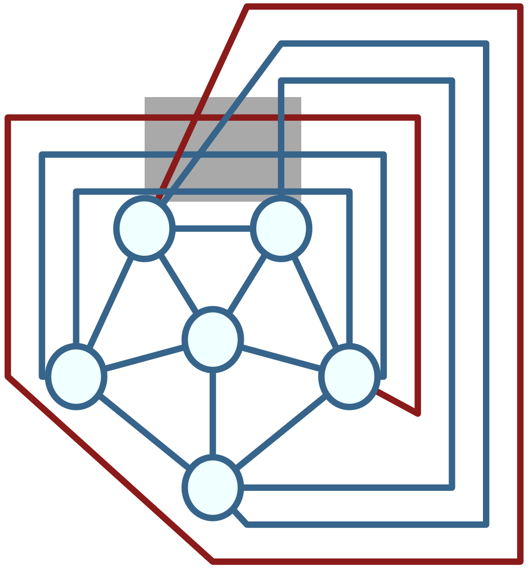
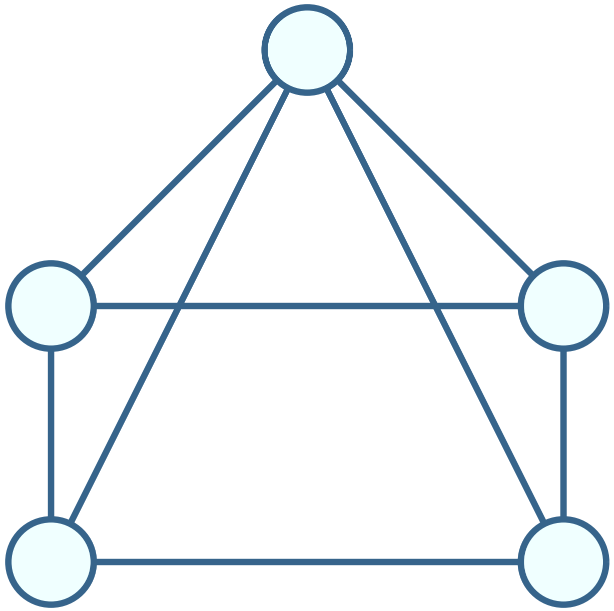
The Bundled Crossing Number

Md. Jawaherul Alam
Martin Fink
Sergey Pupyrev



GD 2016 – Athens
September 20

Edge Crossings



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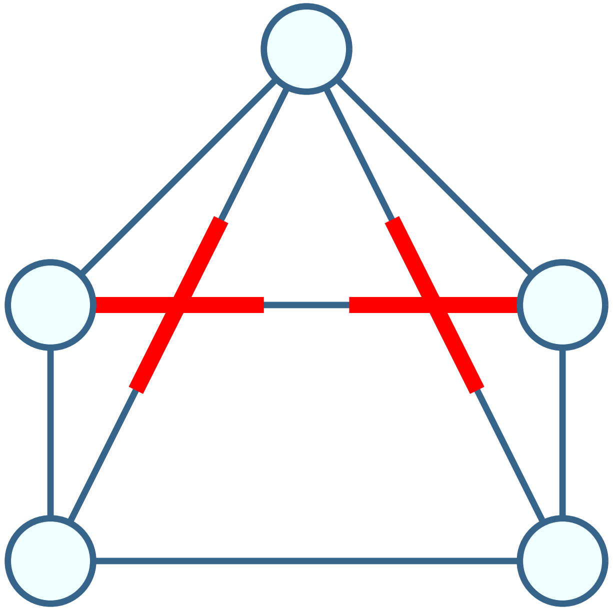

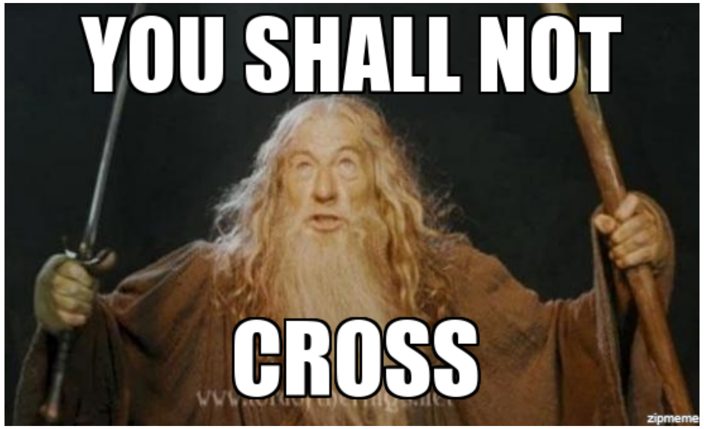

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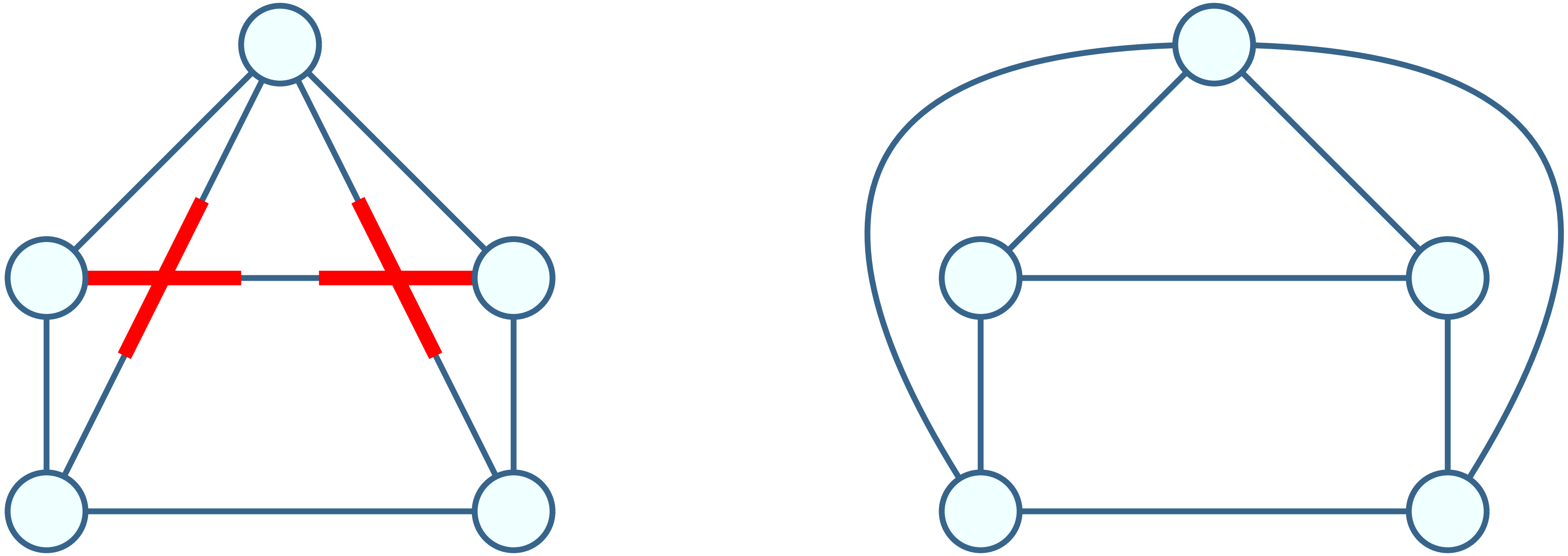

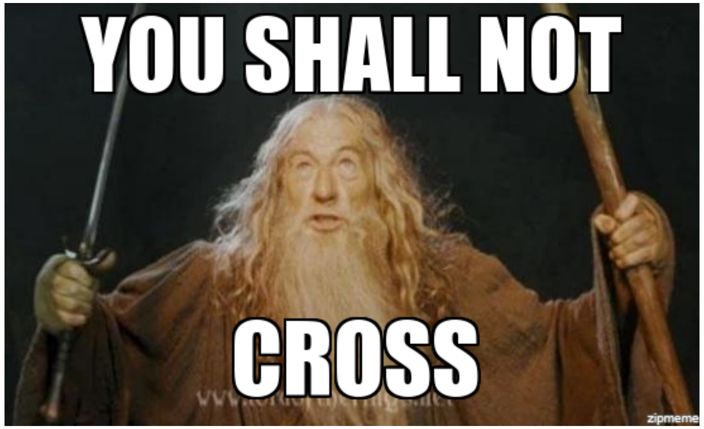

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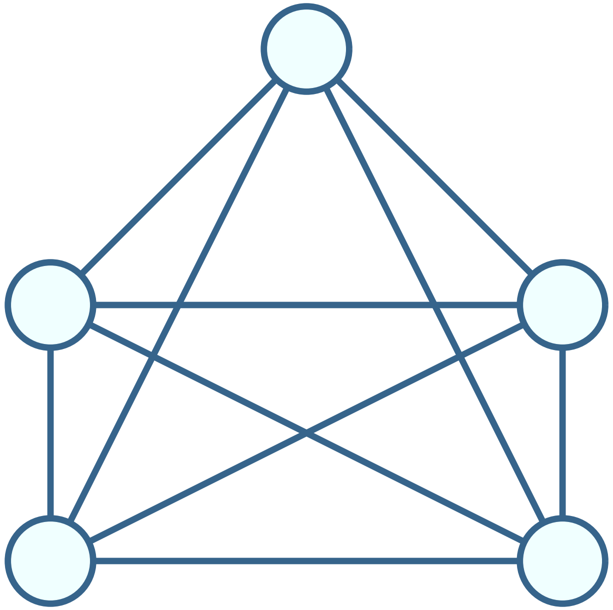

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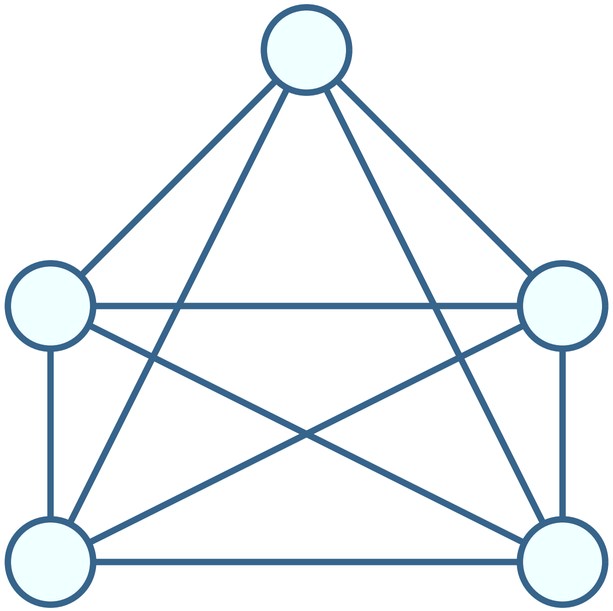

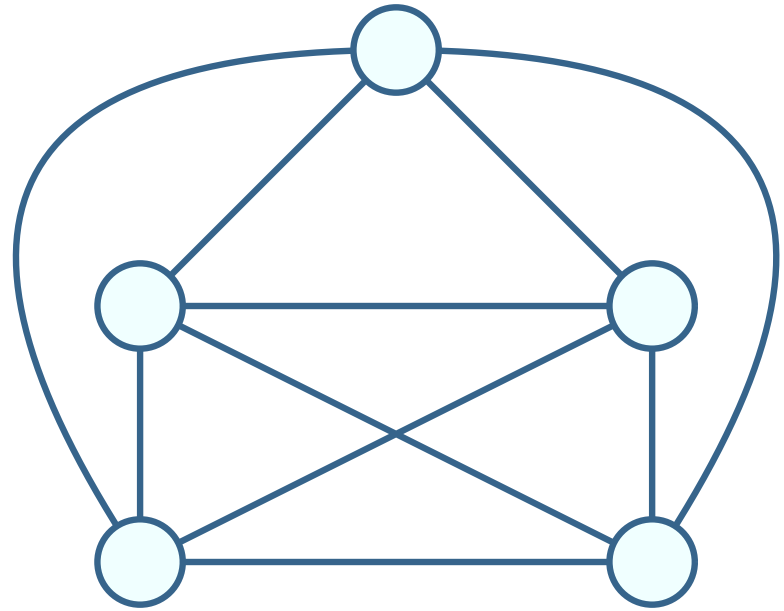

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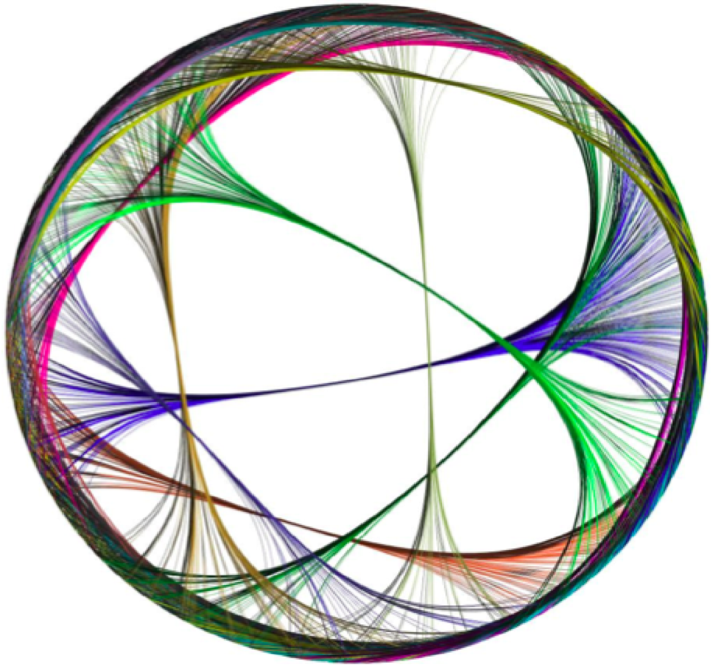

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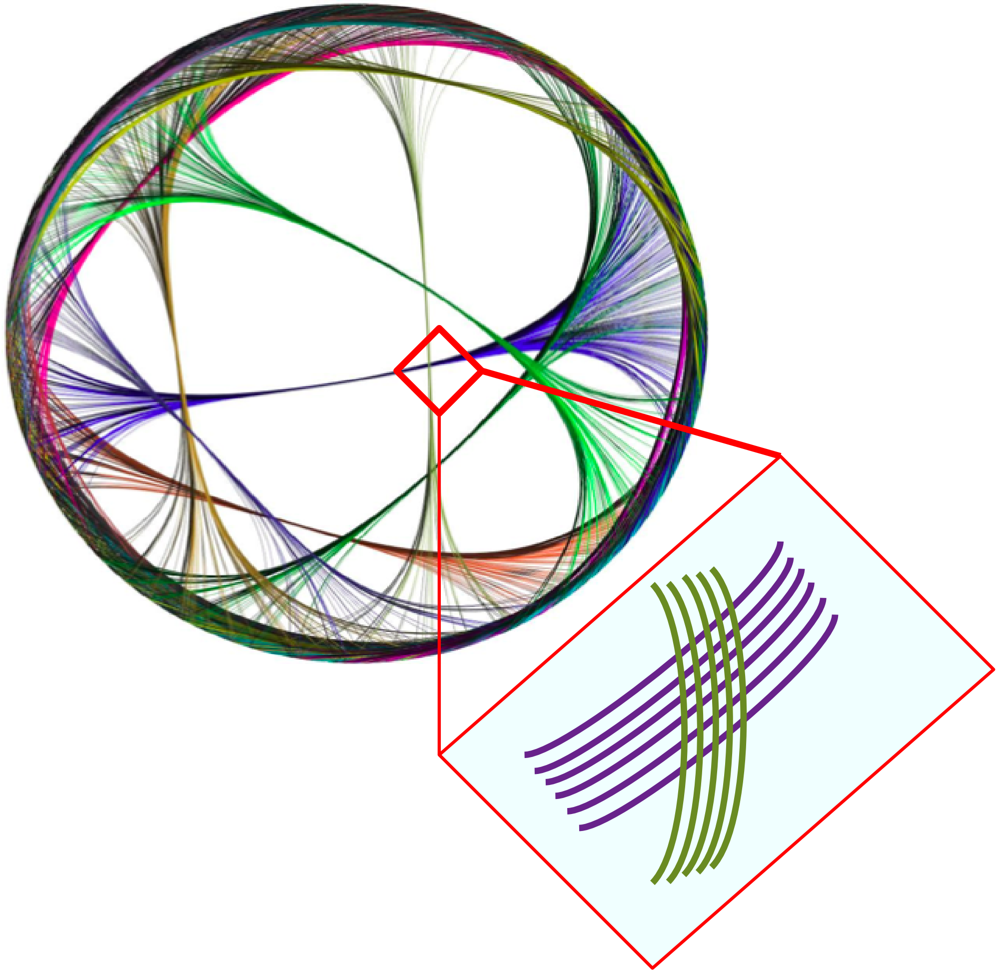
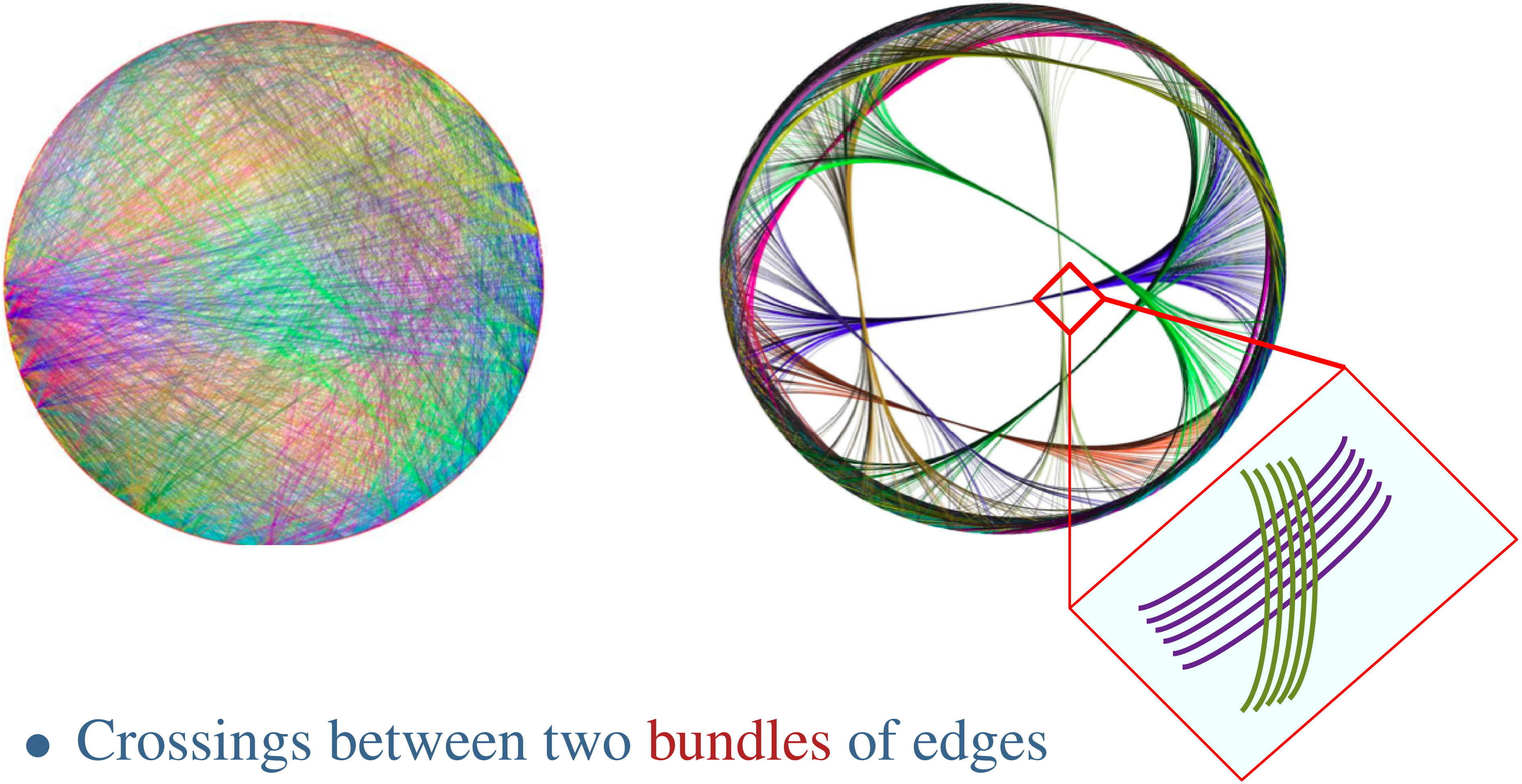


Bundled Crossings



- Crossings between two bundles of edges
- Contained within disjoint pseudodisks

Bundled Crossing Number

minimum number of bundles to group all the crossings

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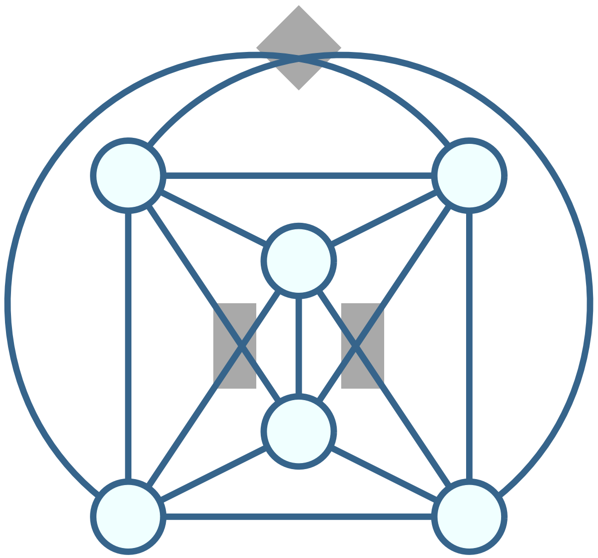

3 bundles

minimum number of bundles to group all the crossings

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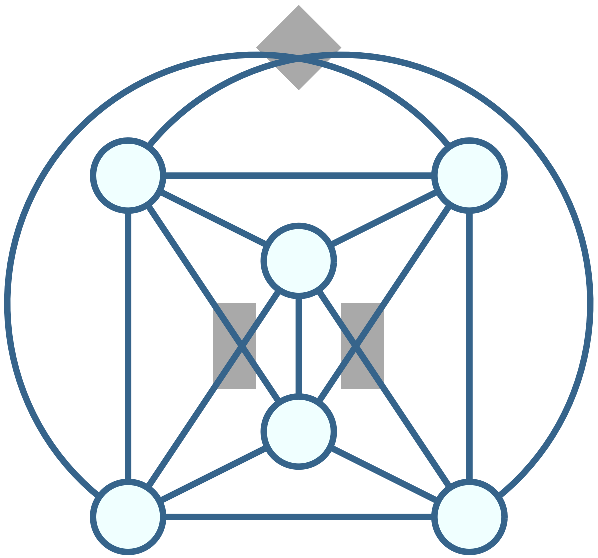

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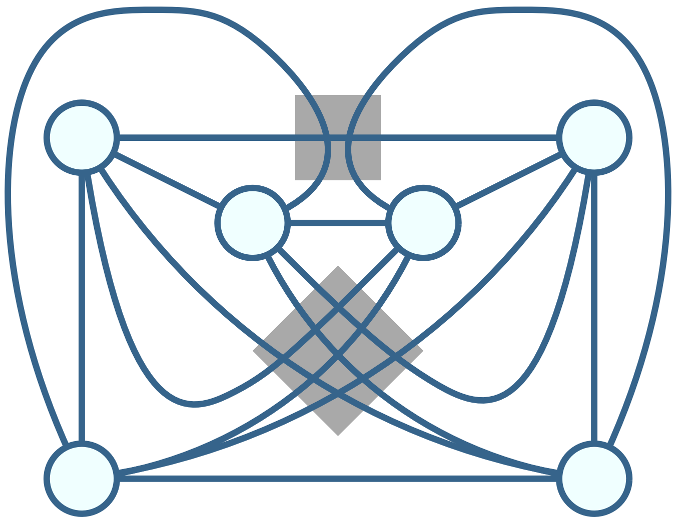

2 bundles

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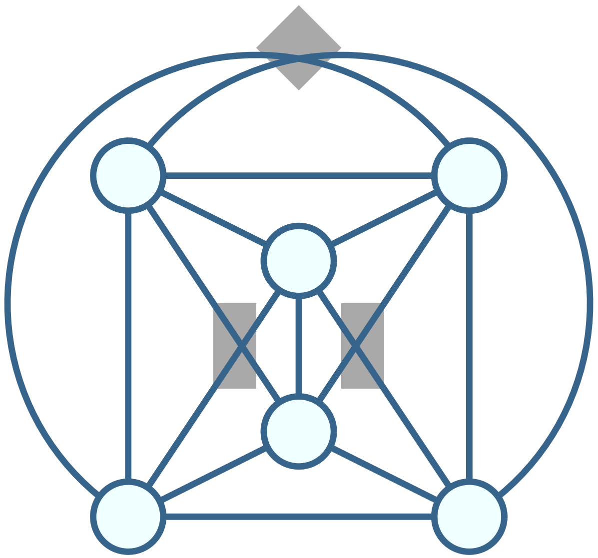

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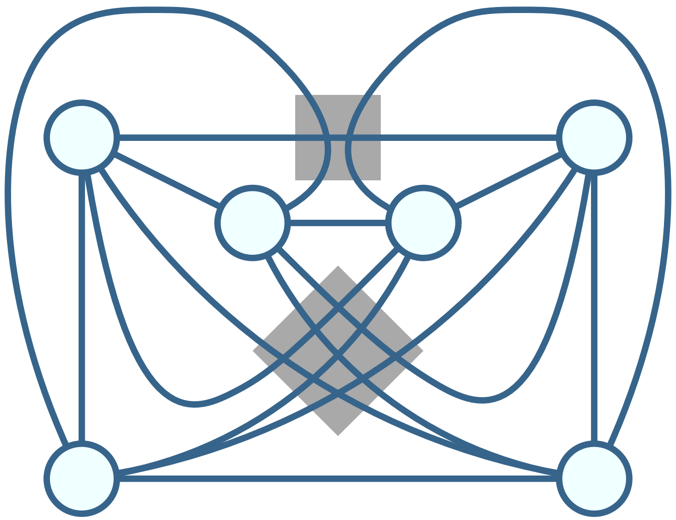

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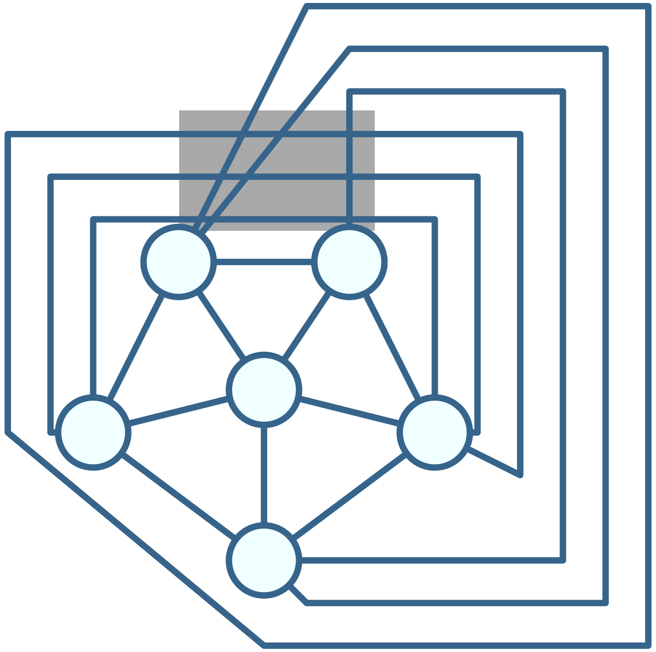

1 bundle

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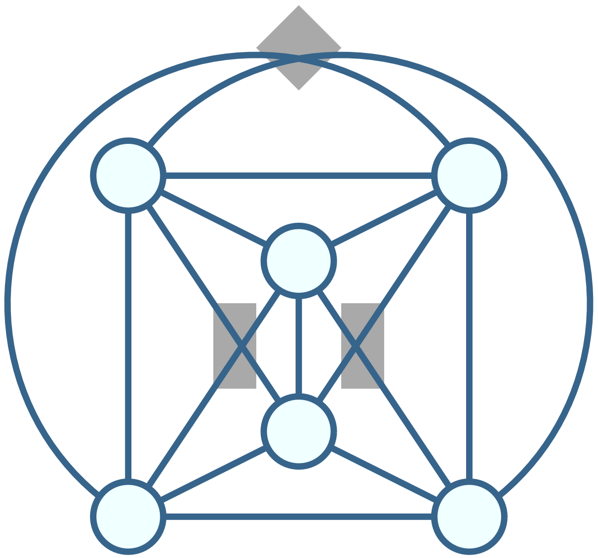
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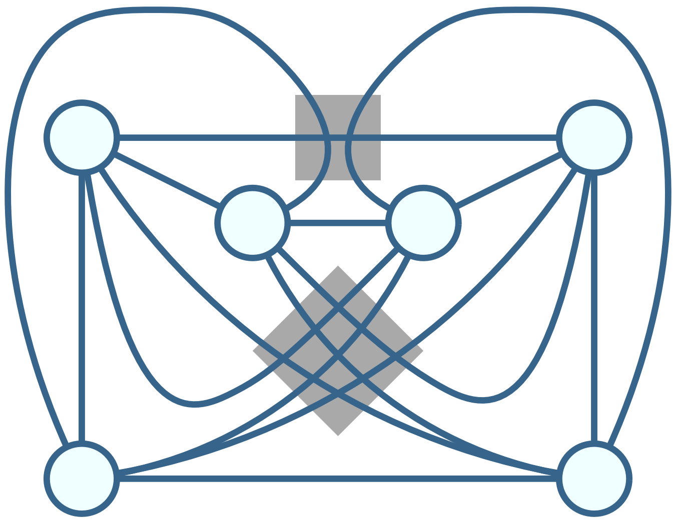
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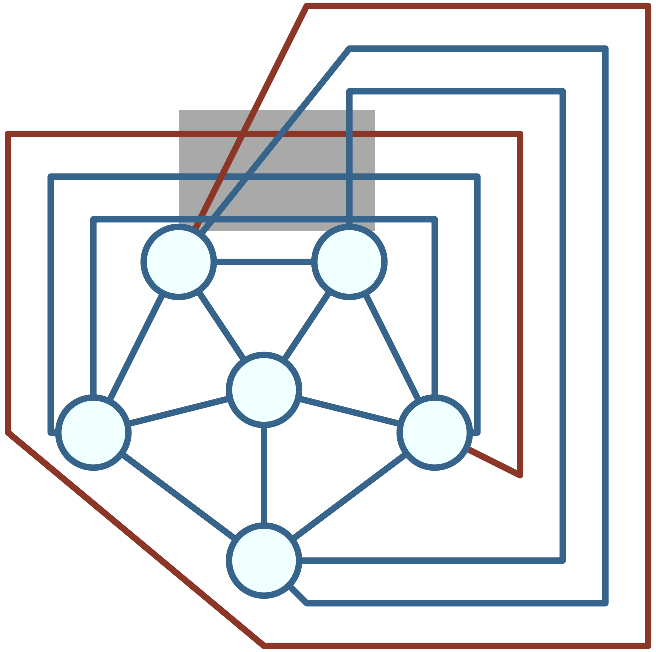
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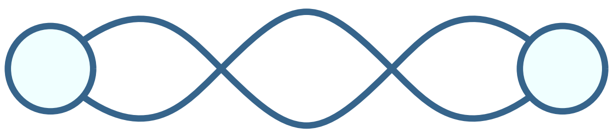
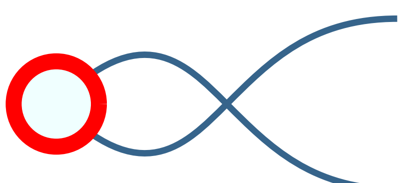
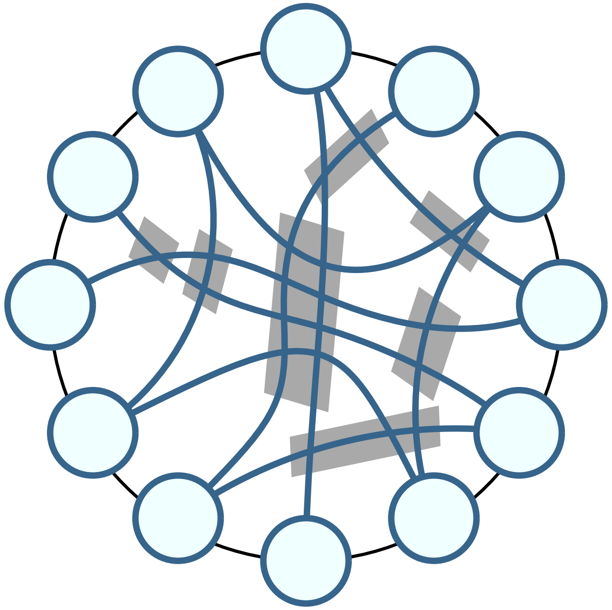
Variants

	Unrestricted Drawing	Simple Drawing
 self crossing	✓	✗
 multiple crossings	✓	✗
 incident edge crossing	✓	✗

Variants



Circular Drawing

- simple drawing
- all vertices on a circle
- all edges inside the circle

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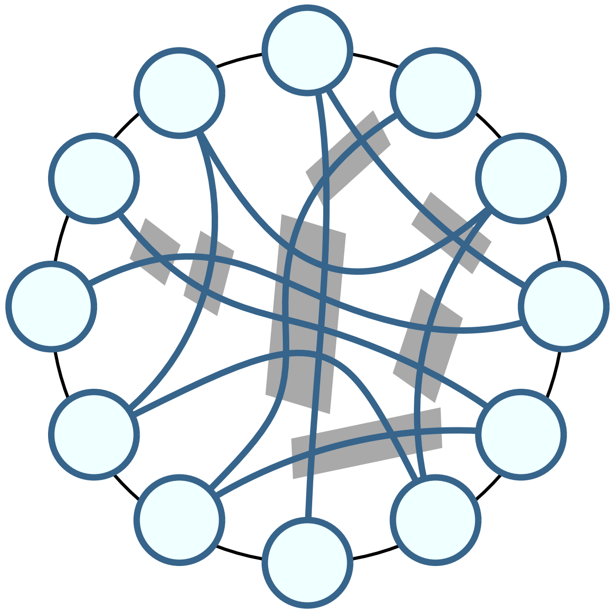

Circular Drawing

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- all vertices on a circle
- all edges inside the circle
- Fixed order vs flexible order of vertices

Related Work

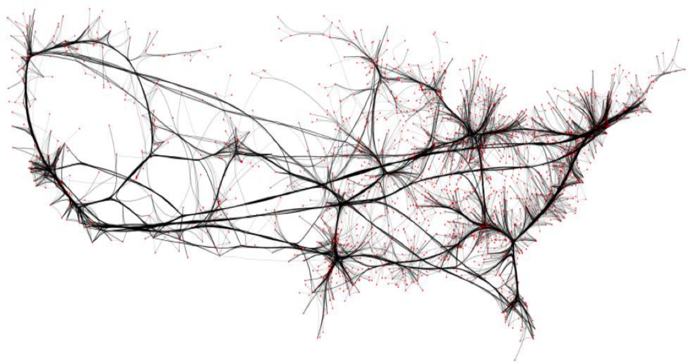
Minimizing Crossings

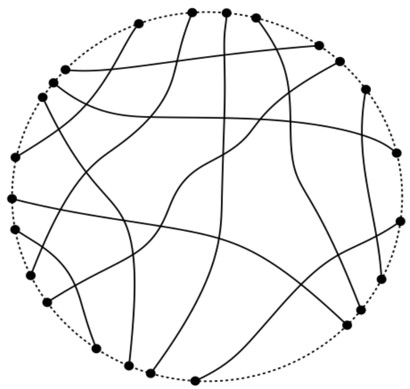
- **NP**-hard, even for cubic graphs
- No constant approximation unless **P≠NP**

[Cabello 2013]

- $O(n^{9/10})$-approximation for bounded degree

[Chuzhoy 2011]





Bundled Crossings

- **NP**-hard, for a fixed embedding
- constant-factor approximation for circular layout

[Fink et al. 2013]

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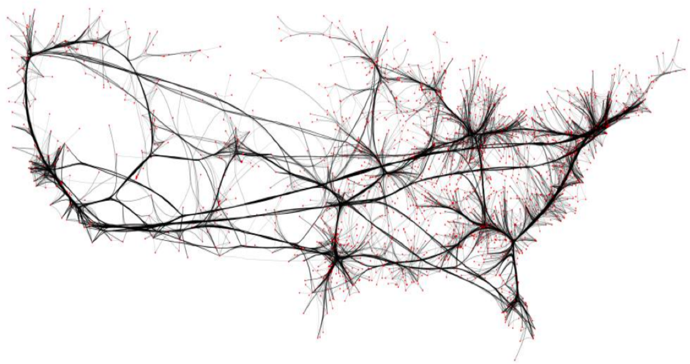

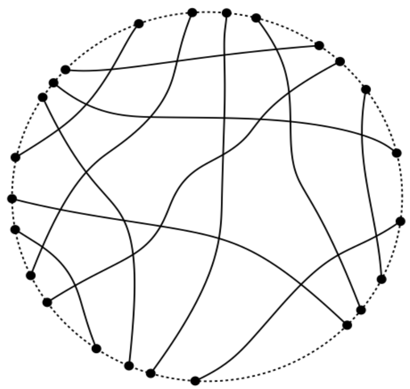

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Our Result

- We address the problem in the fixed embedding setting

	Lower Bound	Upper Bound	Approximation
Unrestricted	g	g	$--$
Simple	g	$--$	$--$
	$\frac{(m-(3n-6))}{6}$	$m-1$	$\frac{6c}{(c-3)}, m > cn$
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Bundled Crossing Number and Genus

Theorem

For unrestricted drawings of G, $bc(G) = g(G)$

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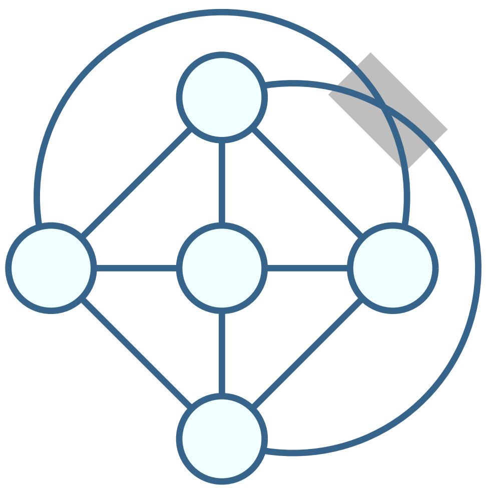

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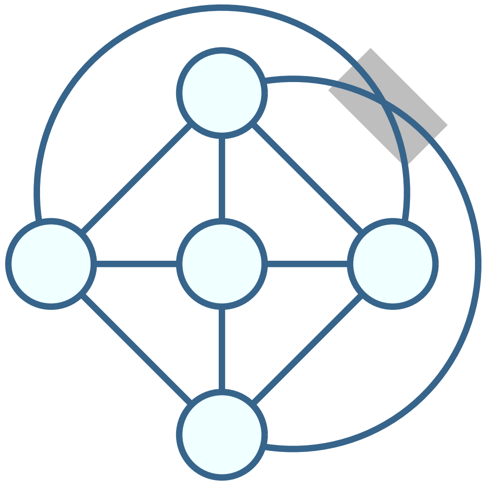

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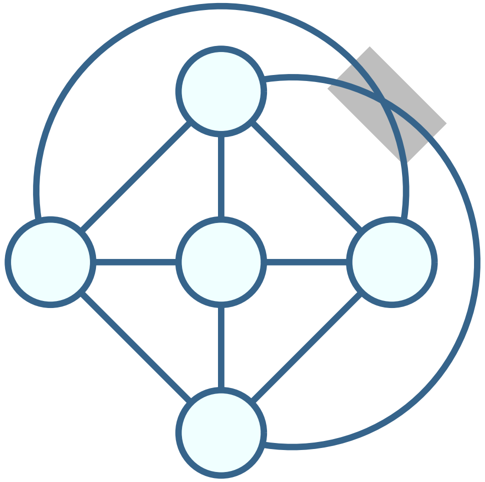

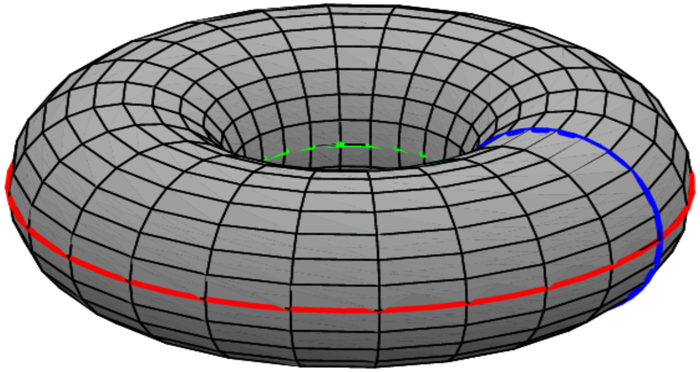

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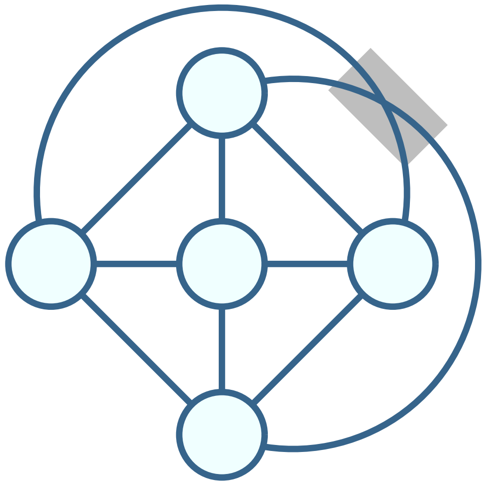

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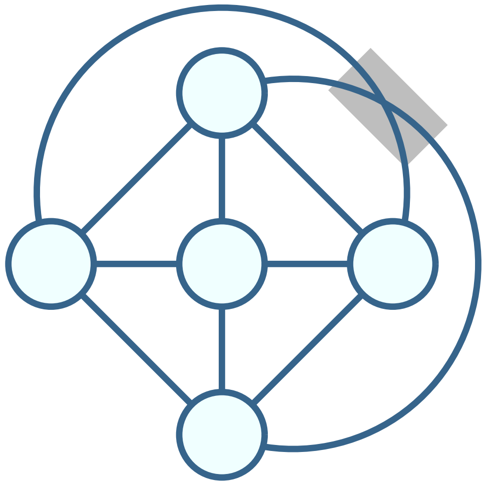

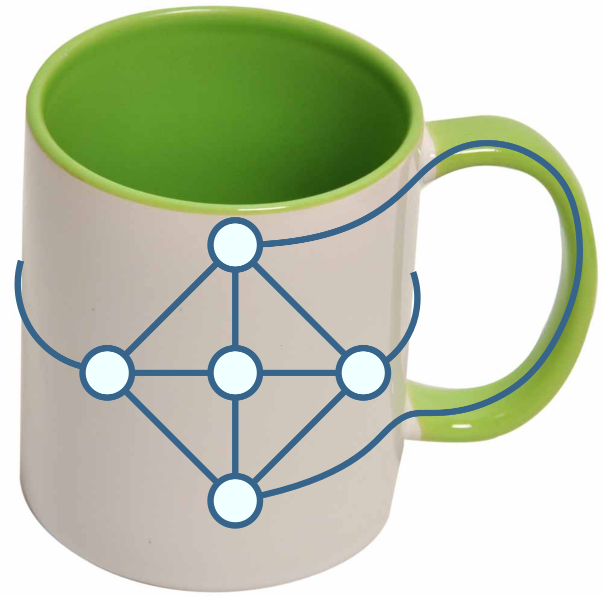

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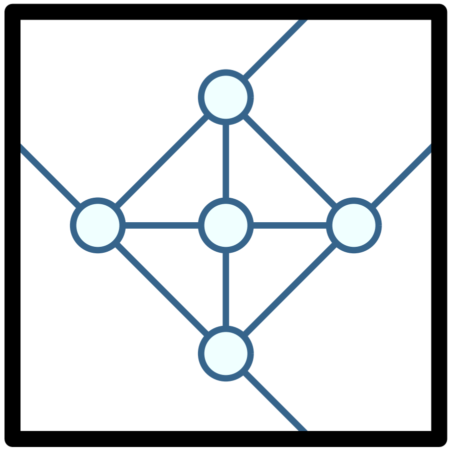

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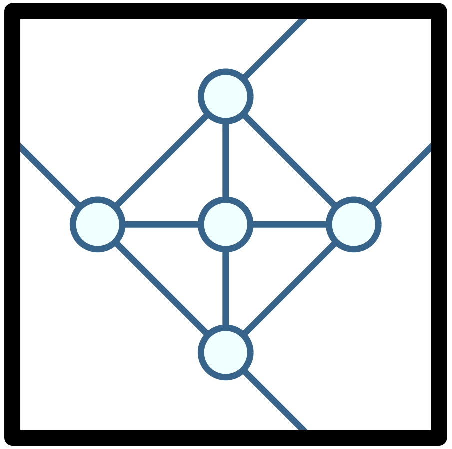

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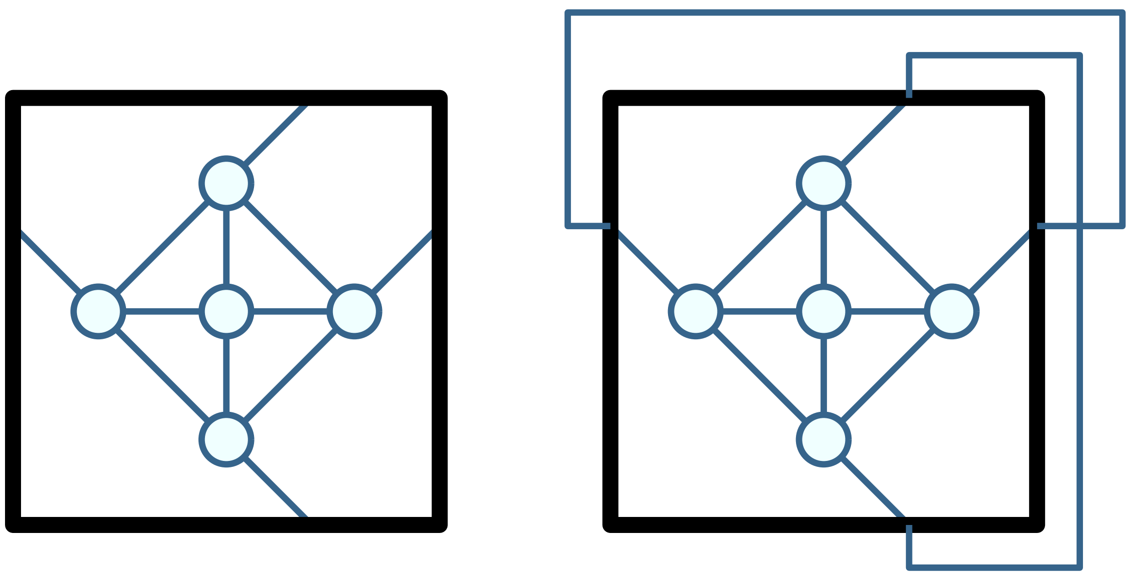

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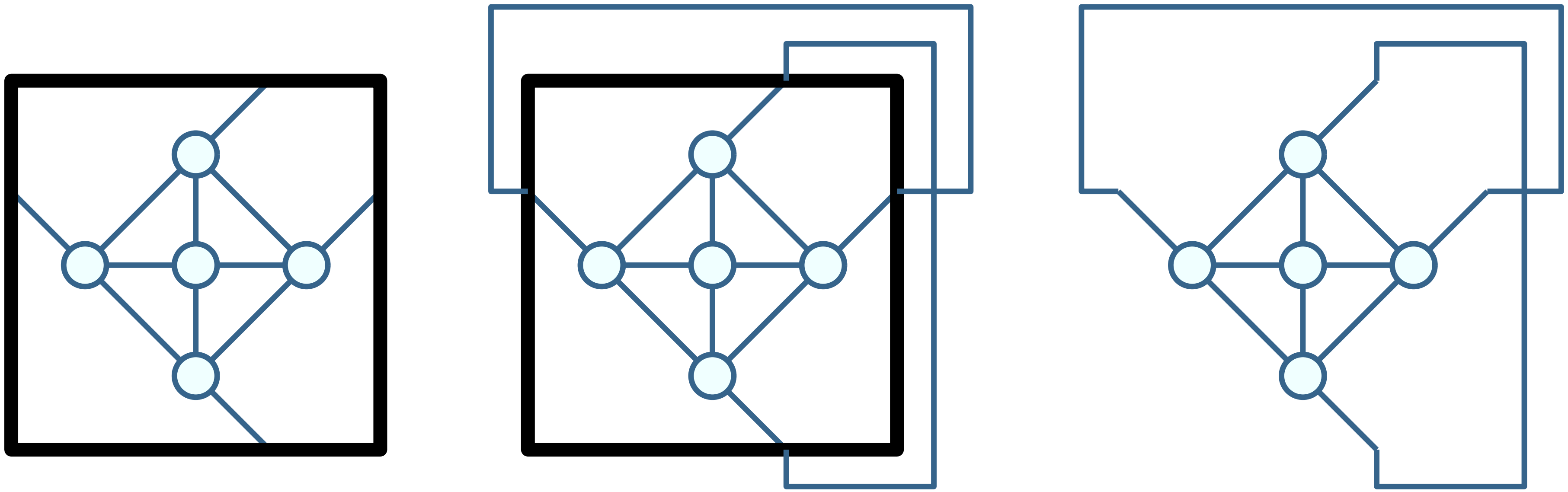

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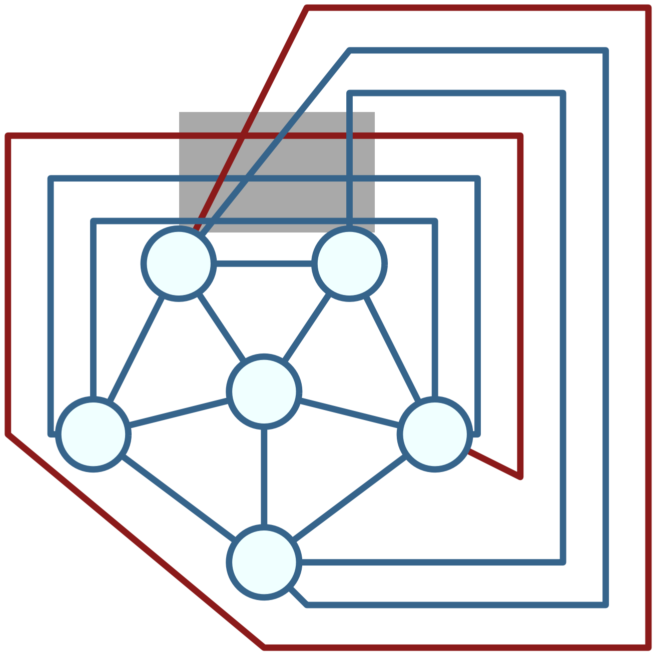
There is some G for which $bc(G) > g(G)$.



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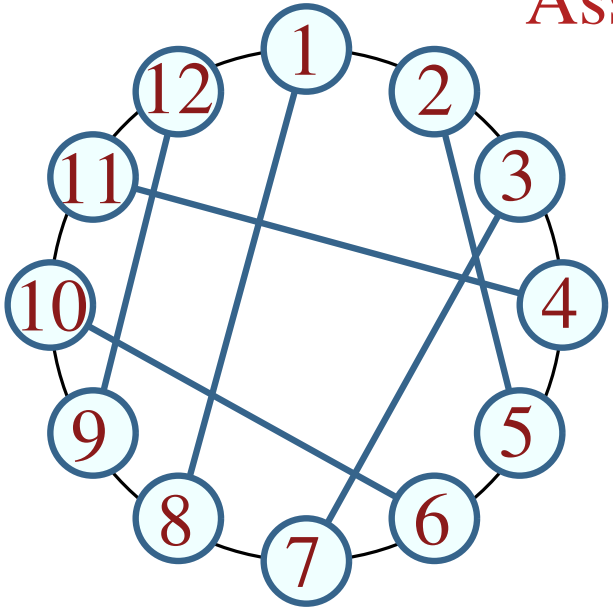

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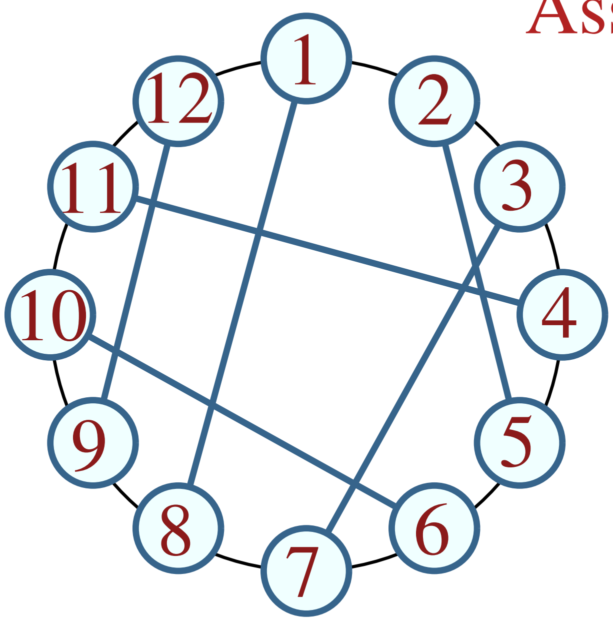

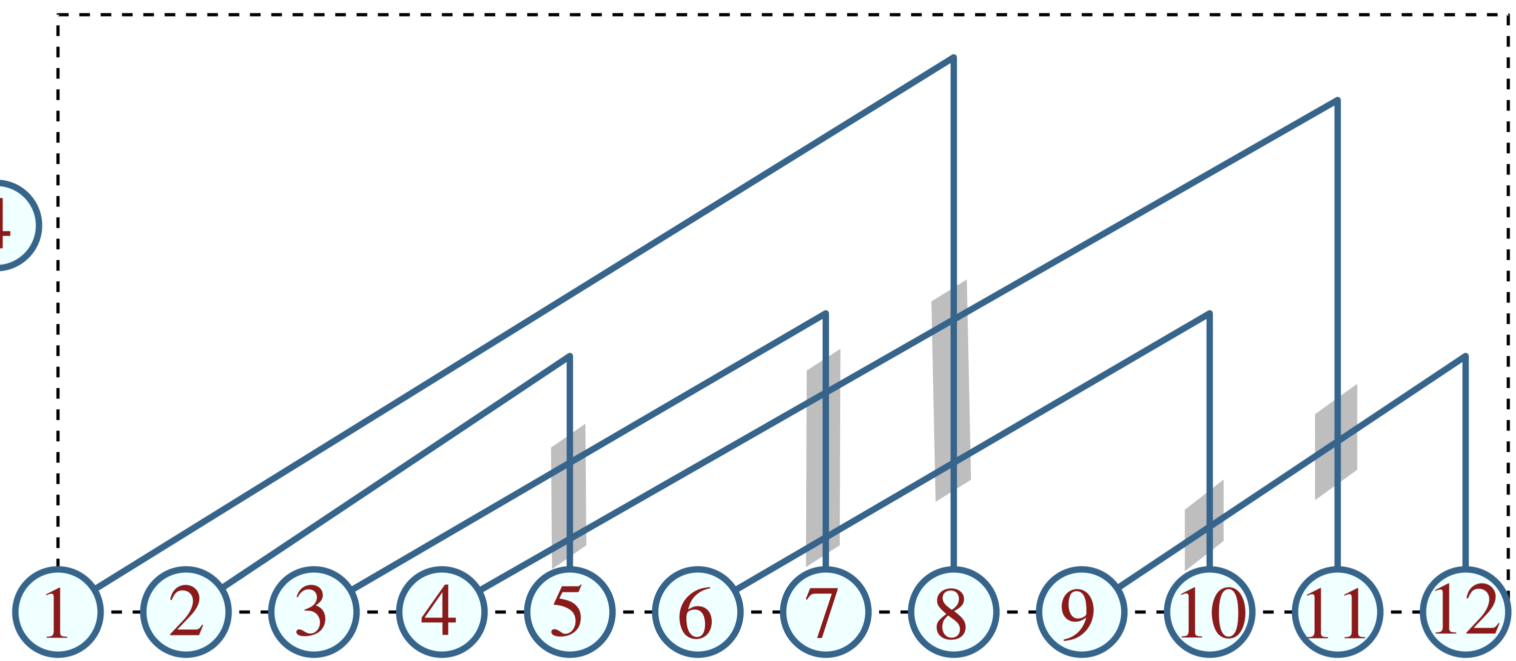

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Future Work

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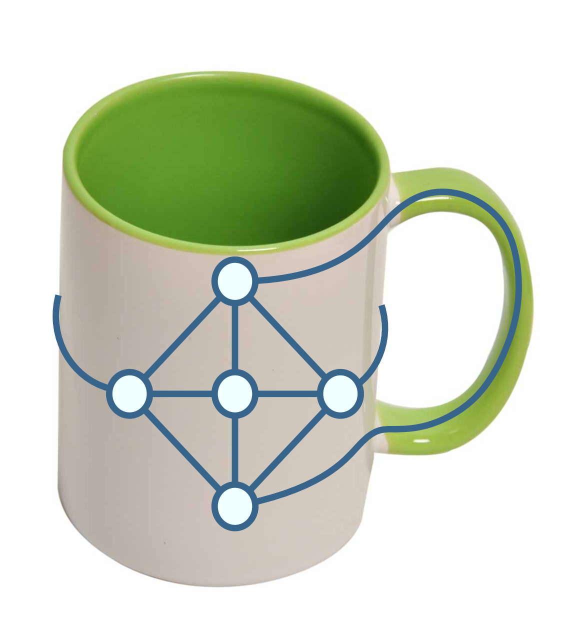
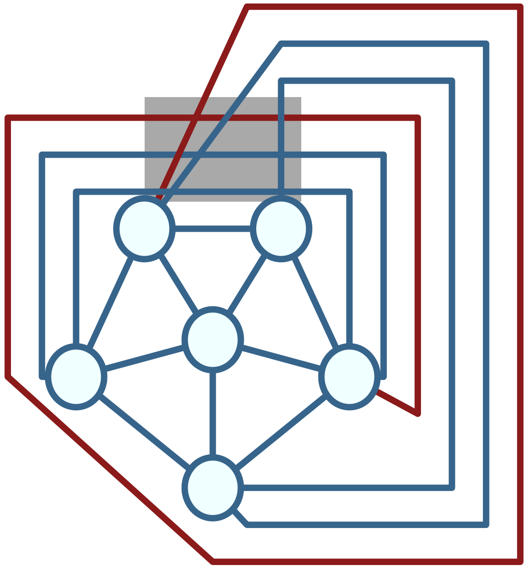
  - **NP**-hard?
- Better approximation for sparse graphs





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