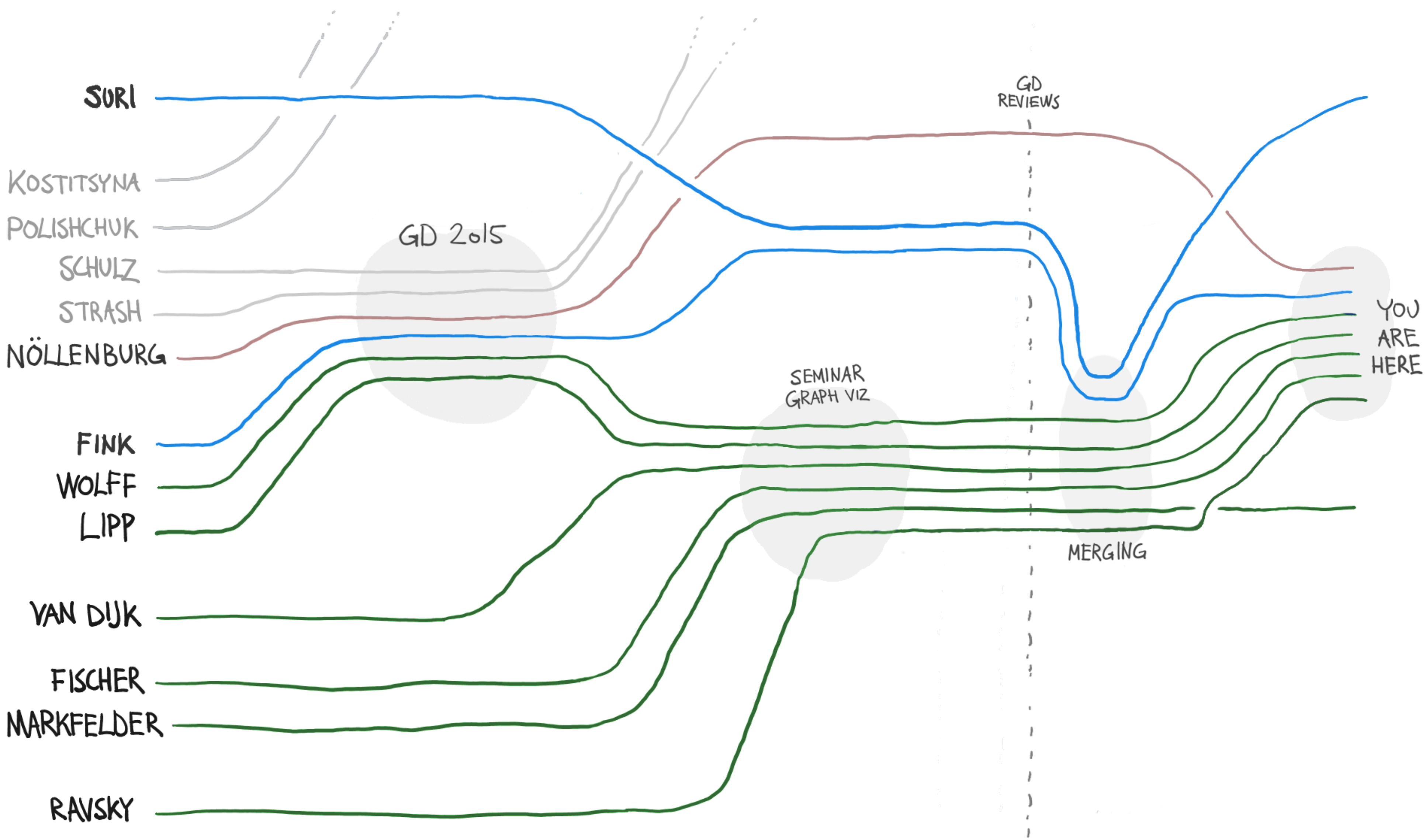
Block Crossings in Storyline Visualizations

Thomas van Dijk, *Martin Fink*, Norbert Fischer, Fabian Lipp, Peter Markfelder, Alexander Ravsky, Subhash Suri, and Alexander Wolff


SURI
KOSTITSYNA
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FINK
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VAN DIJK
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MARKFELDER
RAVSKY
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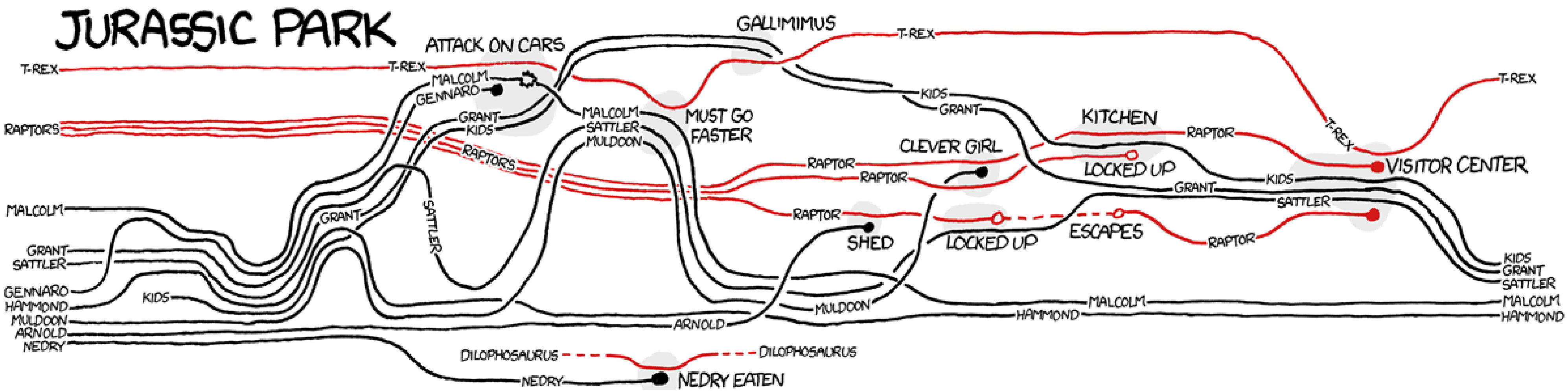
JURASSIC PARK
T-REX
RAPTORS
MALCOLM
GRANT
SATTLER
GENNARO
HAMMOND
MULDOON
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NEDRY
KIDS
ATTACK ON CARS
GENNARO
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MUST GO FASTER
DILOPHOSAURUS
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CLEVER GIRL
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KITCHEN
ESCAPES
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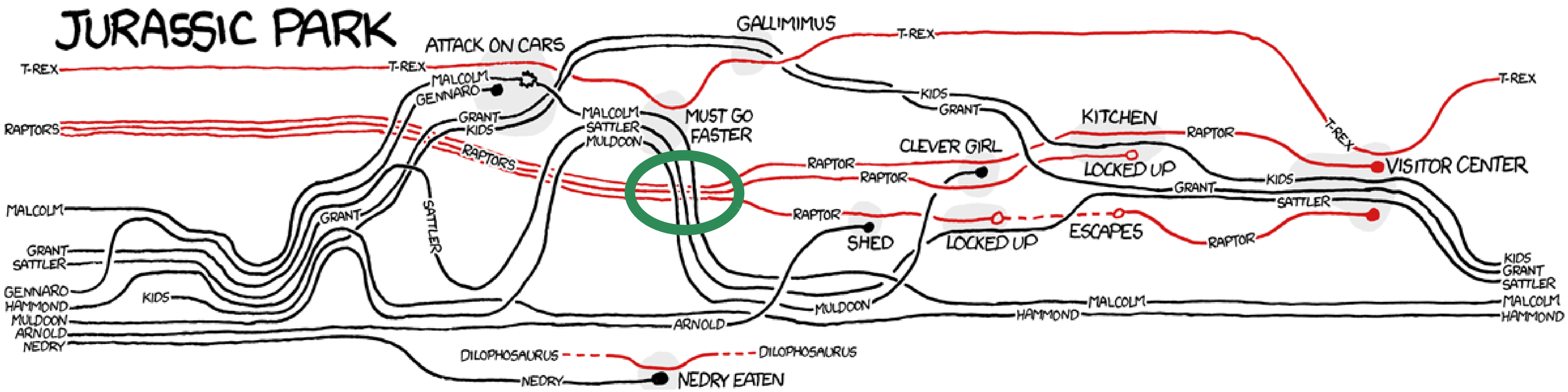
HAMMOND



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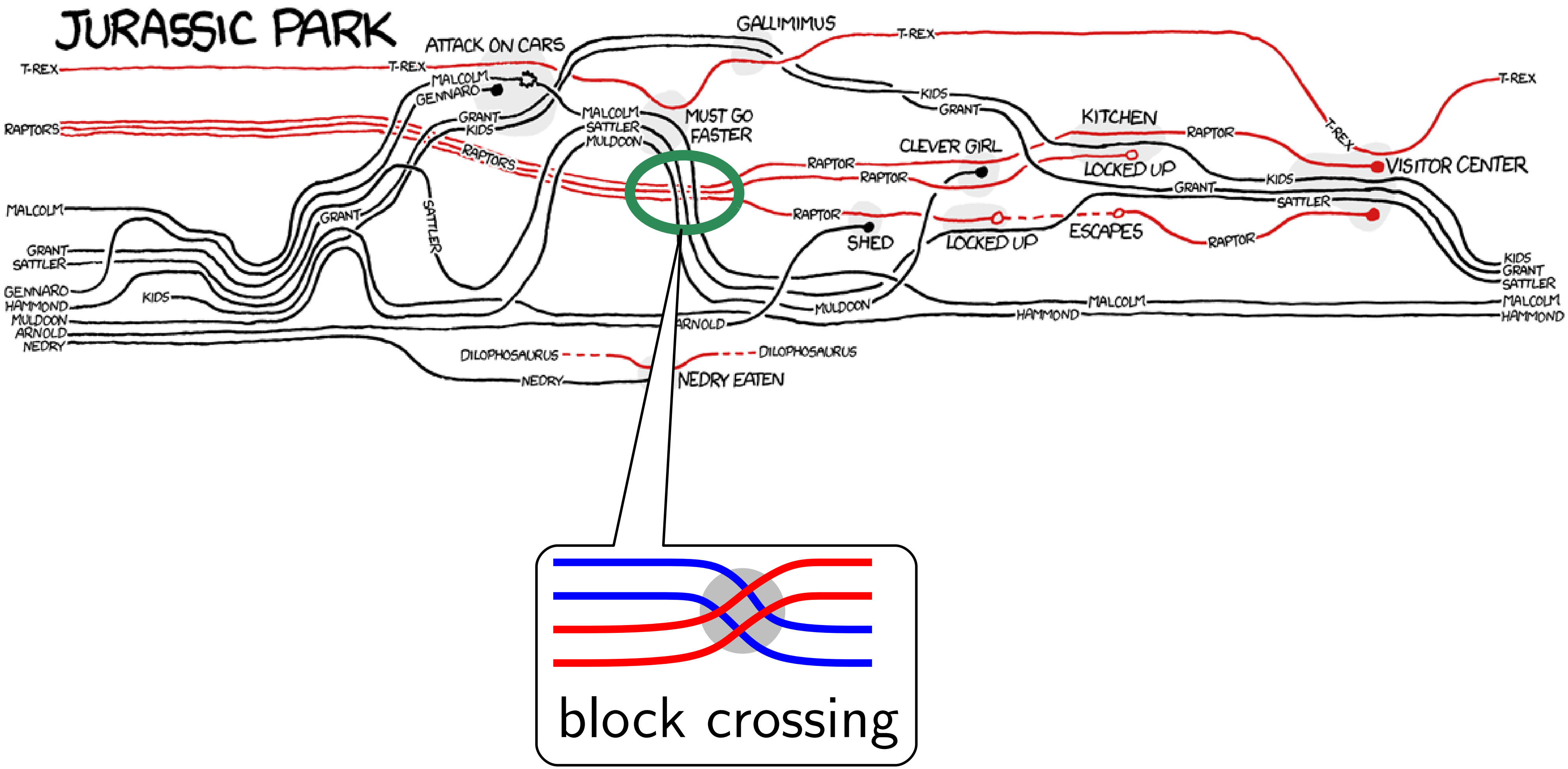
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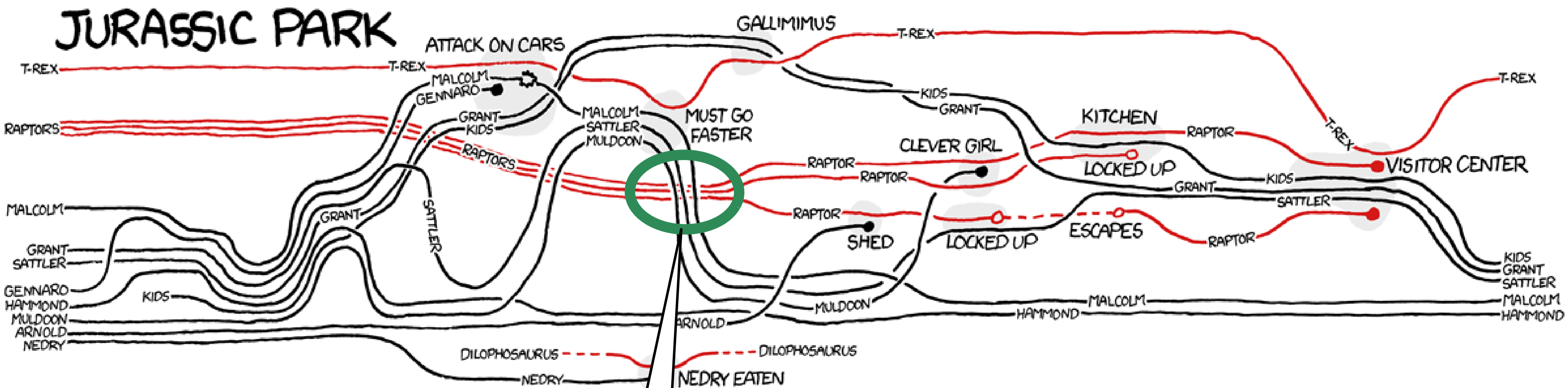
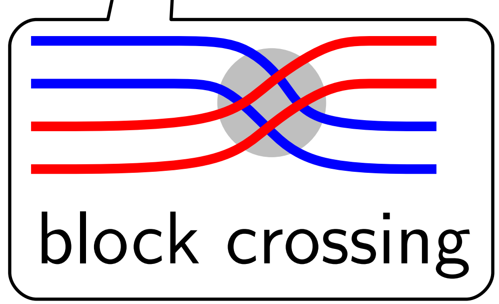
block crossing



We want to minimize block crossings!



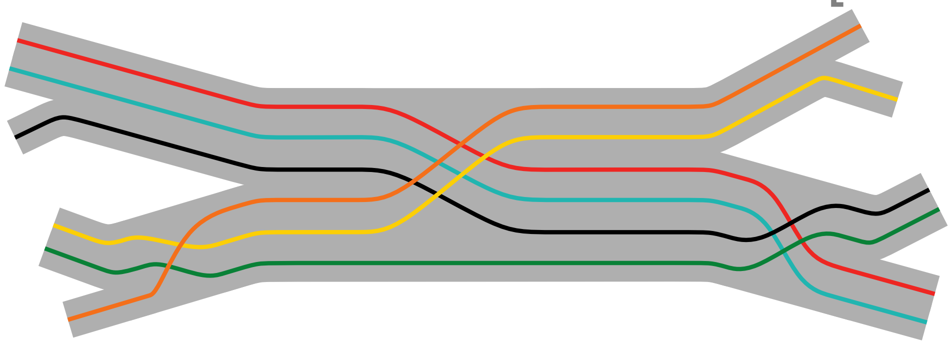
Previous Results – Simple Crossings

[Kostitsyna et al, GD'15]

- NP-hardness
- FPT for #characters
- upper and lower bounds for some cases with pairwise meetings

Related Work

- Block crossings for metro lines [Fink, Pupyrev, Wolff; 2015]



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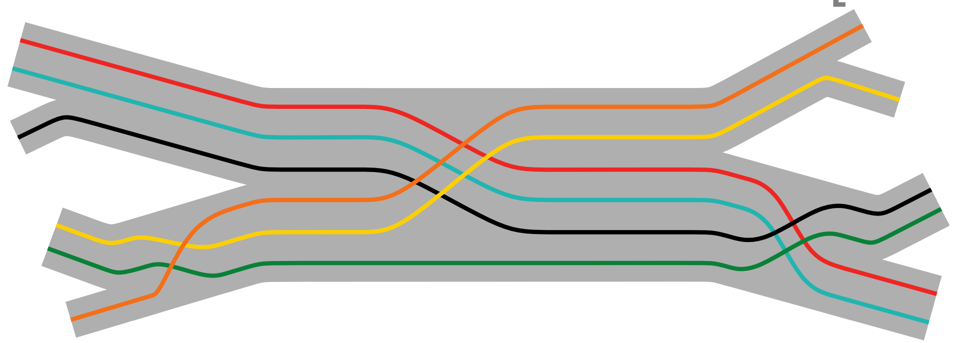
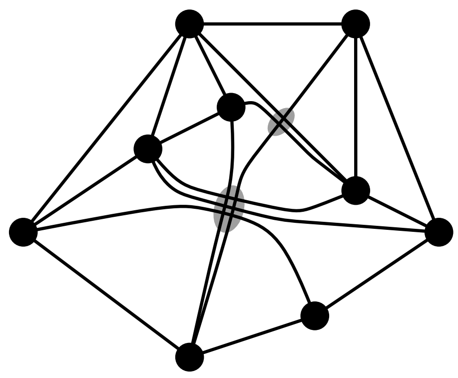


- Bundled Crossings [Fink et al., 2016]



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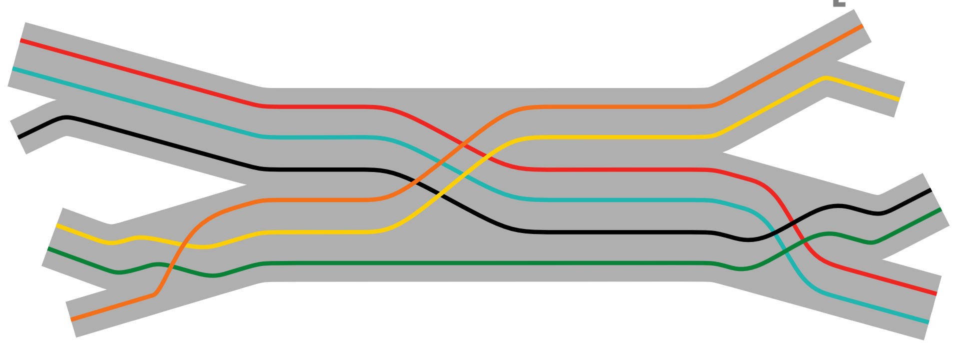

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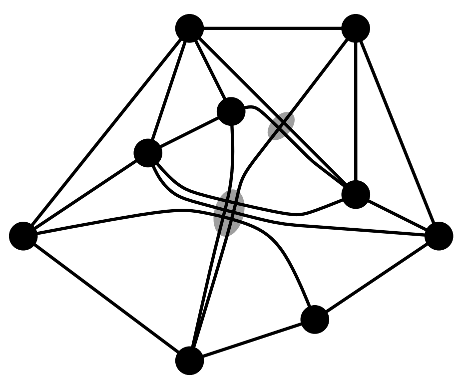
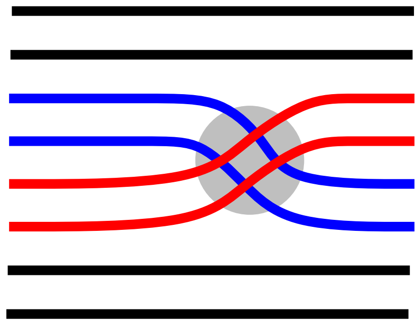


- Bundled Crossing Number [Alam, Fink, Pupyrev; next talk]

Storylines & Block Crossings

- block crossing



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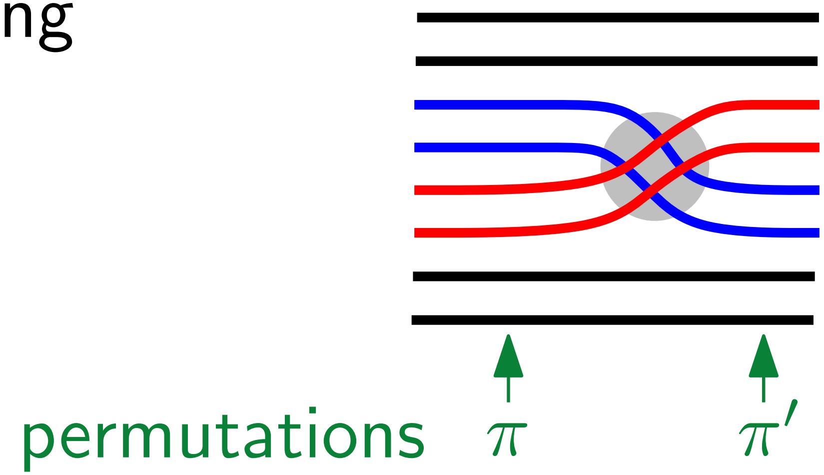

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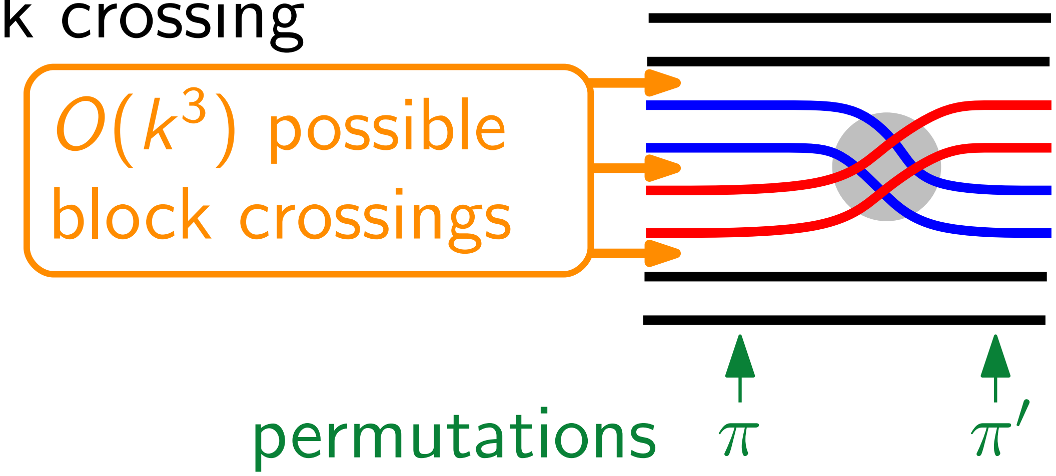

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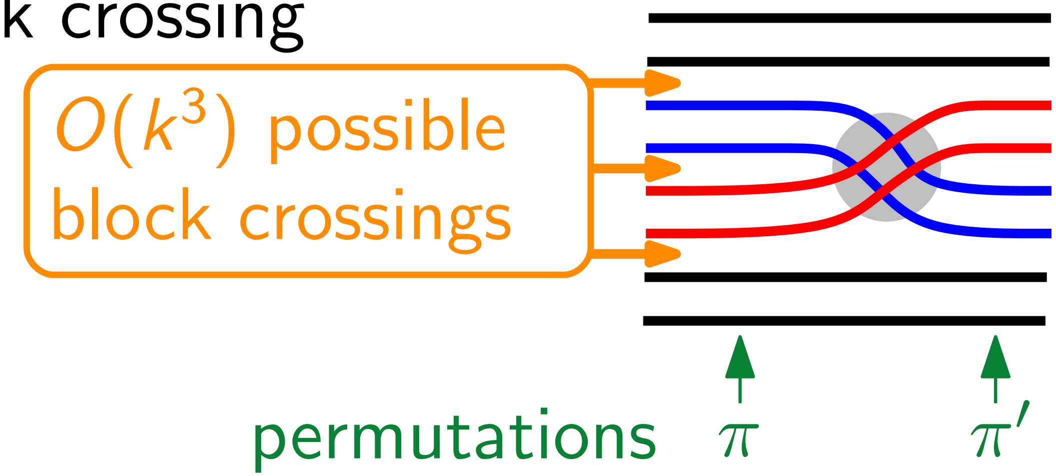

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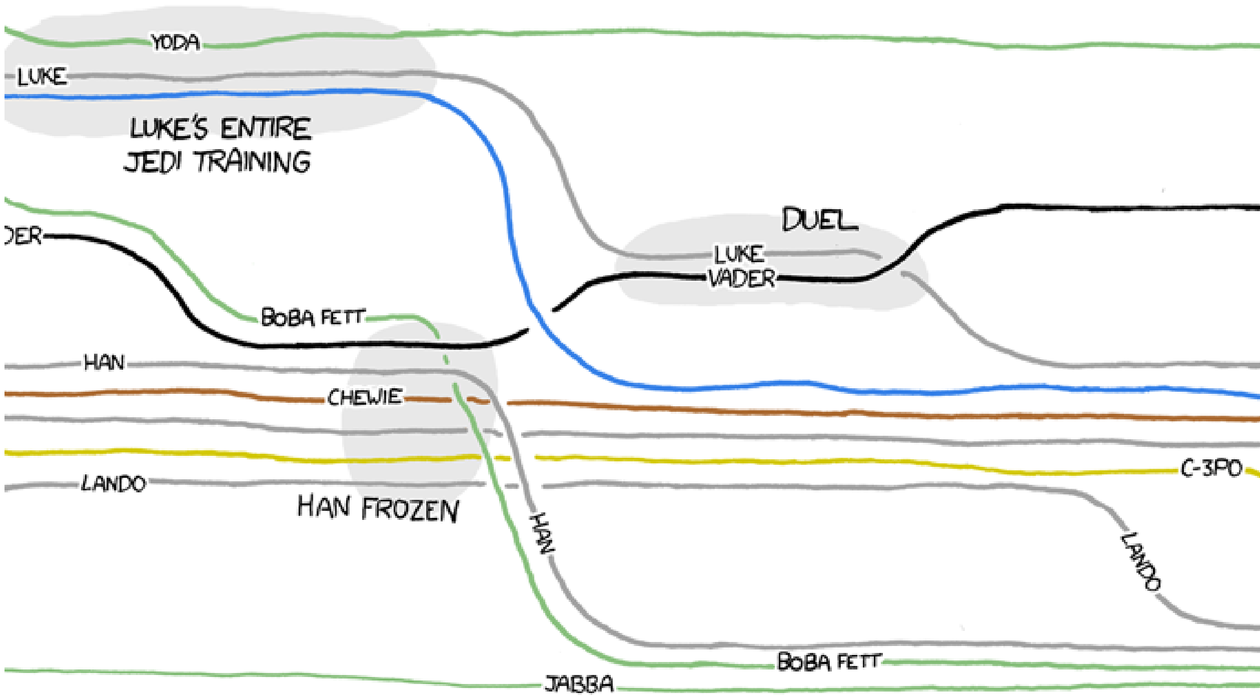

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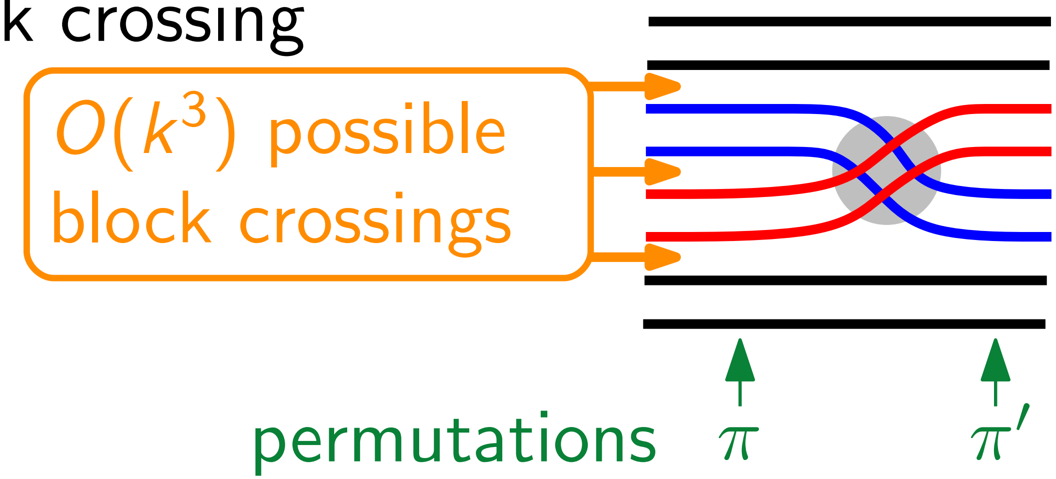

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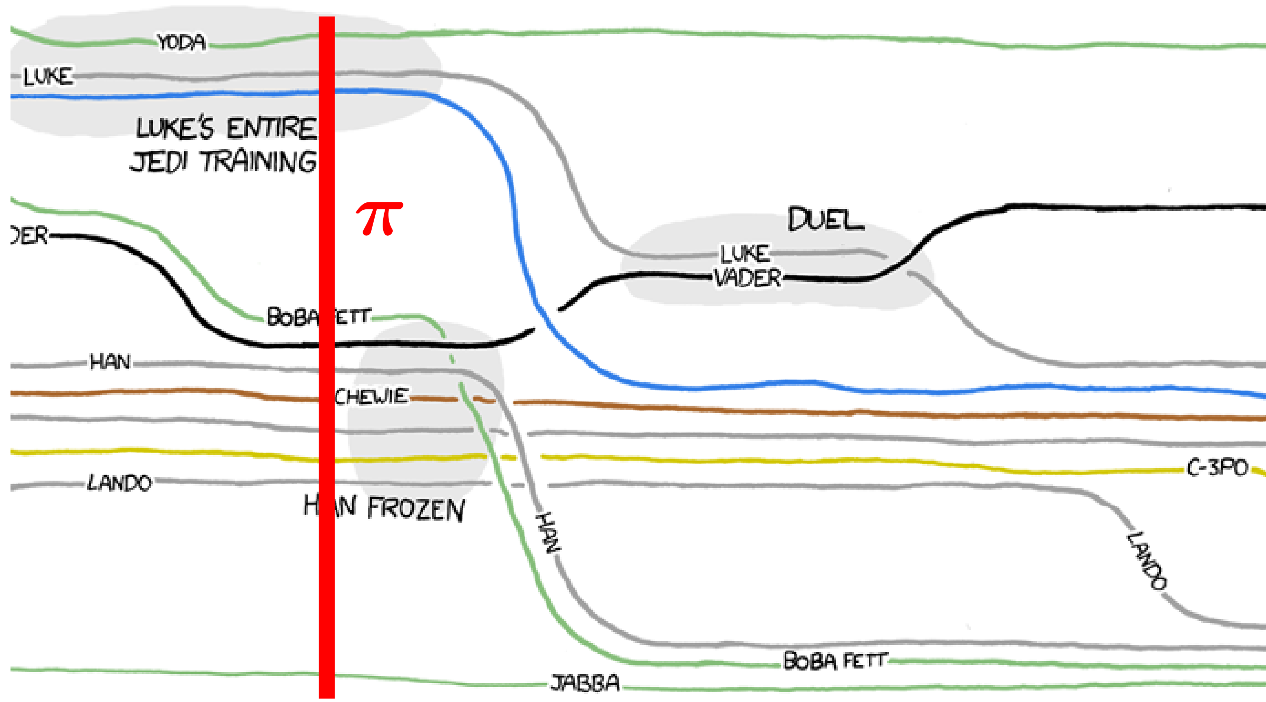

Storylines & Block Crossings

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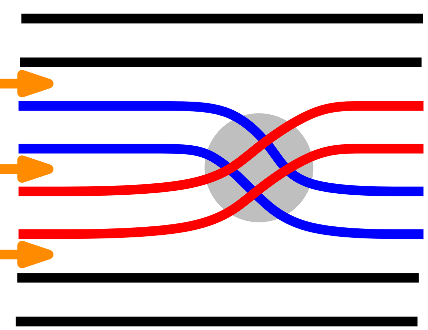
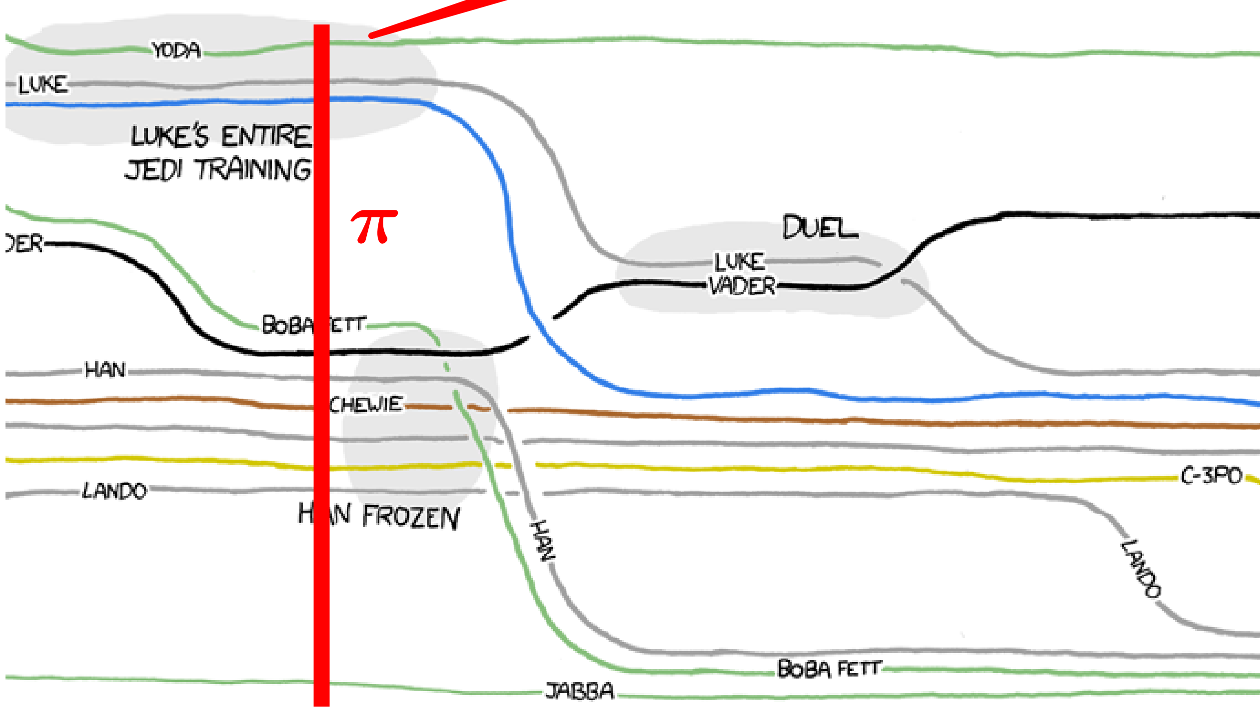
$O(k^3)$ possible block crossings



permutations π π'

π **supports** meeting

- storyline visualization



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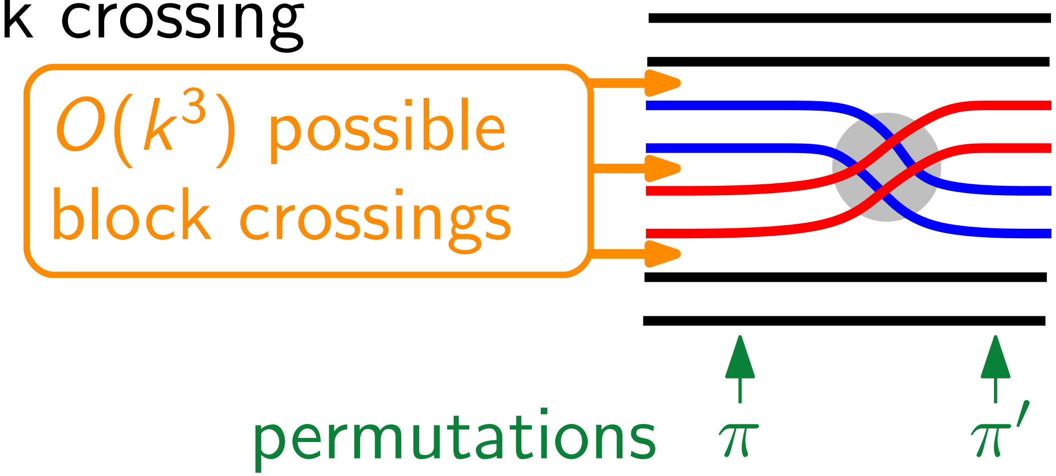

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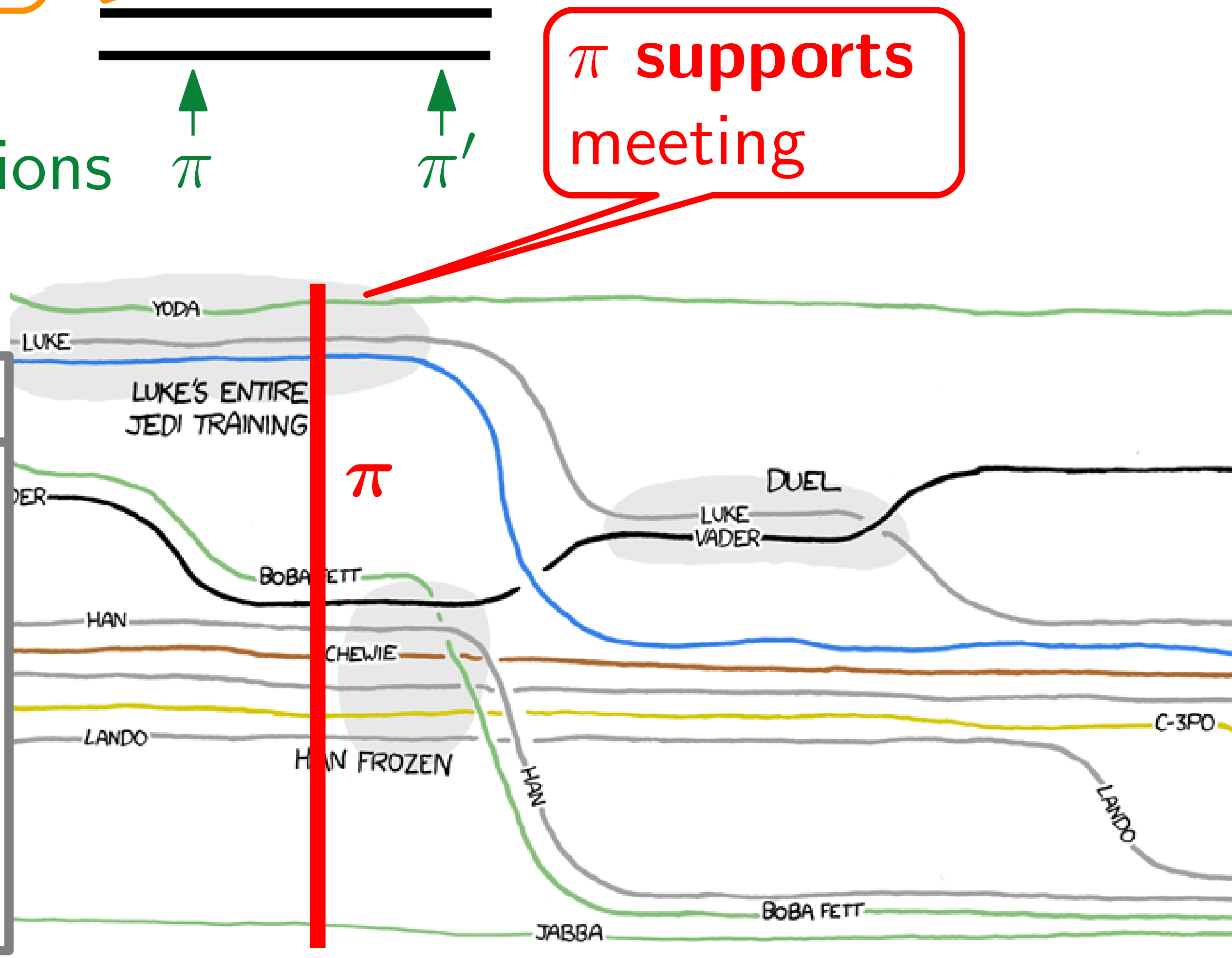

Problem definition:

Storylines & Block Crossings

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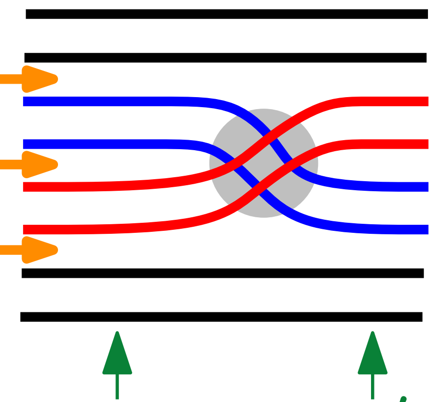

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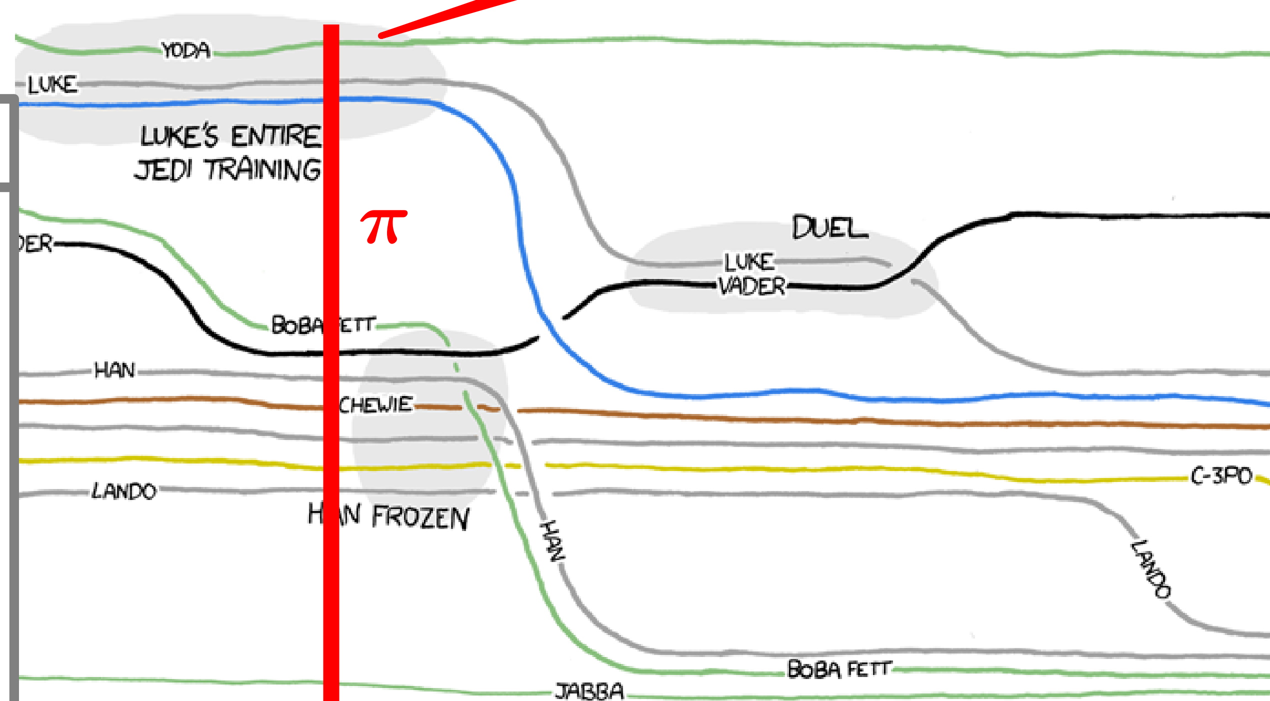

Problem definition:
Given n meetings of k characters, find permutations transformed by min. # block crossings.

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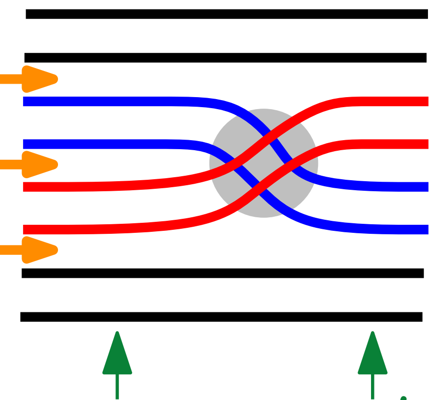

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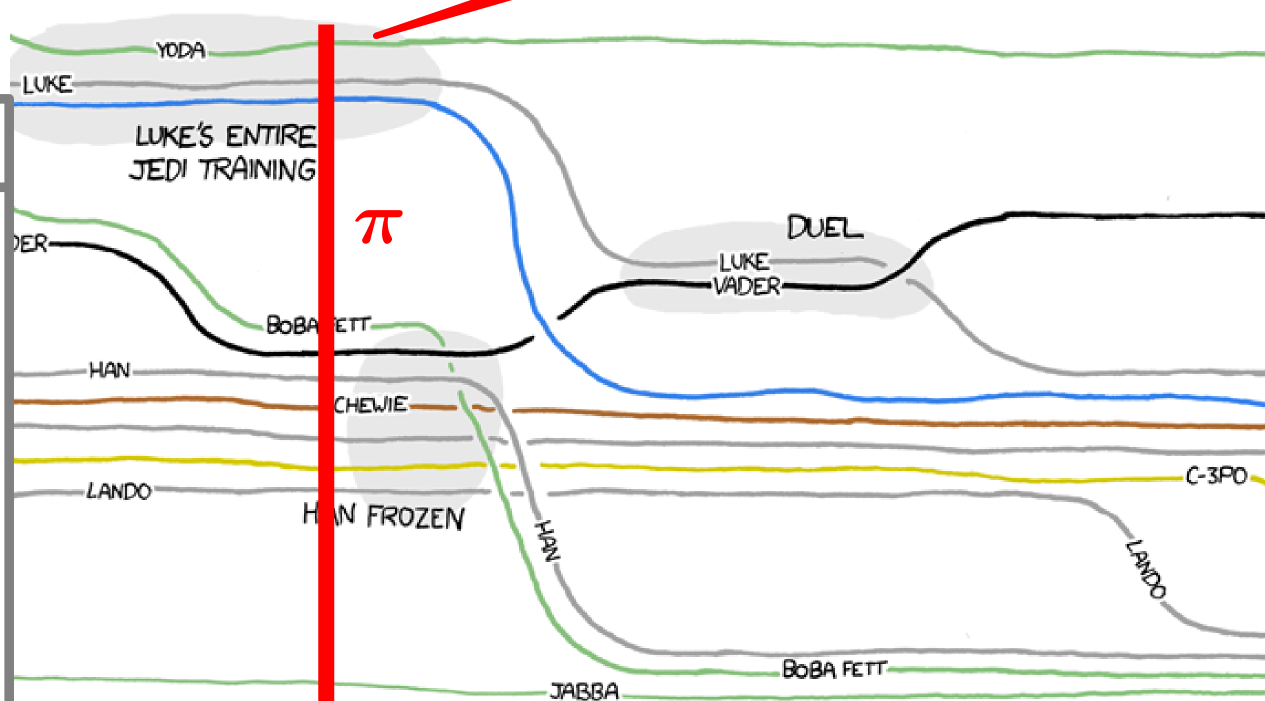

Problem definition:

Given n meetings of k characters, find permutations transformed by min. # block crossings. (Must support all meetings.)

Our Results

- recognize crossing-free instances
- NP-hardness
- approximation
- FPT/exact algorithms
- greedy heuristic for pairwise meetings

Crossing-Free Storylines Visualizations

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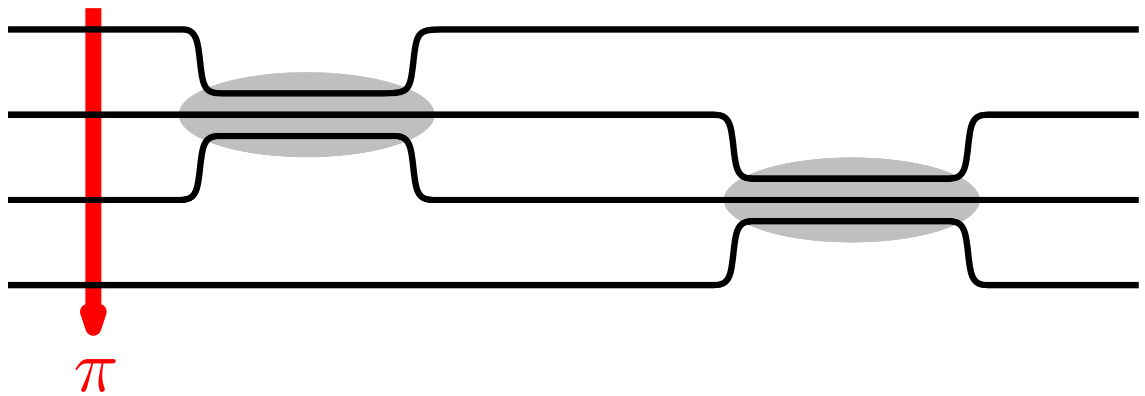

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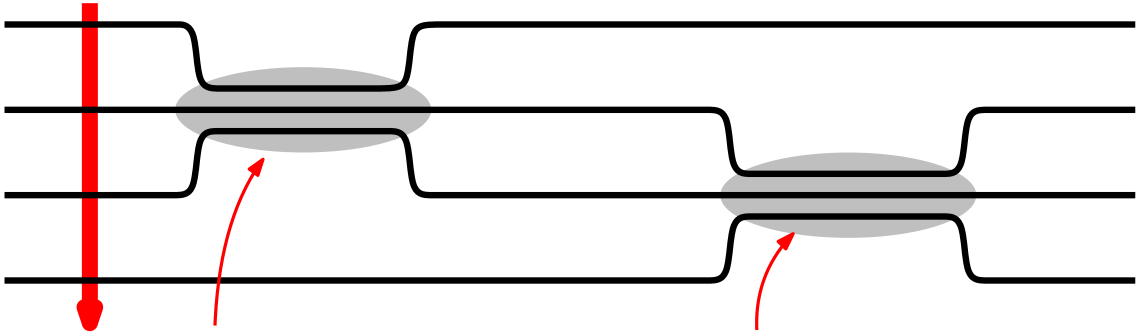

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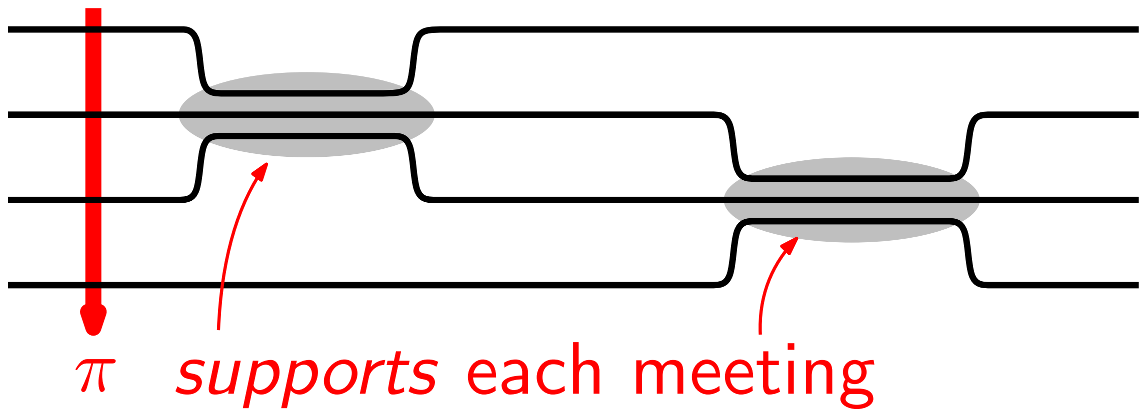

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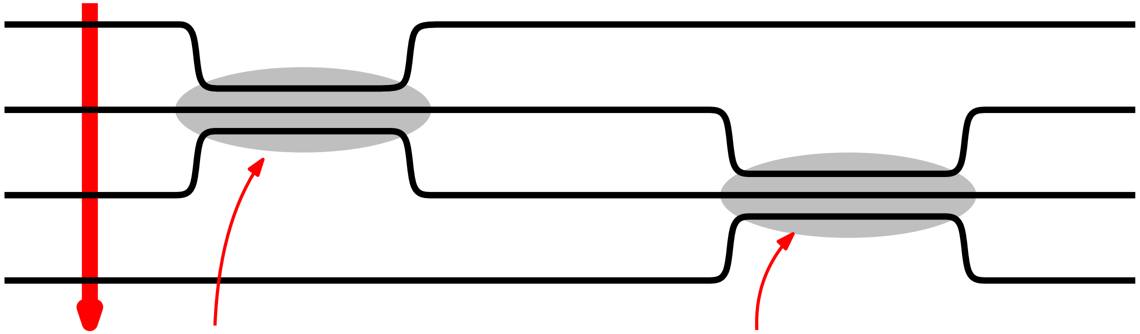

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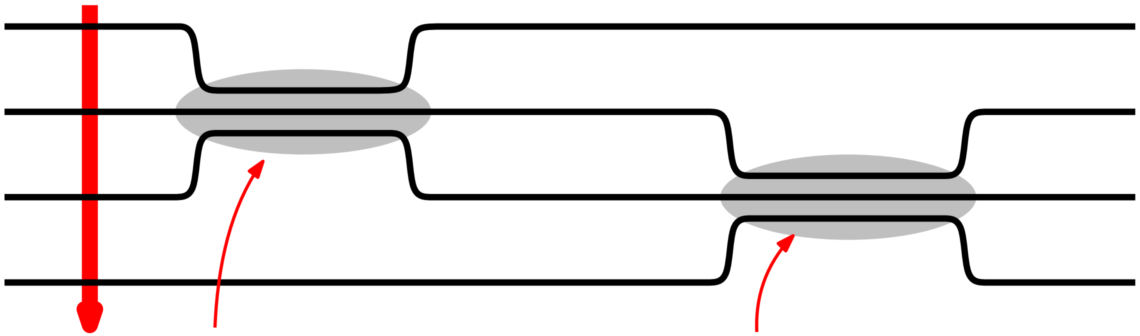

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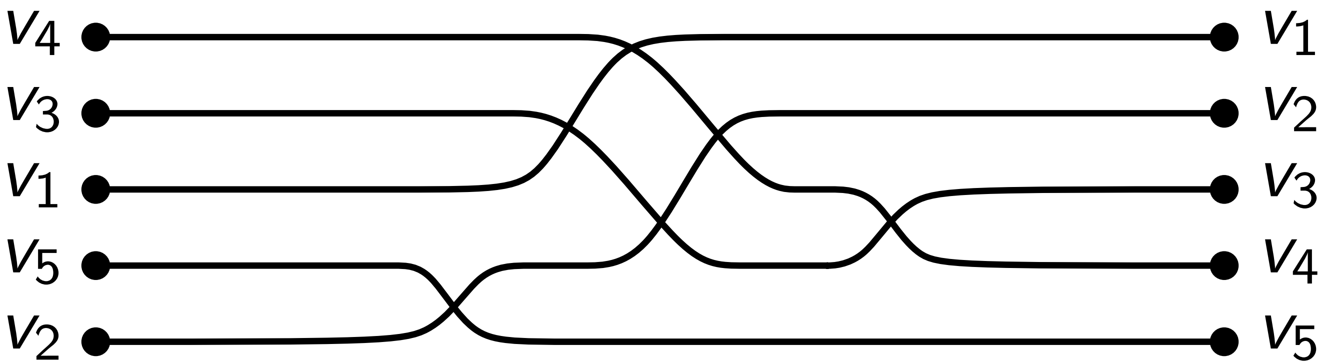
  groups that meet

- interval hypergraph property can be checked in $O(k^2)$ time

  [Trotter, Moore, 1976]

Minimizing Block Crossings is NP-hard

- Reduction from *Sorting by Transpositions*



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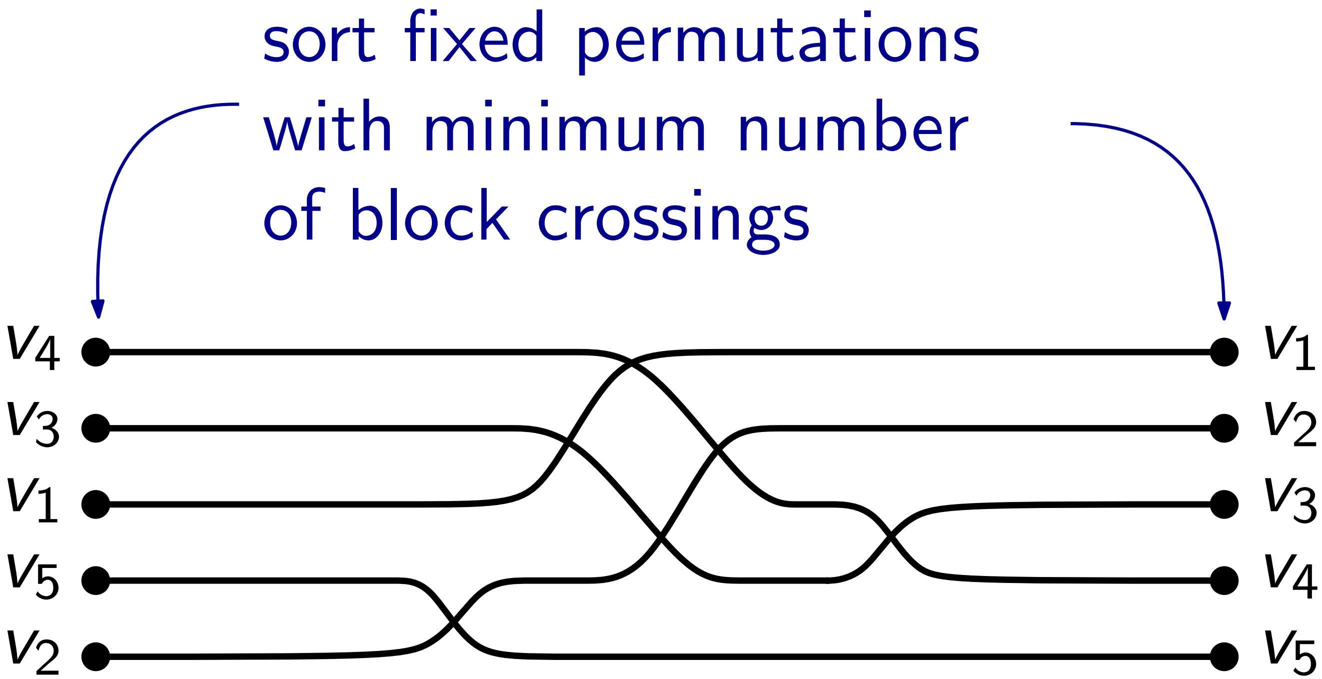

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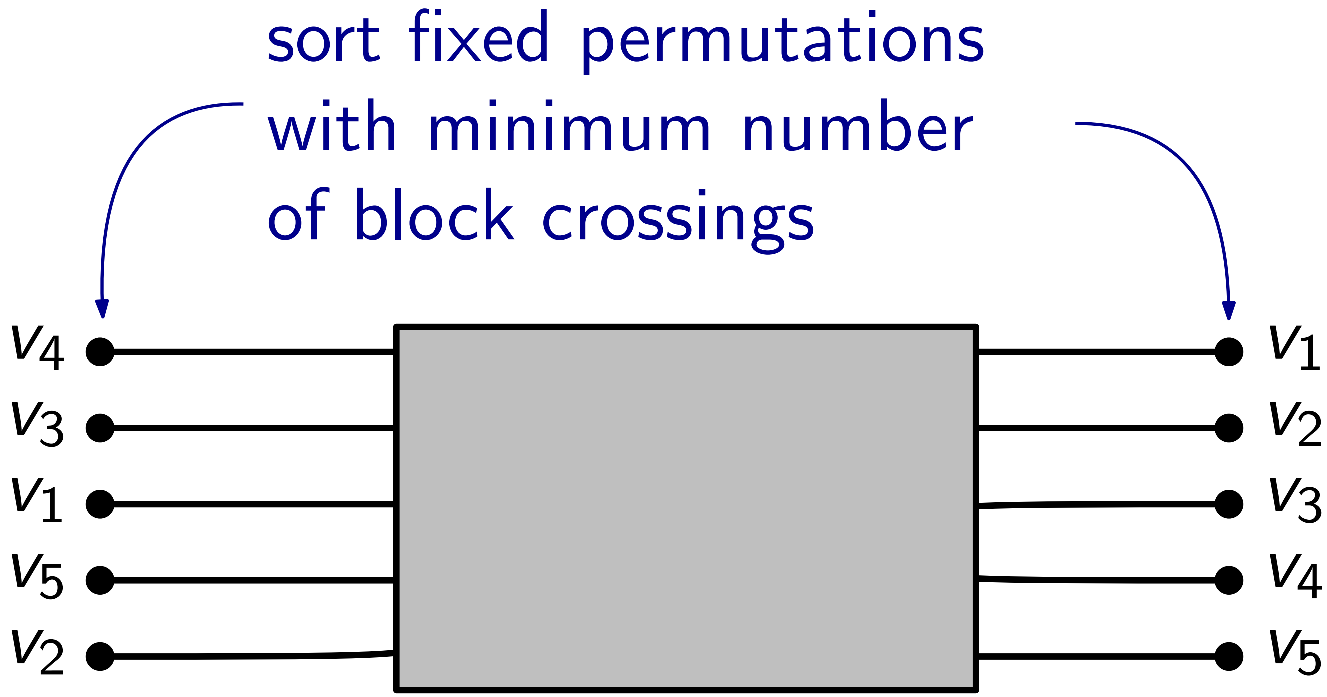

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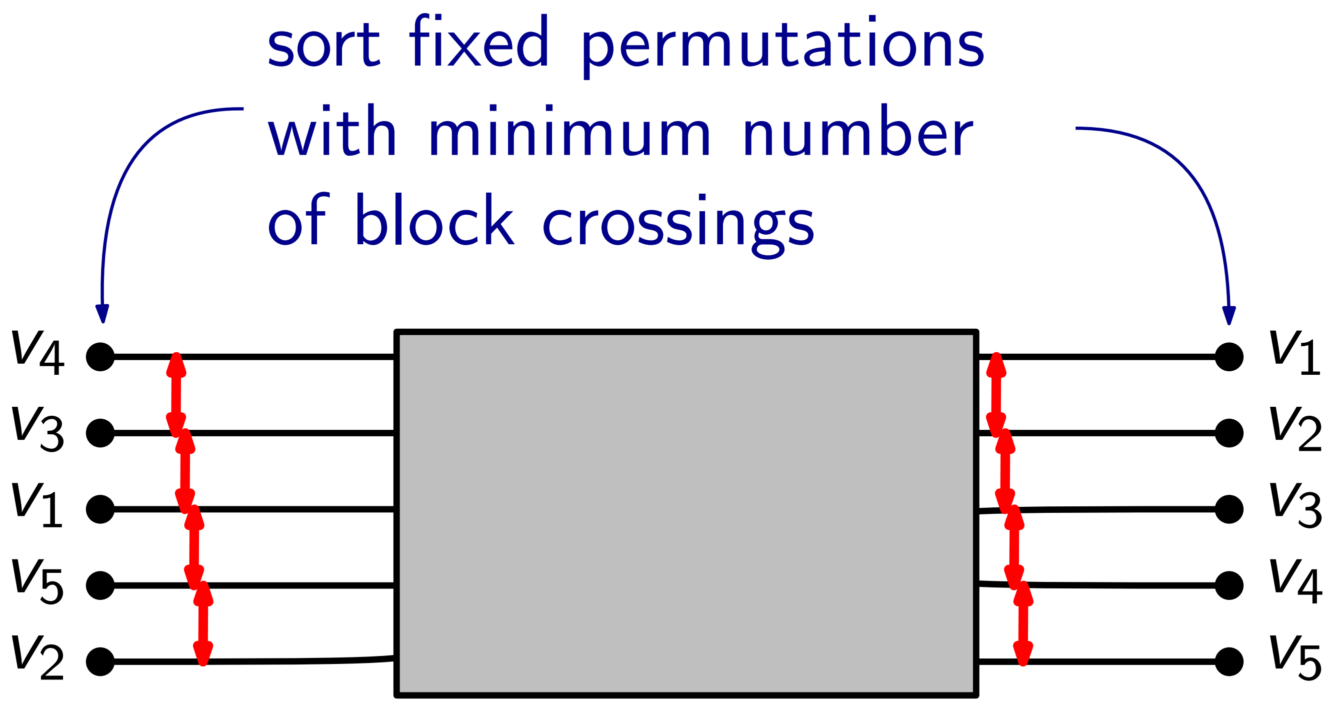

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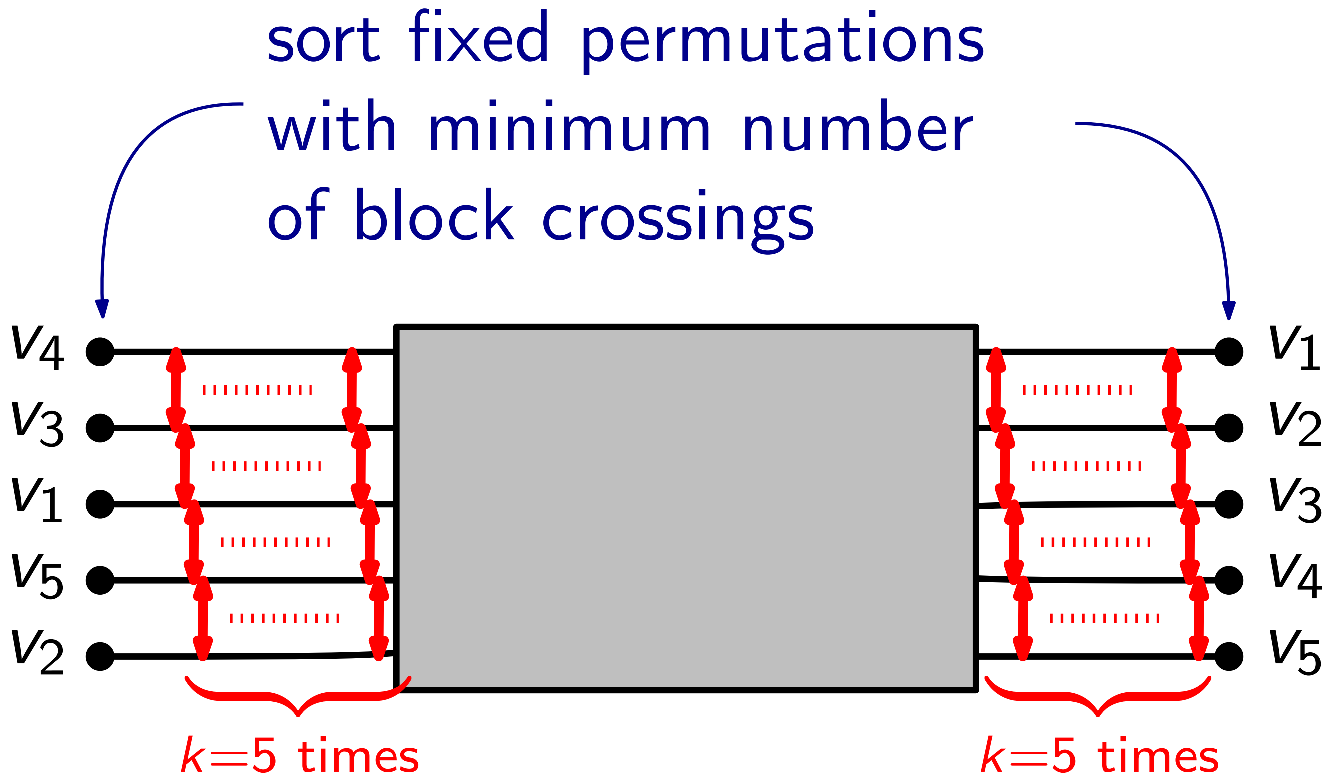

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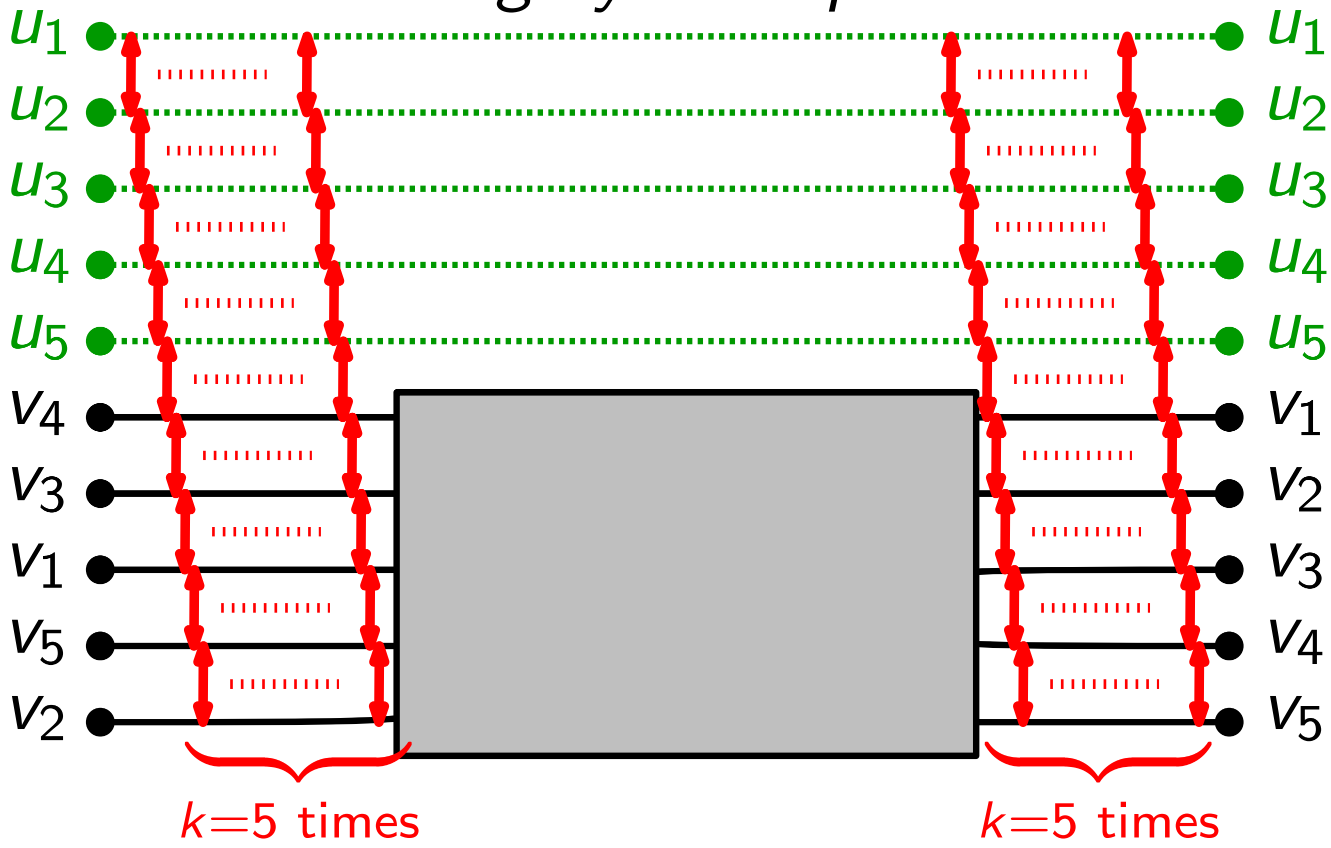

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Approximation Algorithm

- all meetings of size $\leq d$ (constant)
- no repeated meetings

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idea:

1. choose starting order π that supports many meetings
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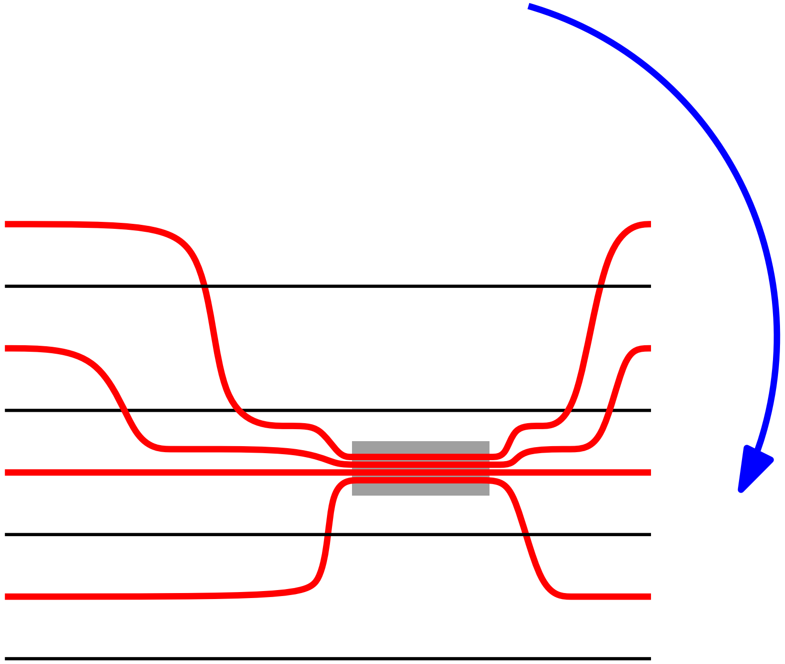

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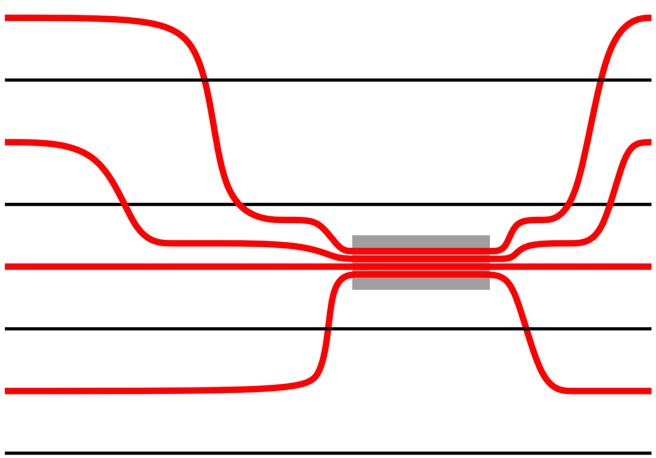

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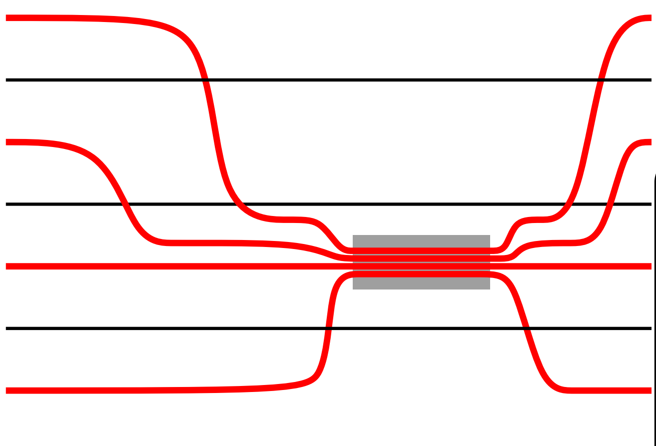

Lemma: starting order π has α unsupported meetings $\Rightarrow$ at least $4\alpha/(3d^2)$ block crossings necessary

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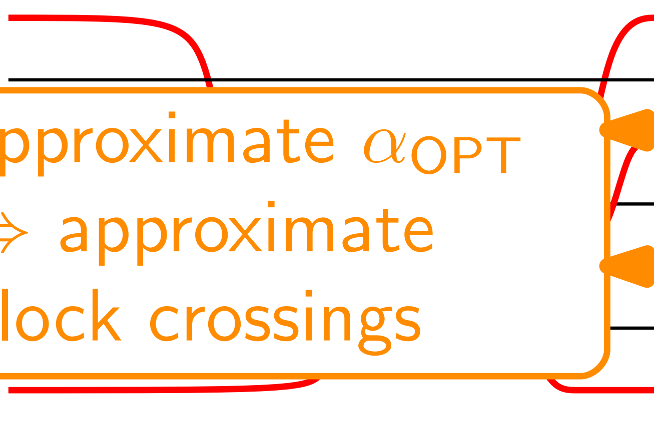

approximate α_{OPT} $\Rightarrow$ approximate block crossings

Lemma: starting order π has α unsupported meetings $\Rightarrow$ at least $4\alpha/(3d^2)$ block crossings necessary

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find π minimizing $\#$unsupported meetings

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$\leftrightarrow$ remove minimum #meetings so that storyline crossing-free

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Theorem: Interval Hypergraph Edge Deletion admits a $(d+1)$-approximation (constant rank d).

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Theorem: INTERVAL HYPERGRAPH EDGE DELETION admits a $(d+1)$-approximation (constant rank d).

Theorem: We can find a $(3(d^2-1)d^2/2)$-approximation for the minimum number of block crossings in storyline visualizations in $O(kn)$ time.

Interval Hypergraph Edge Deletion

- Remove minimum number of hyperedges so that $\mathcal{H} = (V, E)$ becomes interval hypergraph

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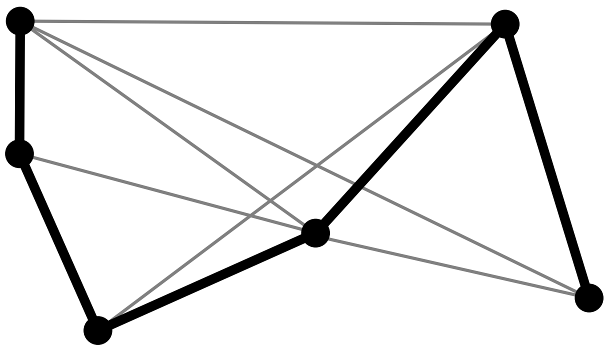
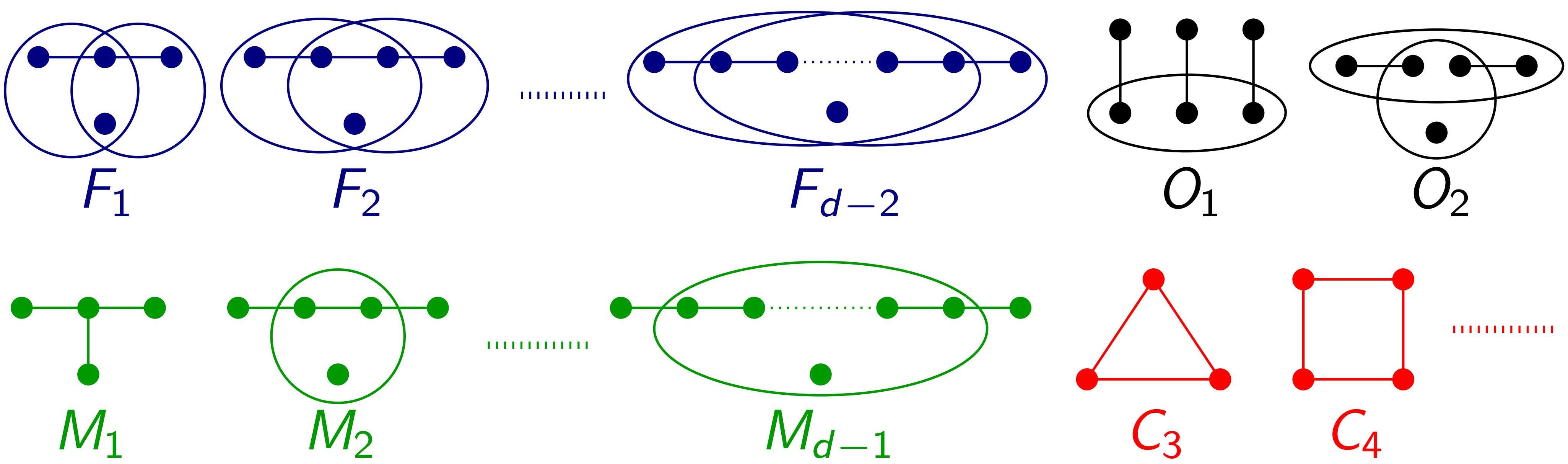
NP-hard for graphs:
remove all but $n - 1$ edges $\rightarrow$
Hamiltonian path



Interval Hypergraph Edge Deletion

- Remove minimum number of hyperedges so that $\mathcal{H} = (V, E)$ becomes interval hypergraph
- characterization of interval hypergraphs by forbidden subhypergraphs



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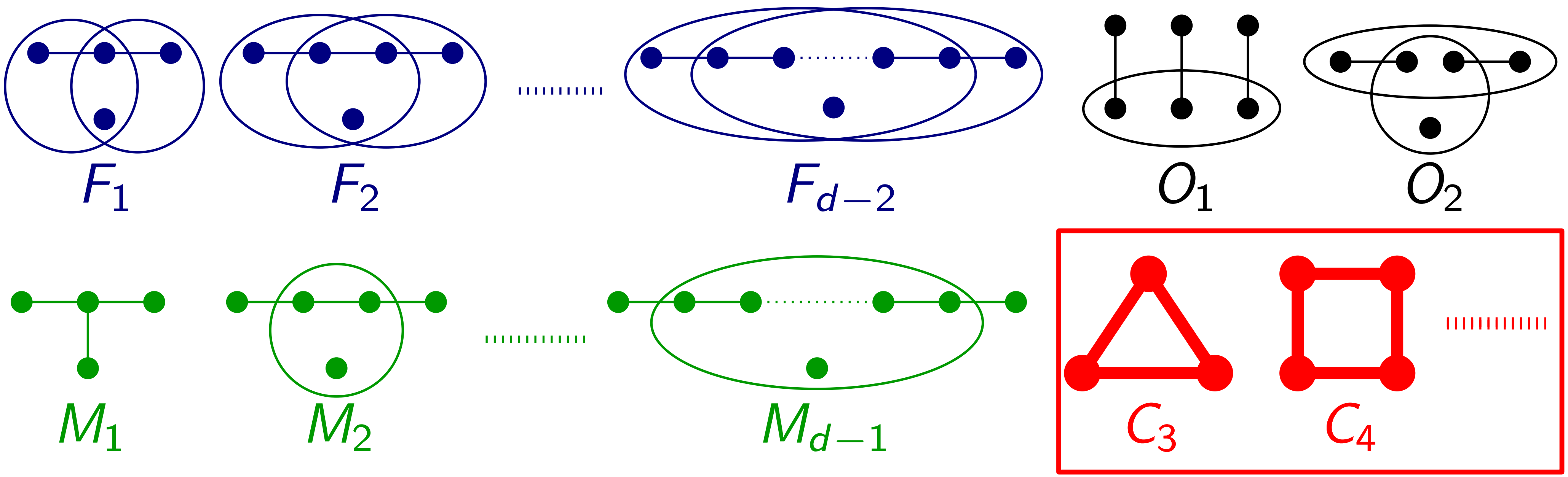
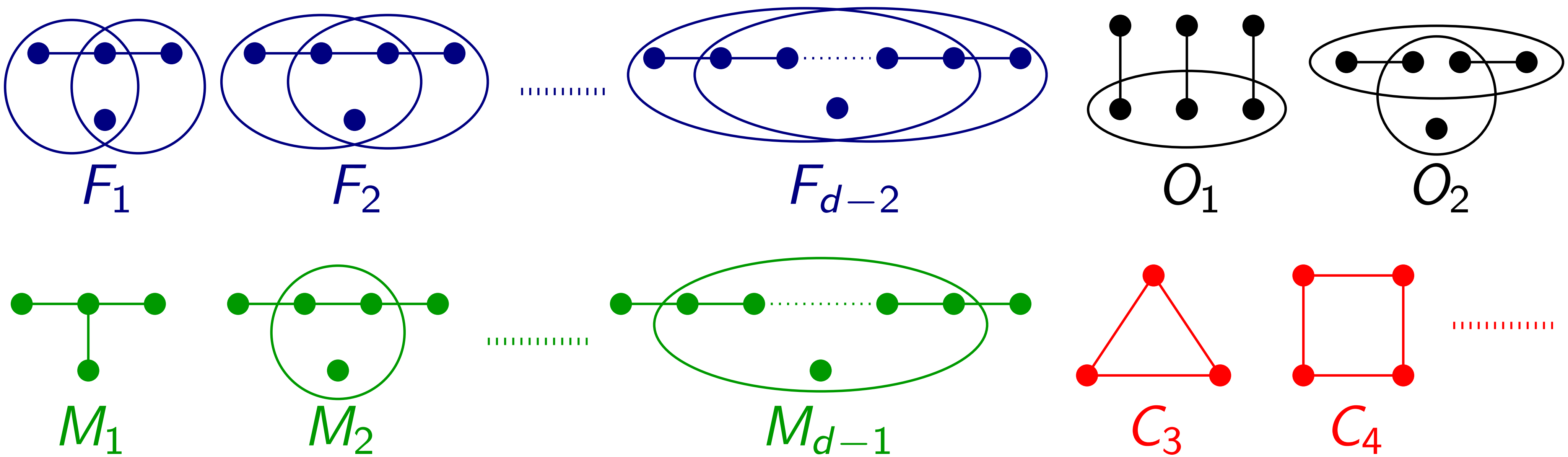


Interval Hypergraph Edge Deletion

- Rem
  $\mathcal{H} =$
- char … len
  subh

outline:
– iteratively: search for forbidden subhypergraphs except $C_{d+2}, \ldots$ & completely remove them

– result: cyclic generalization of interval hypergraph; break optimally



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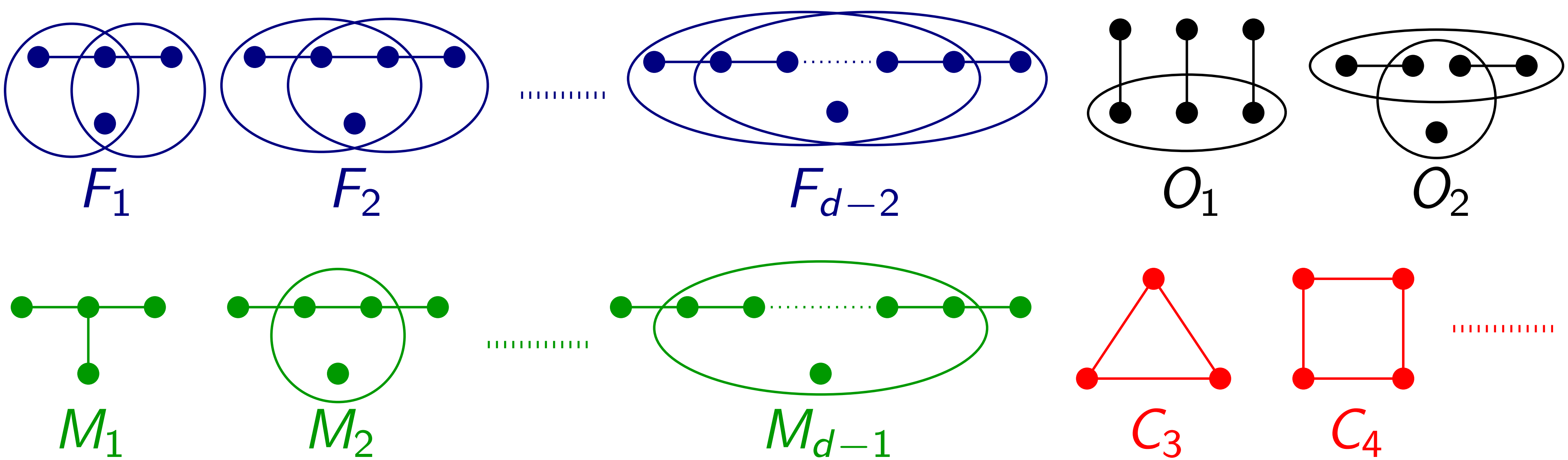

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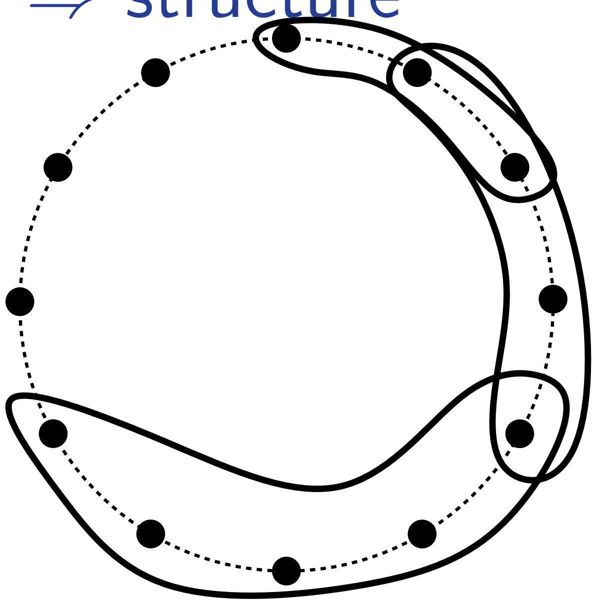
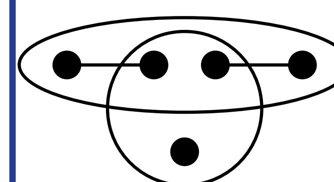
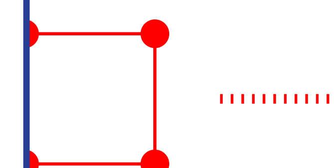


proof skipped (several lemmas & case distinctions)



O_2



C_4

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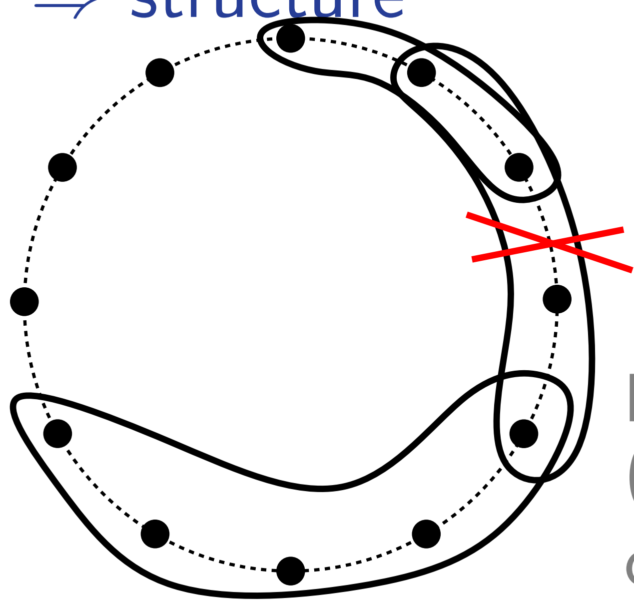
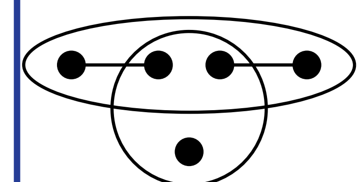
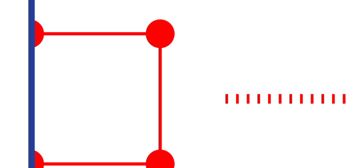


can cut optimally

proof skipped (several lemmas & case distinctions)



O_2



C_4

Exact Algorithm

- first idea: modify FPT of Kostitsyna et al. for block crossings

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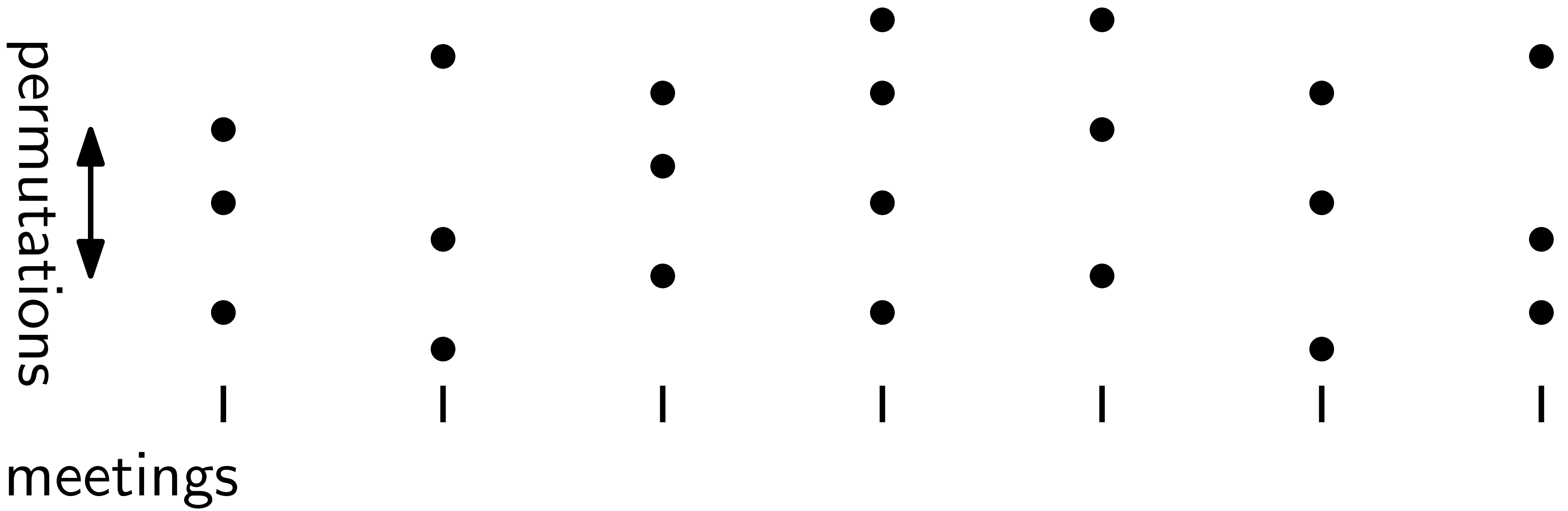

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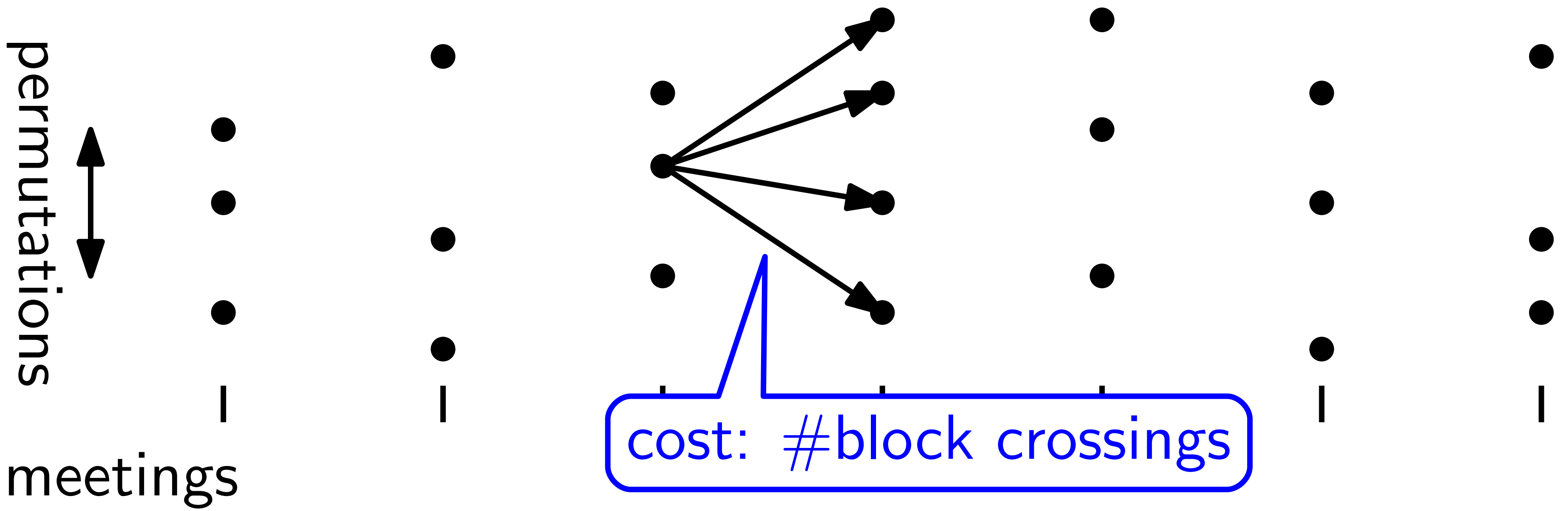

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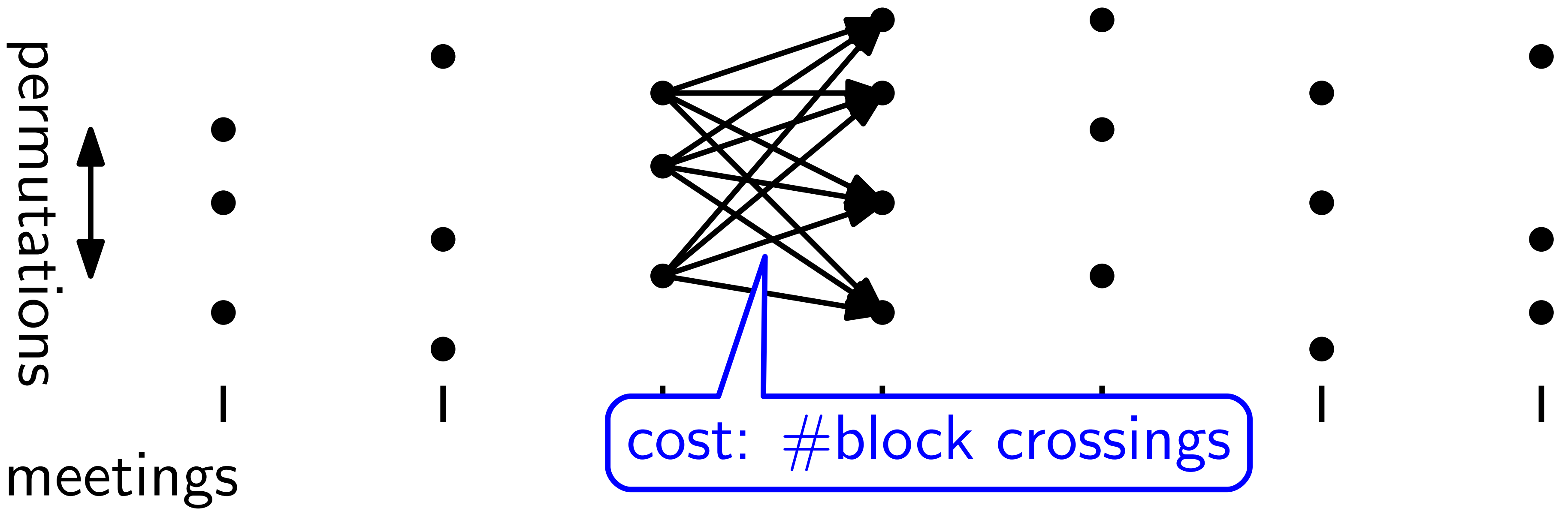

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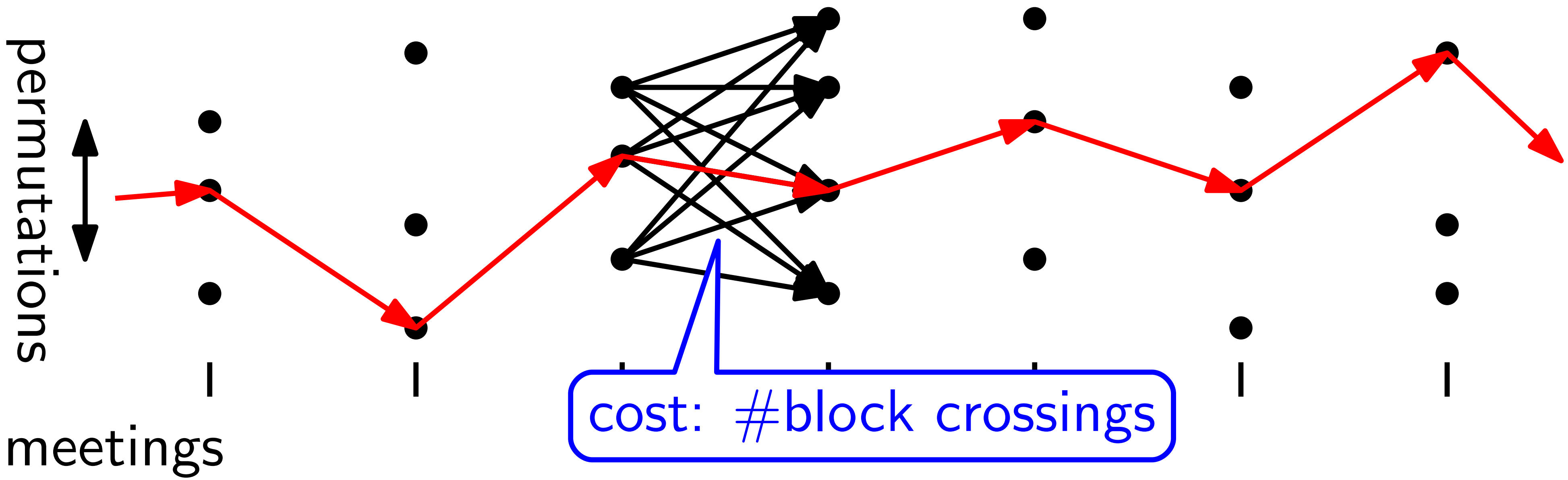

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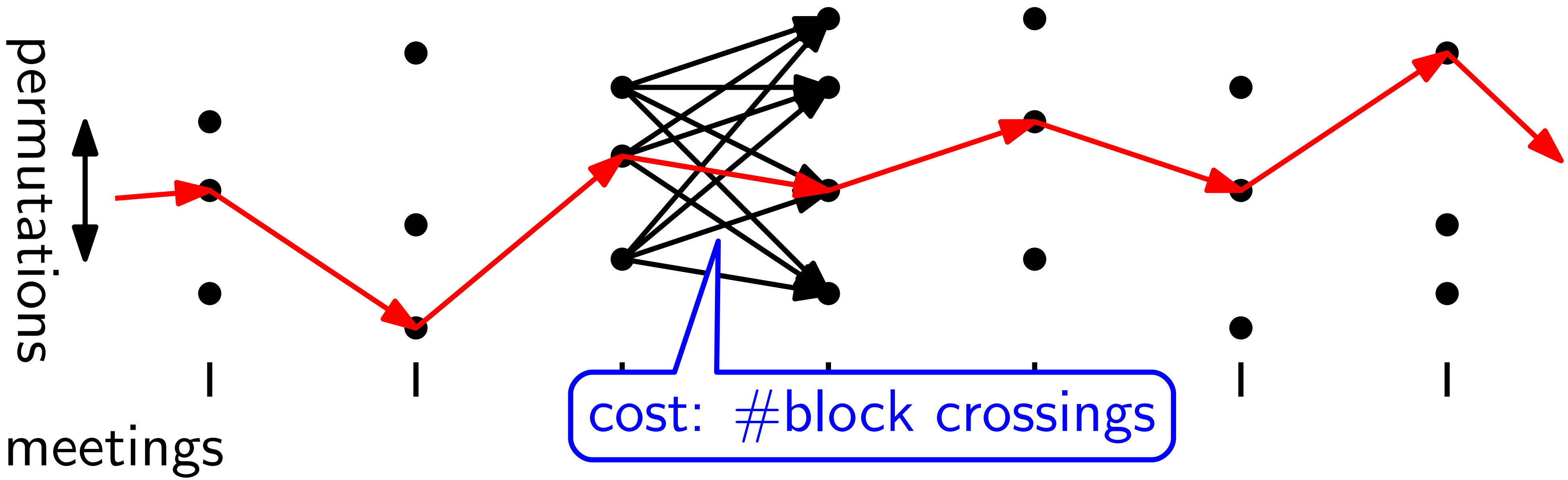

- find minimum-cost path
- runtime: $O(k!^{\mathbf{2}}n)$

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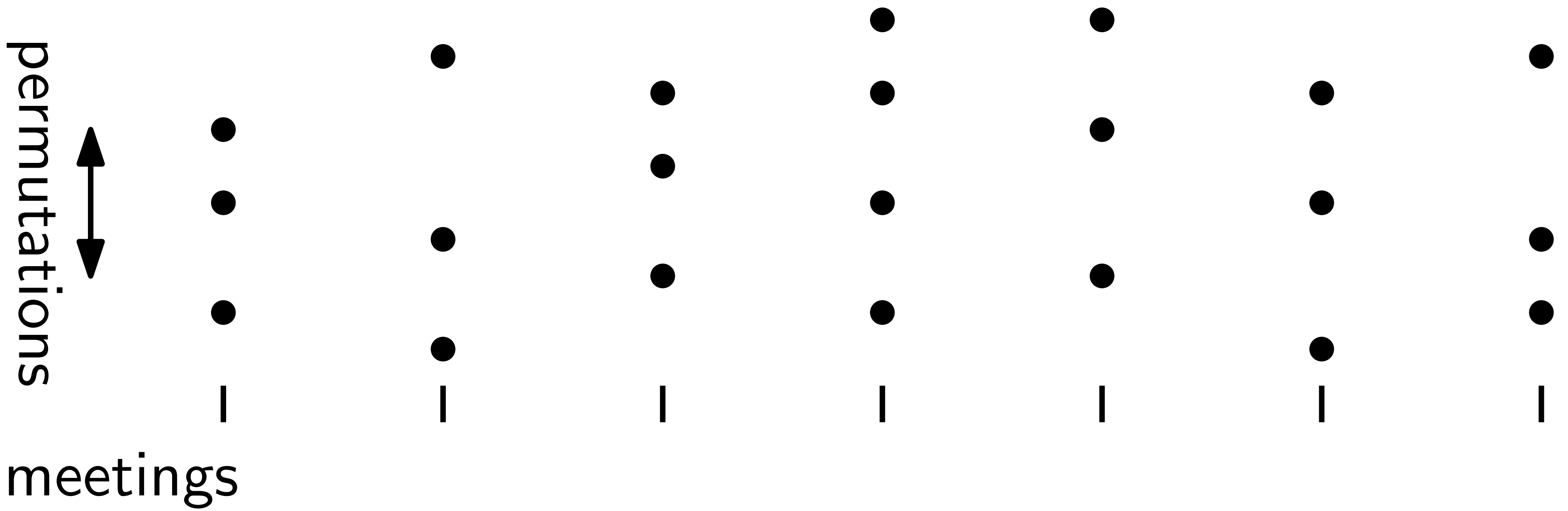

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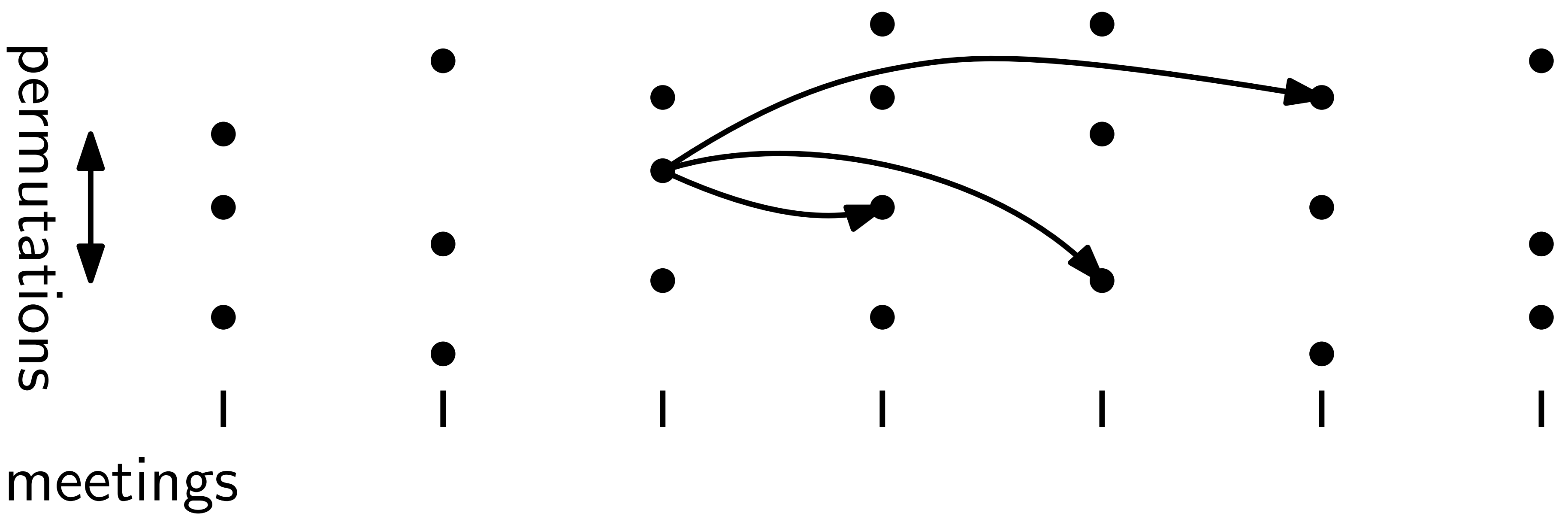

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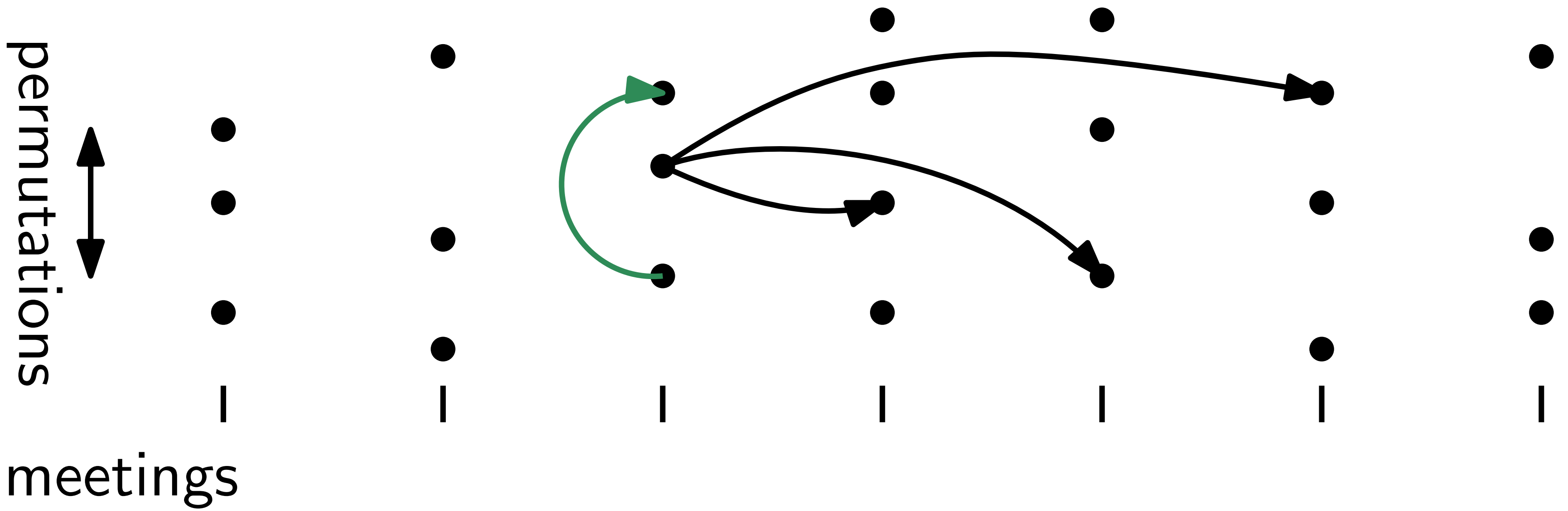

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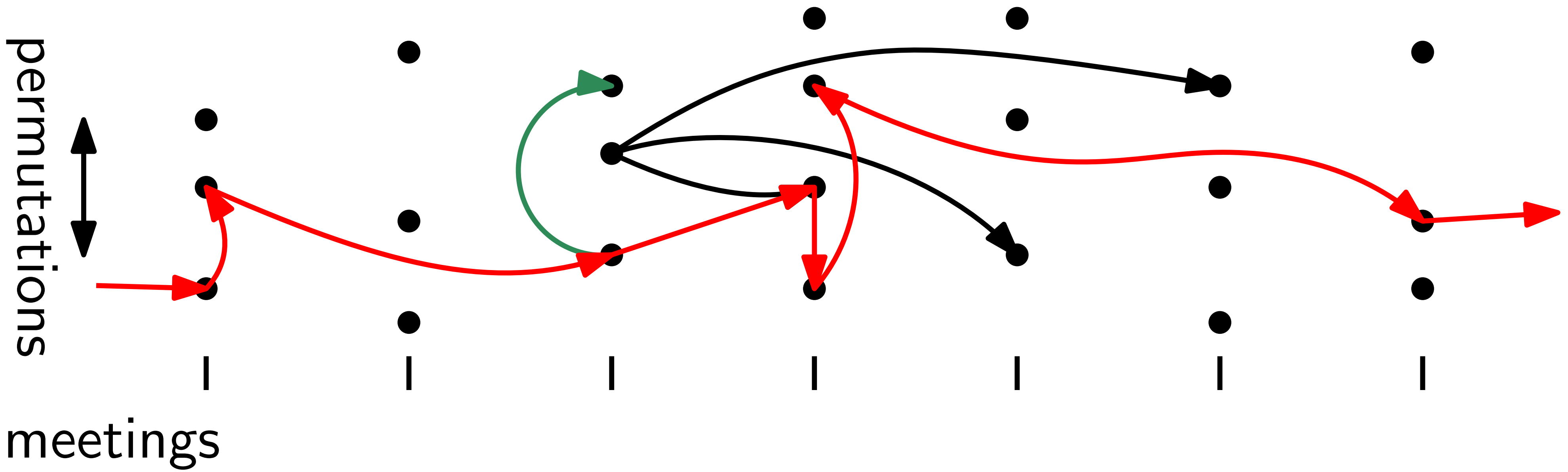

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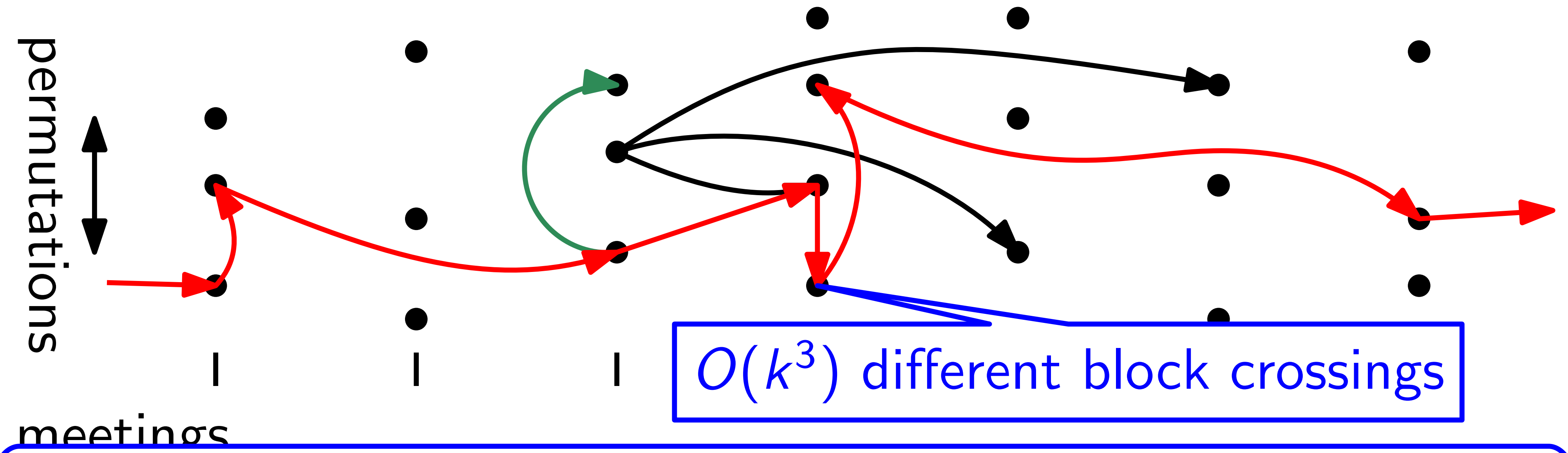

Theorem: We can minimize block crossings in $O(k!k^3n)$ time and $O(k!kn)$ space.

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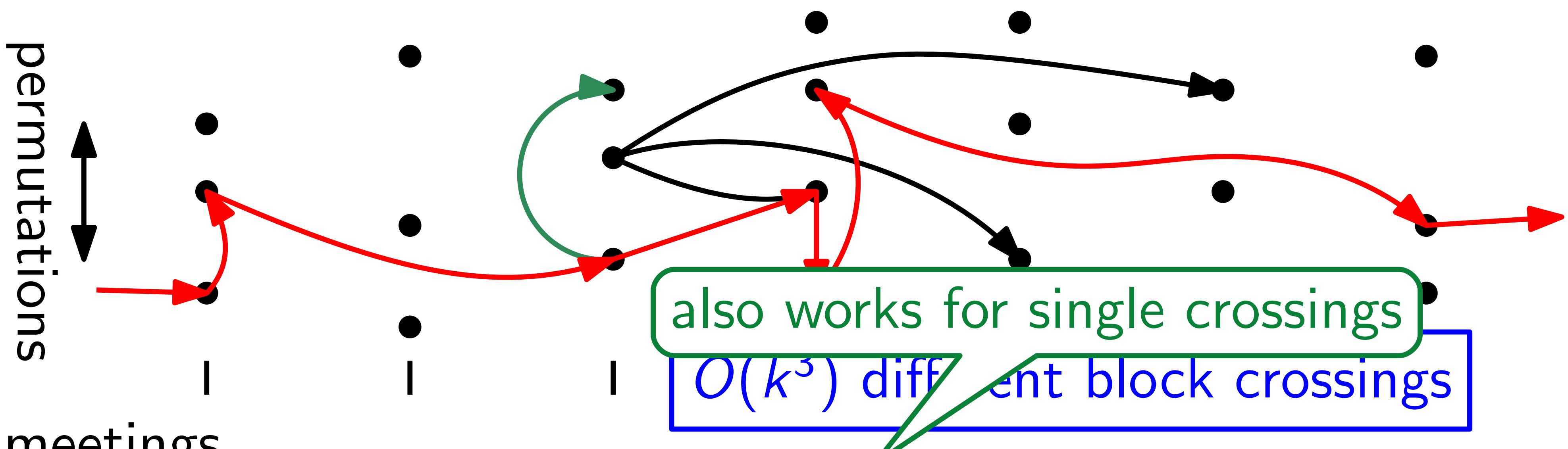

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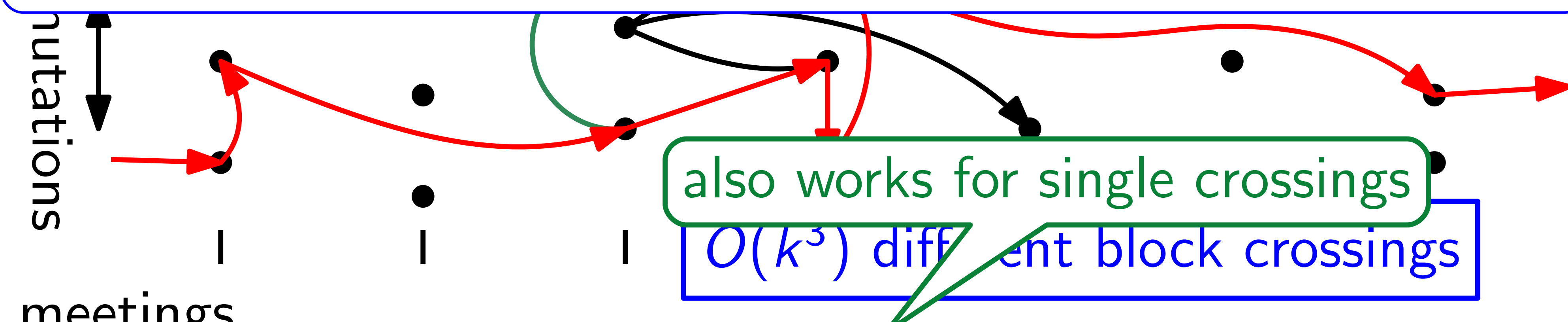
Alternative: Can minimize block crossings in $O(k!k^{\beta}(\beta + kn))$ time and $O(\beta k)$ space, where $\beta =$ opt. #block crossings



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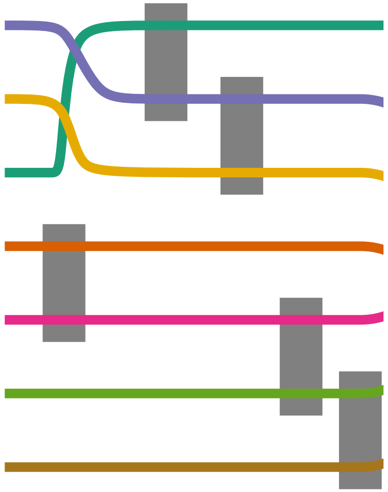

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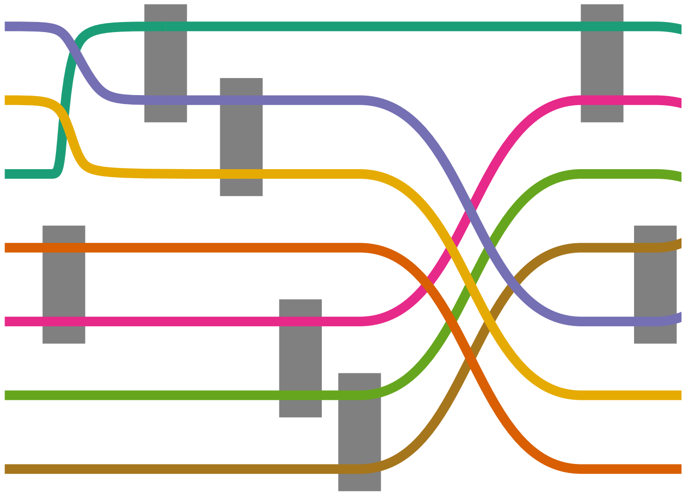

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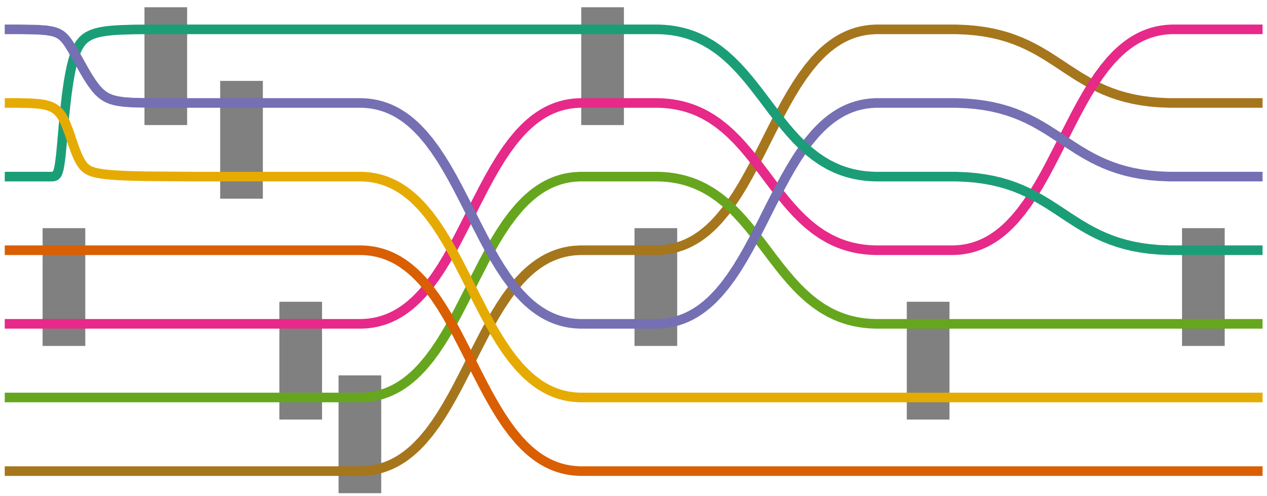

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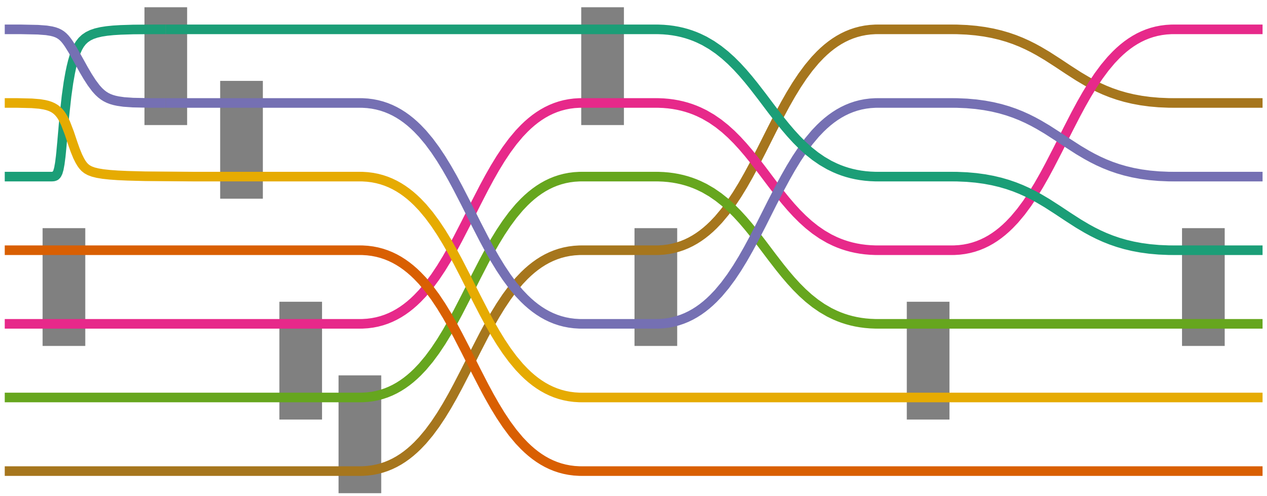

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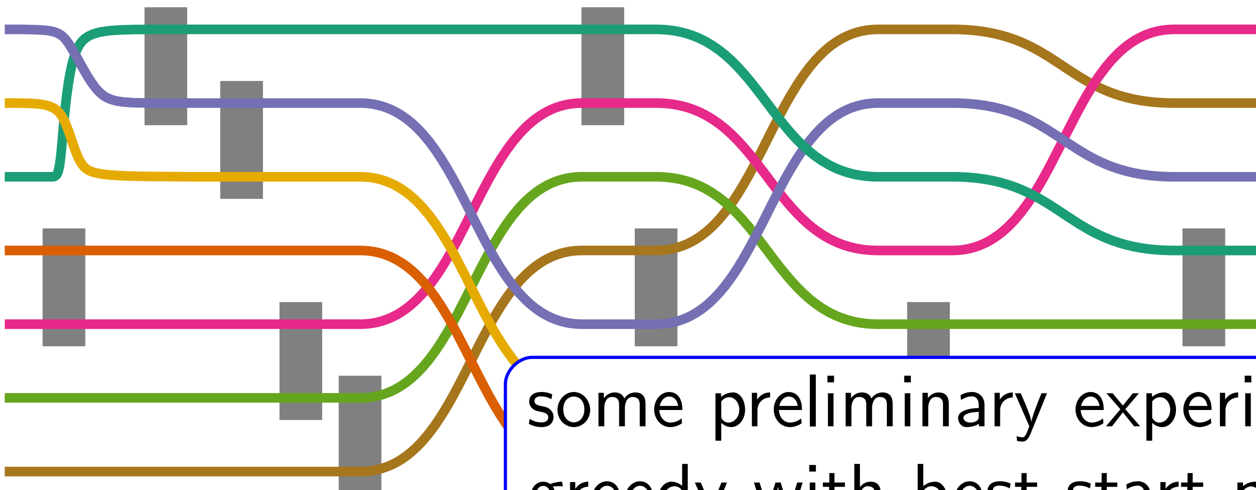

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- greedily try to sup
  with single block c
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some preliminary experiments; e.g.:
greedy with best start permutation
for $k = 5, n = 12$:
56% opt., 38% + 1bc, 5% + 2bc,
1% + 3bc

Conclusion

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- approximation algorithm
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Thank you!