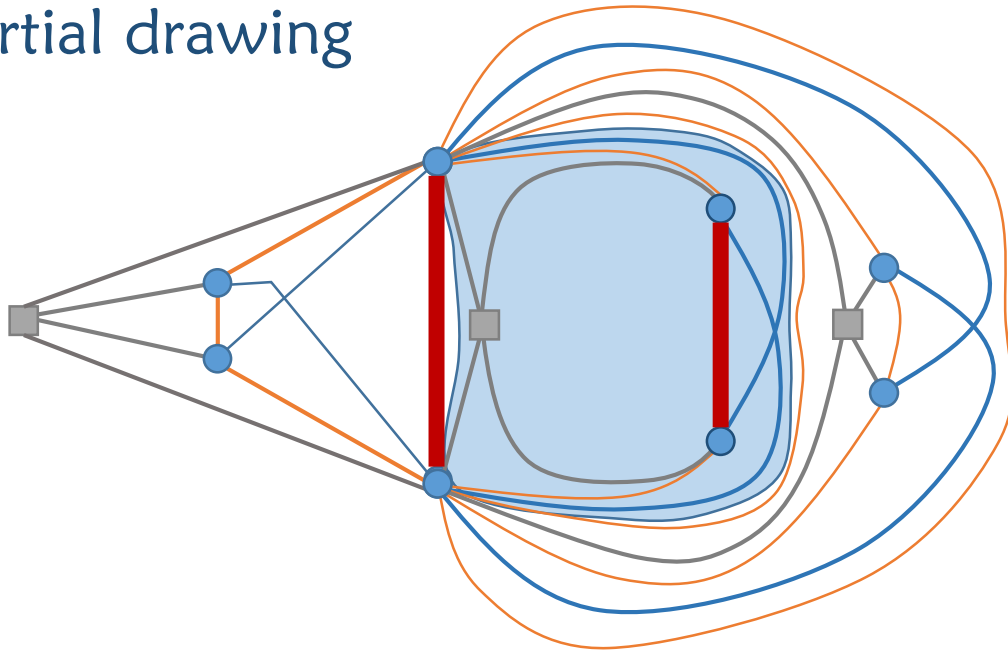


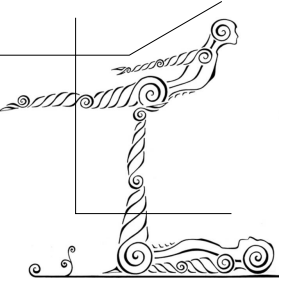
reinsert
crossing edges



Drawing procedure

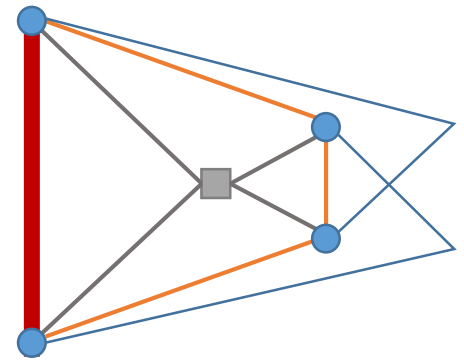
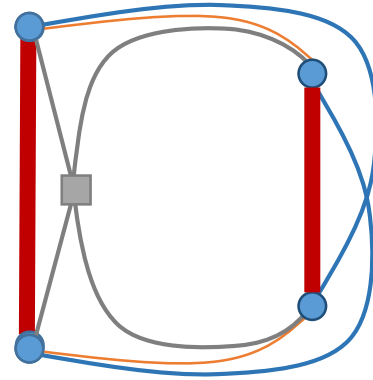
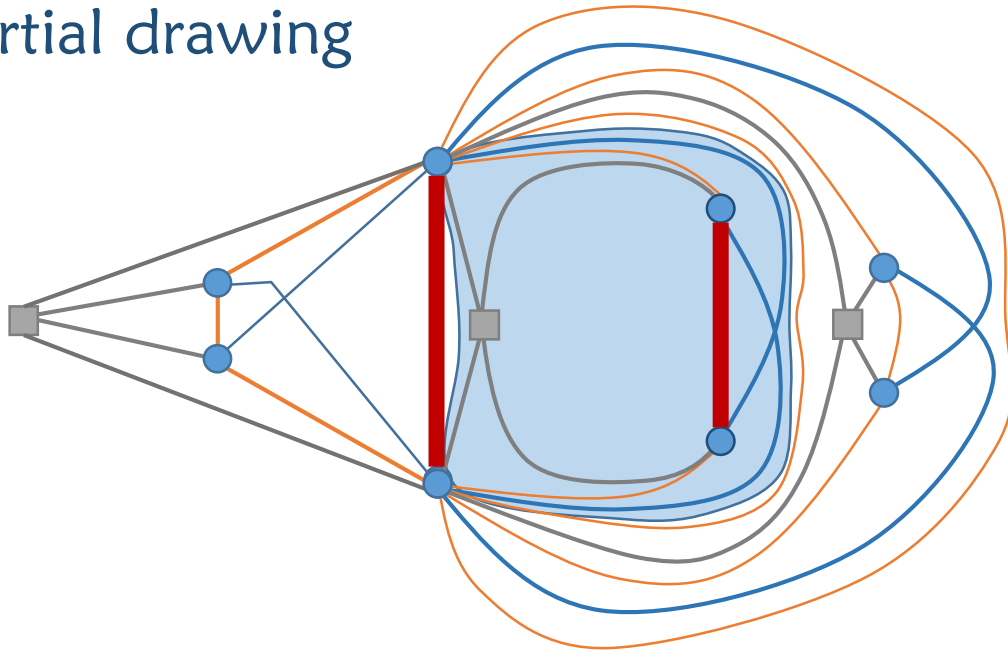
partial drawing



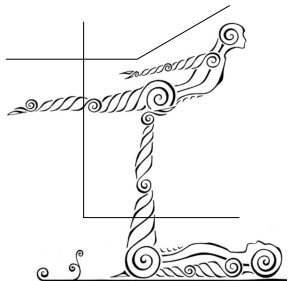


Drawing procedure

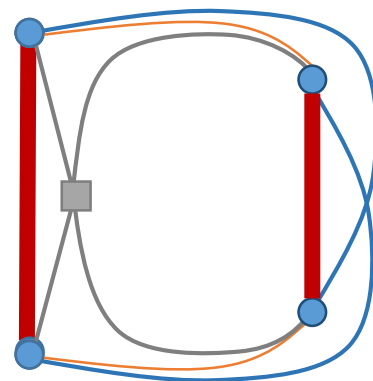
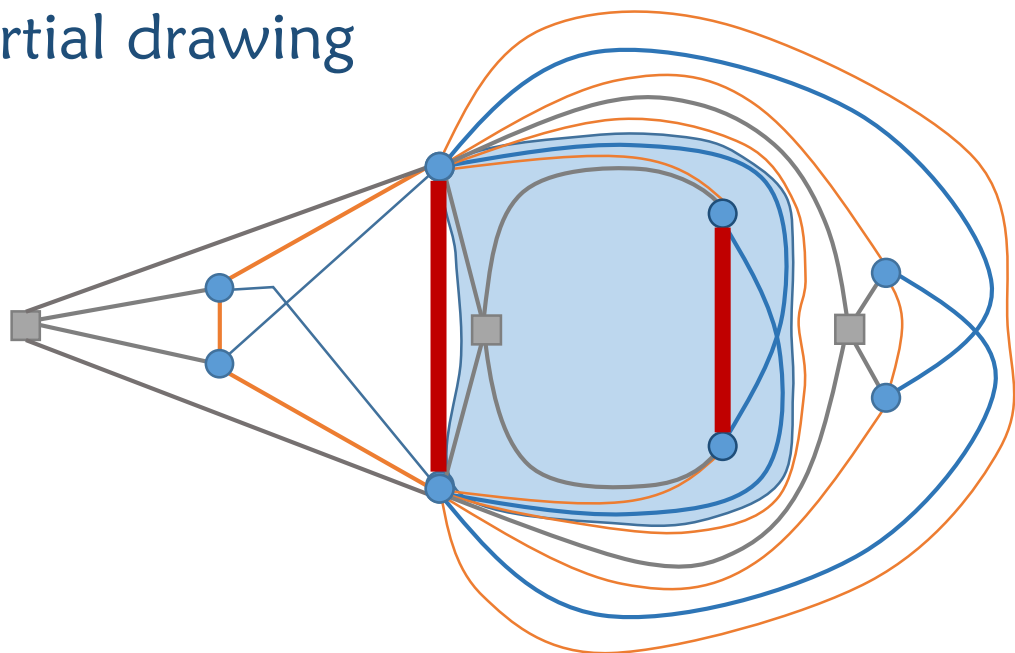
partial drawing



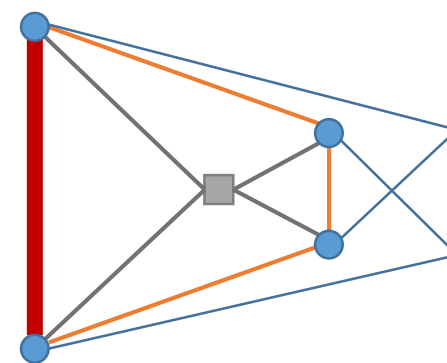
Drawing procedure

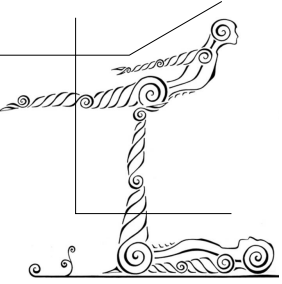


partial drawing



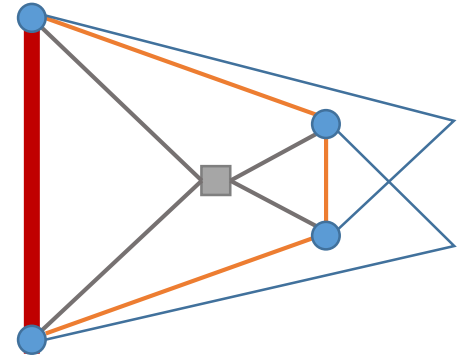
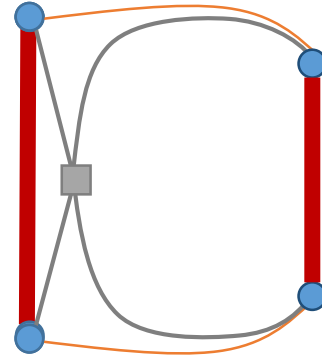
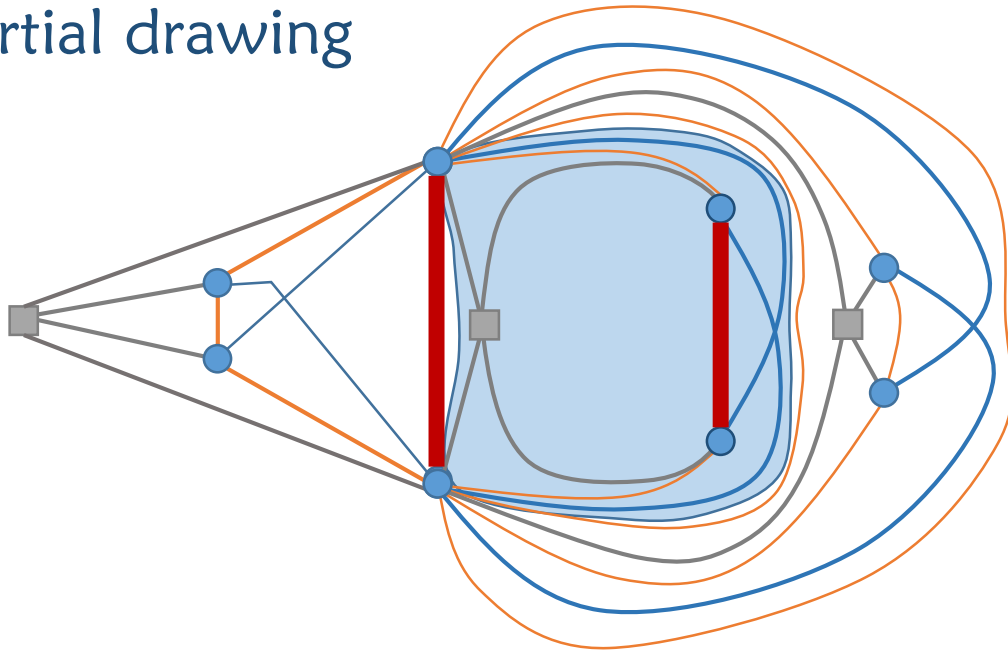
remove
crossing edges

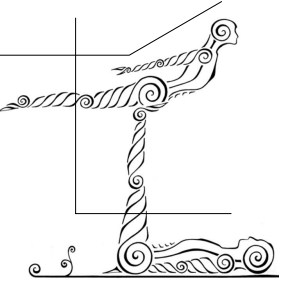




Drawing procedure

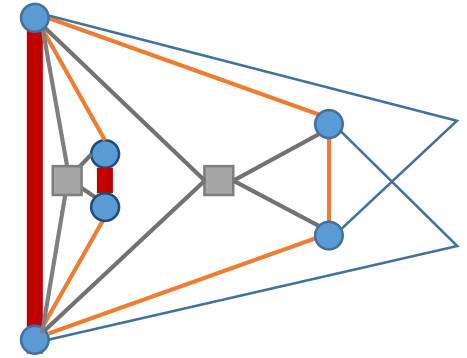
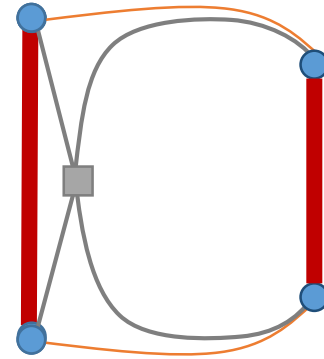
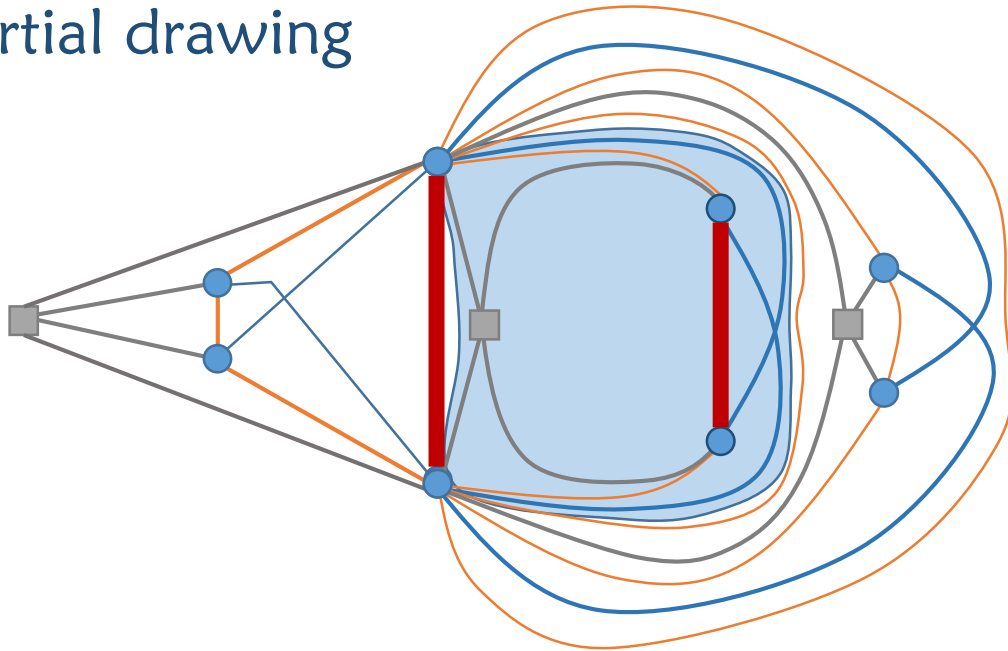
partial drawing



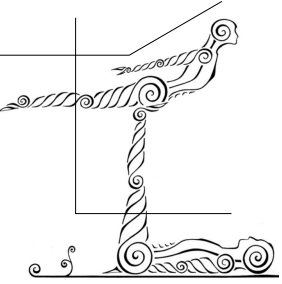


Drawing procedure

partial drawing

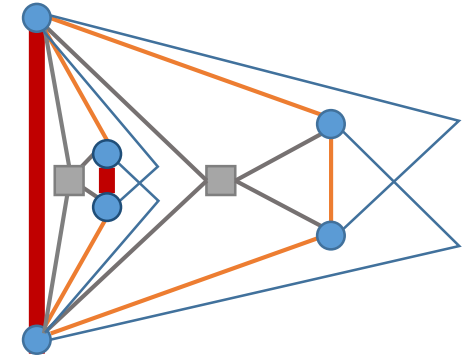
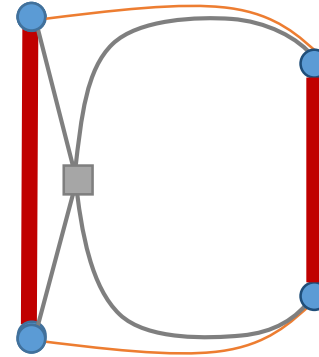
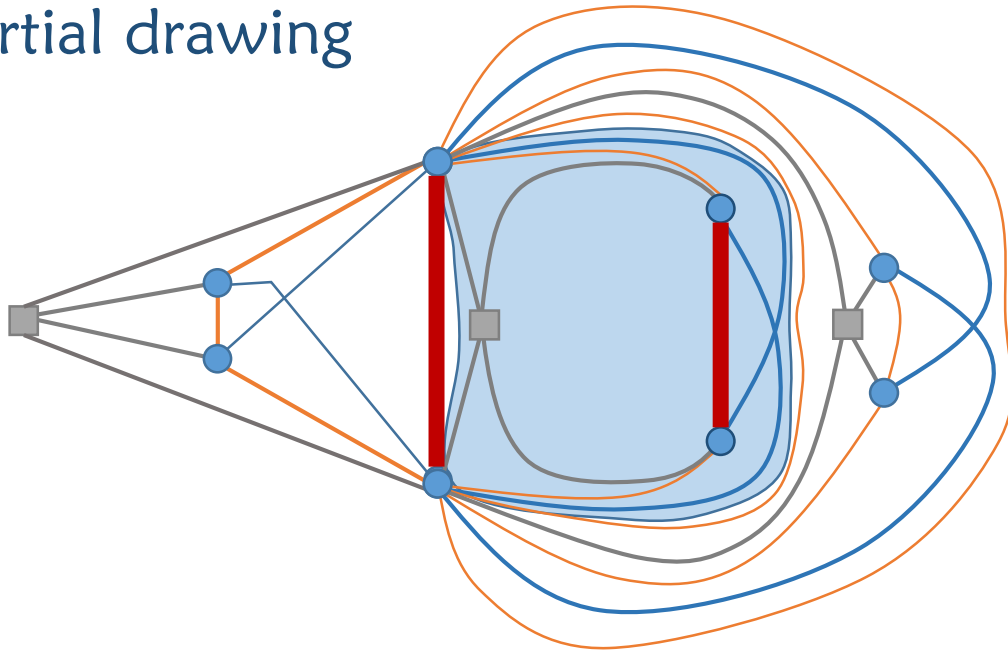


apply Chiba et al. 1984

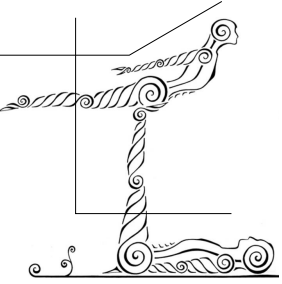


Drawing procedure

partial drawing

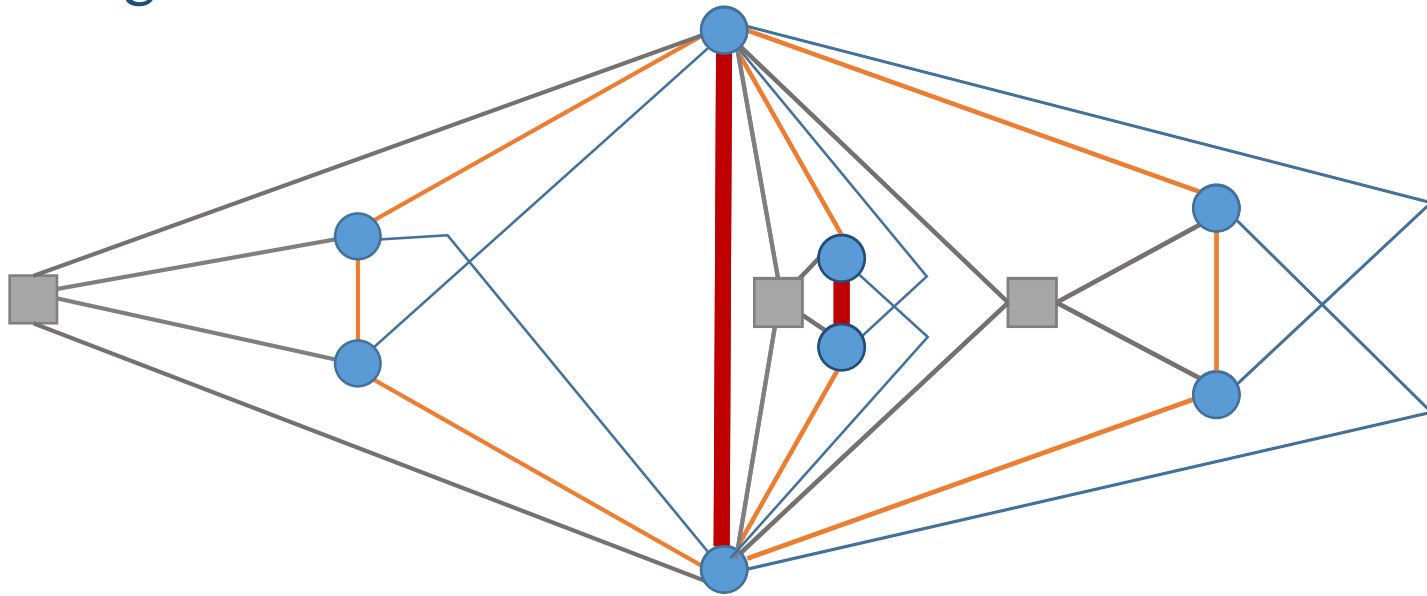


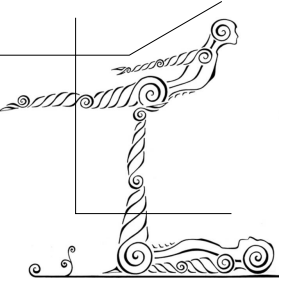
reinsert
crossing edges



Drawing procedure

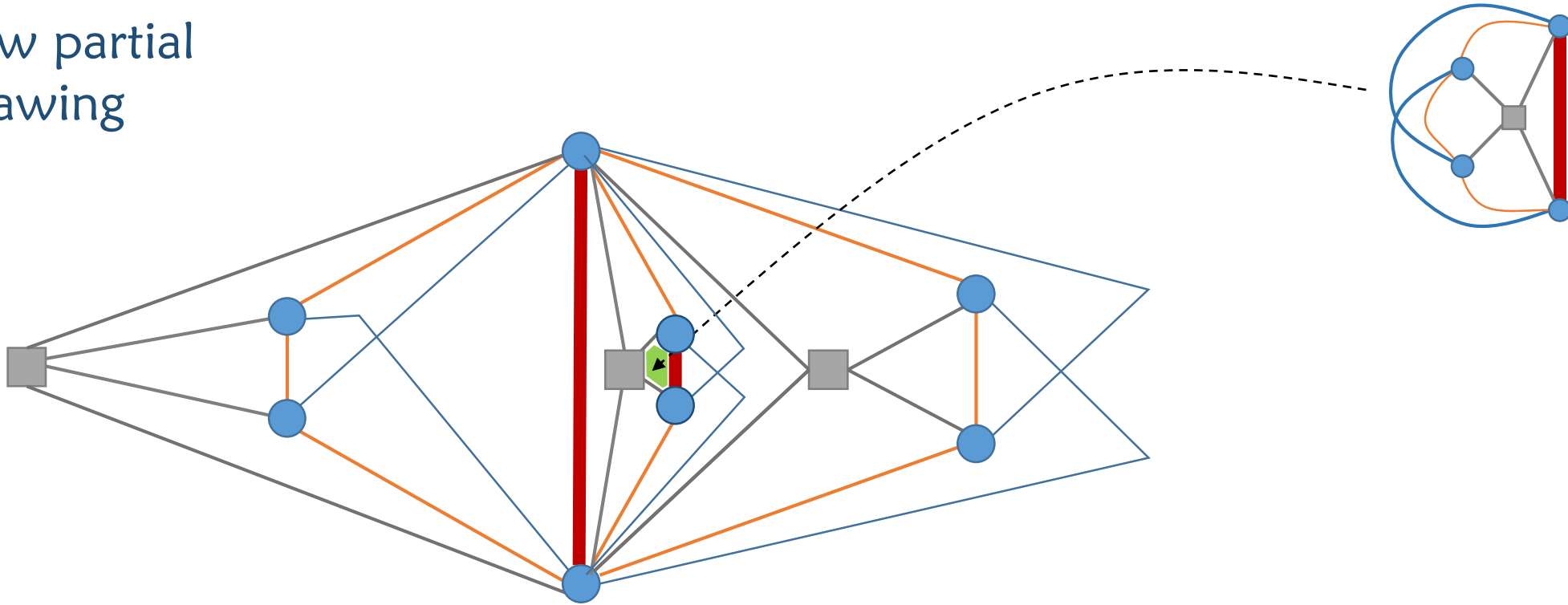
new partial
drawing

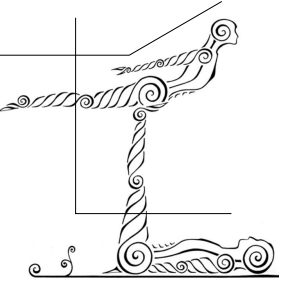




Drawing procedure

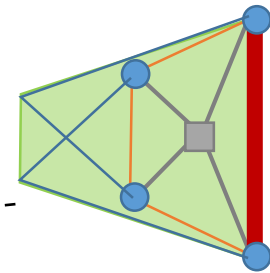
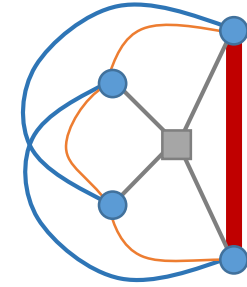
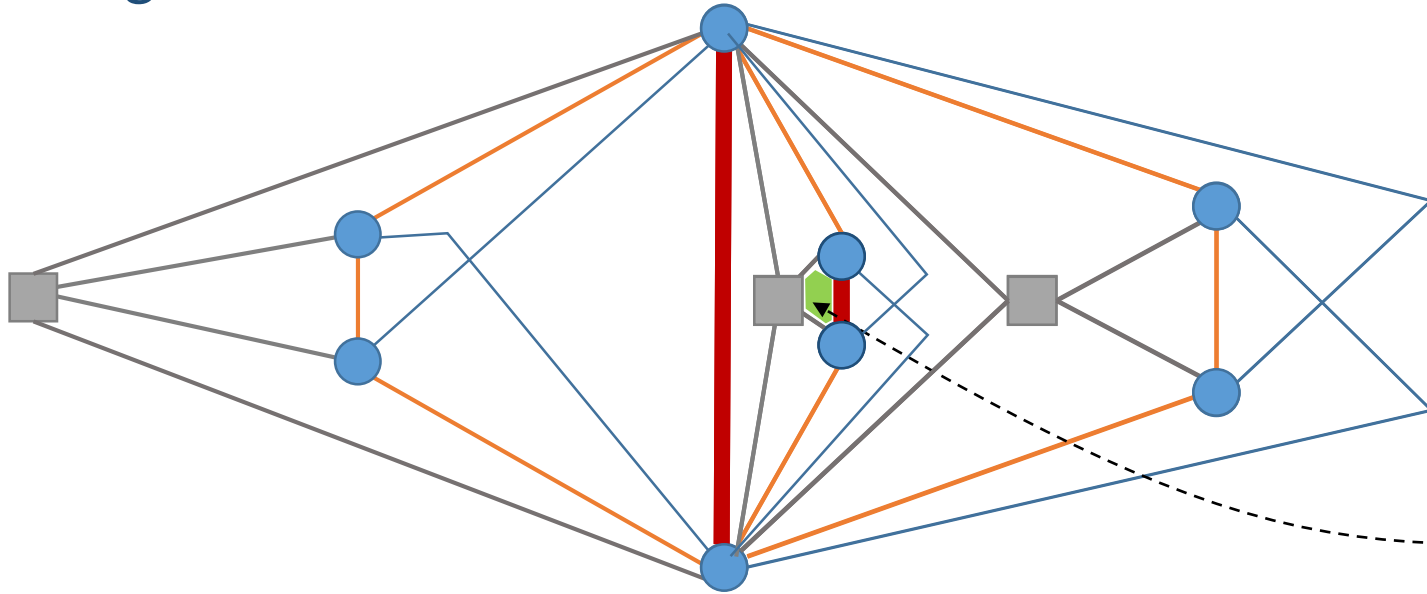
new partial
drawing



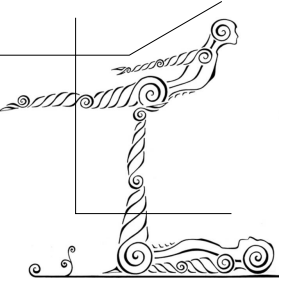


Drawing procedure

new partial
drawing



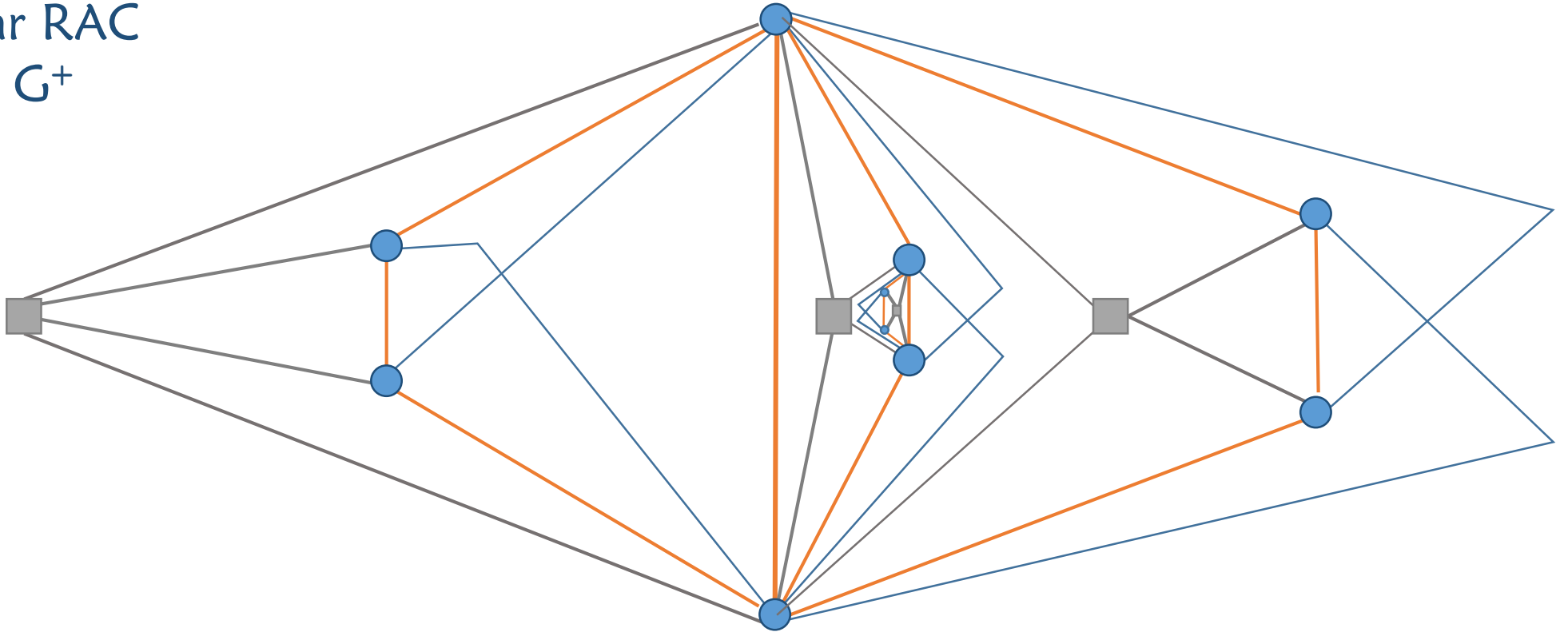
draw it
as usual

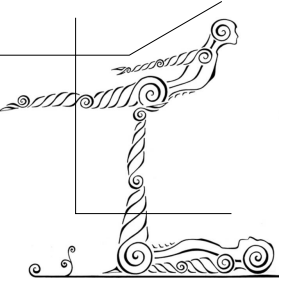


Drawing procedure

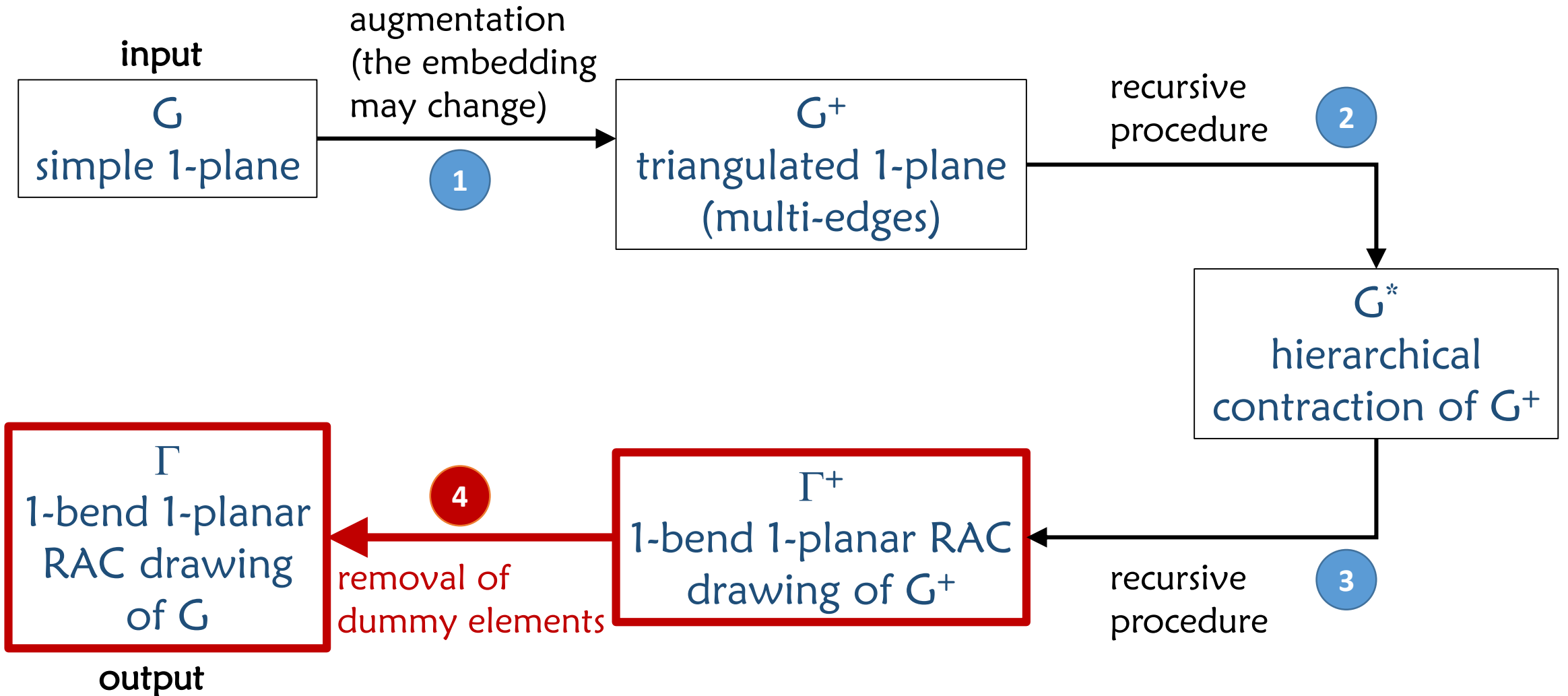
Γ^+

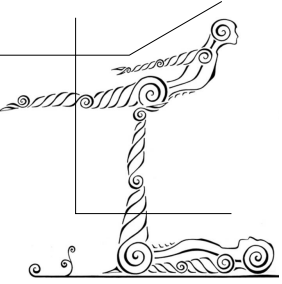
1-bend 1-planar RAC
drawing of G^+





Algorithm Outline

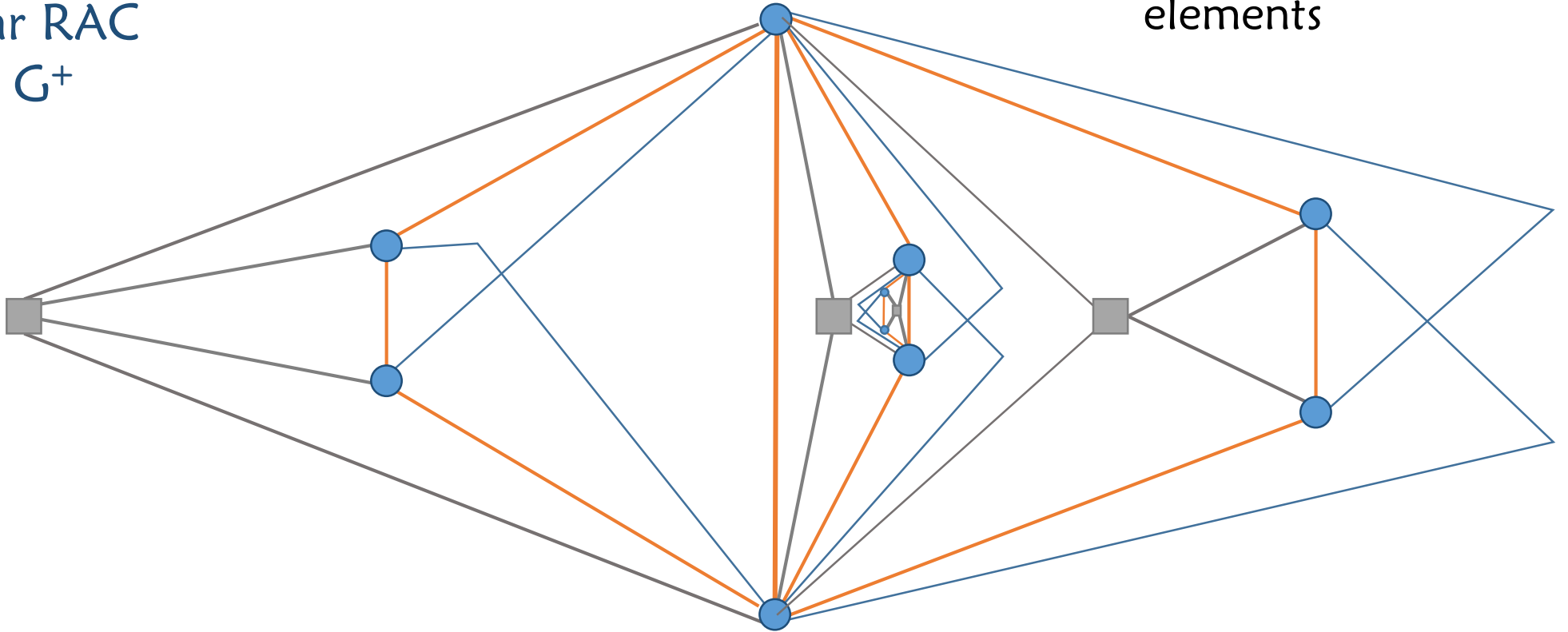


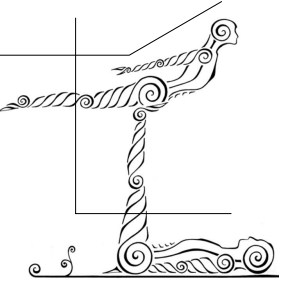


Drawing procedure

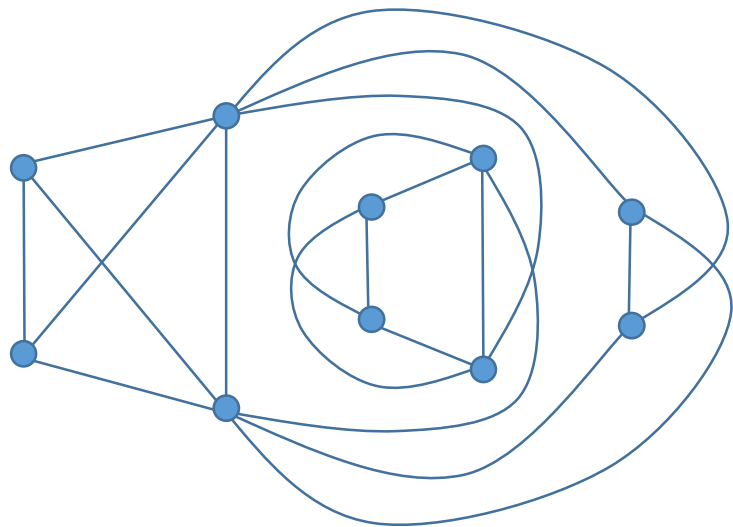
Γ^+
1-bend 1-planar RAC
drawing of G^+

remove
dummy
elements

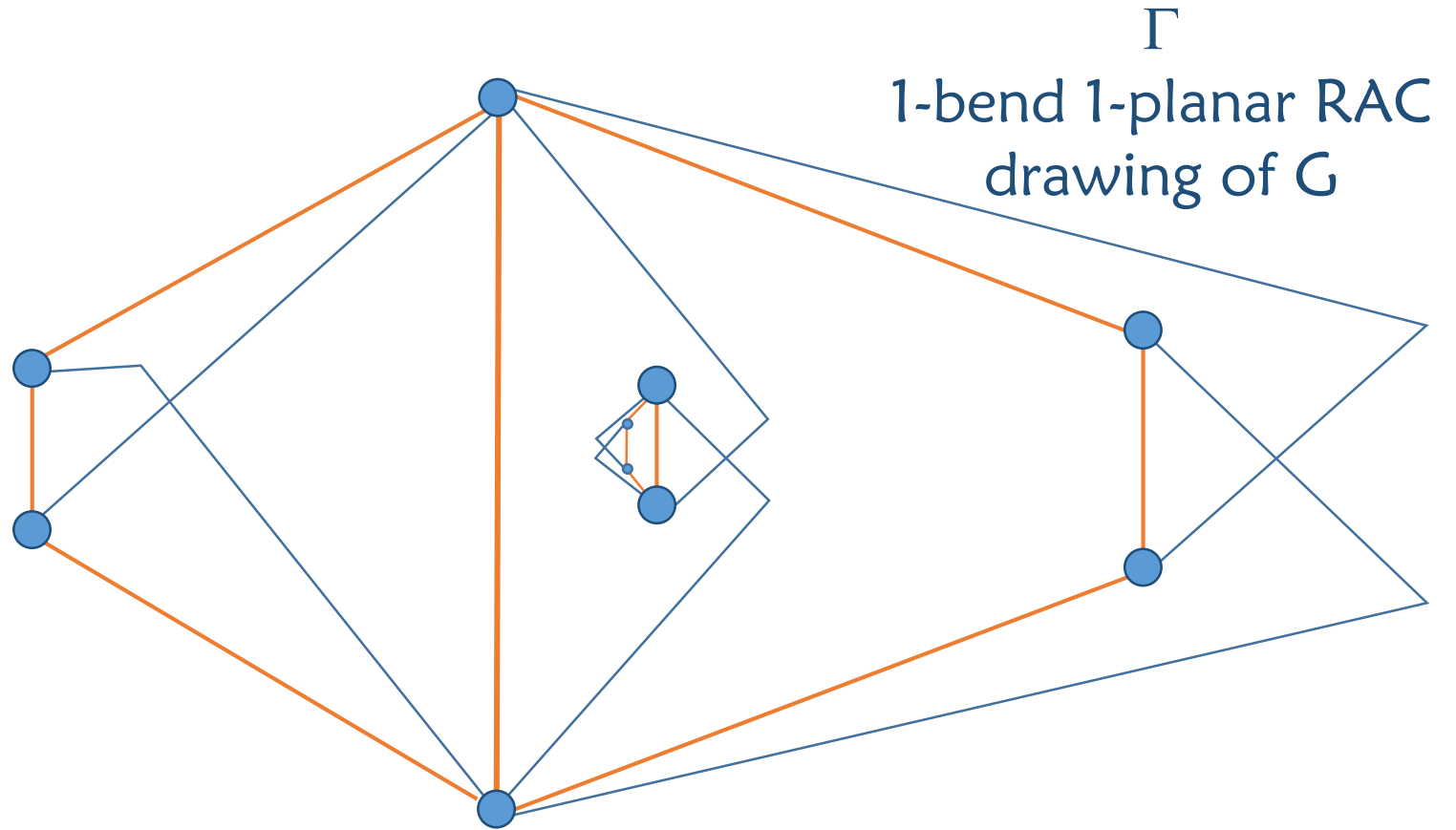




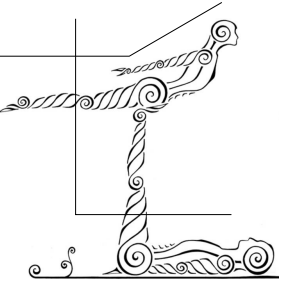
Drawing procedure



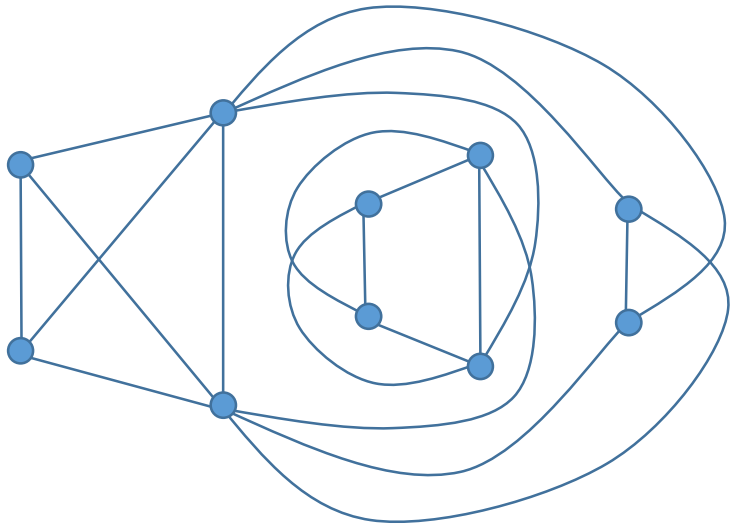
input graph G



Γ
1-bend 1-planar RAC
drawing of G

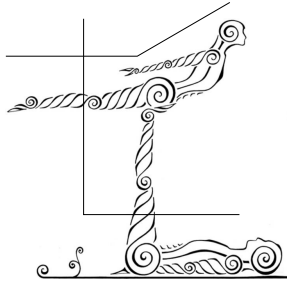


Drawing procedure



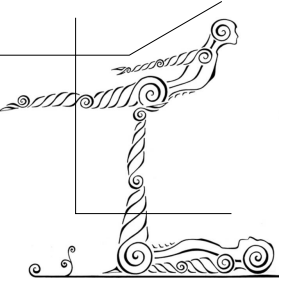
input graph G





Open problems

- Our algorithm may give rise to drawings with exponential area:
is such an area necessary in some cases?
- Our algorithm is allowed to change the initial embedding:
What if we cannot?
- Still missing:
Characterization of straight-line 1-planar RAC graphs



Thank you

