Track Layout is Hard

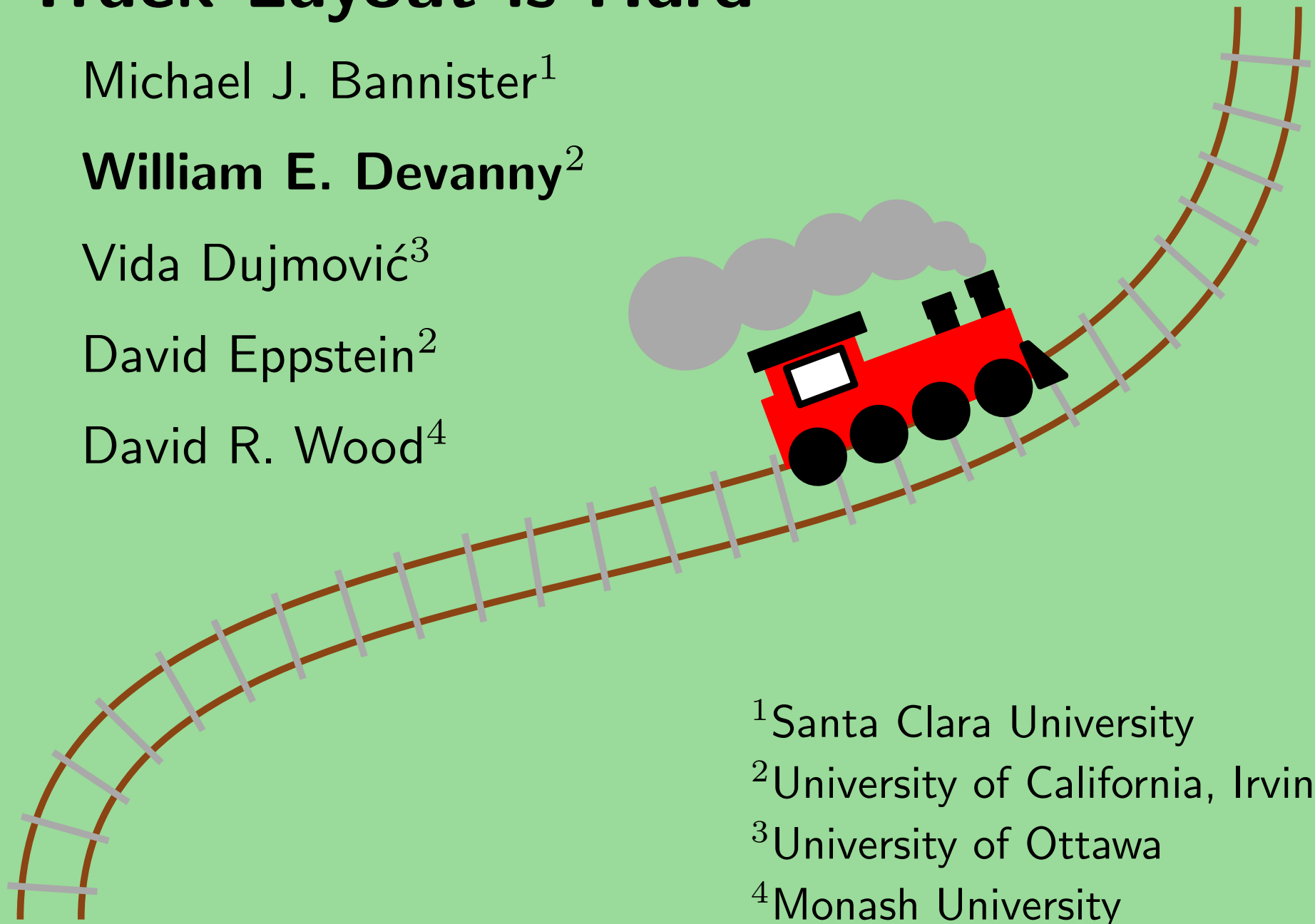
Michael J. Bannister[1]

William E. Devanny[2]

Vida Dujmović[3]

David Eppstein[2]

David R. Wood[4]



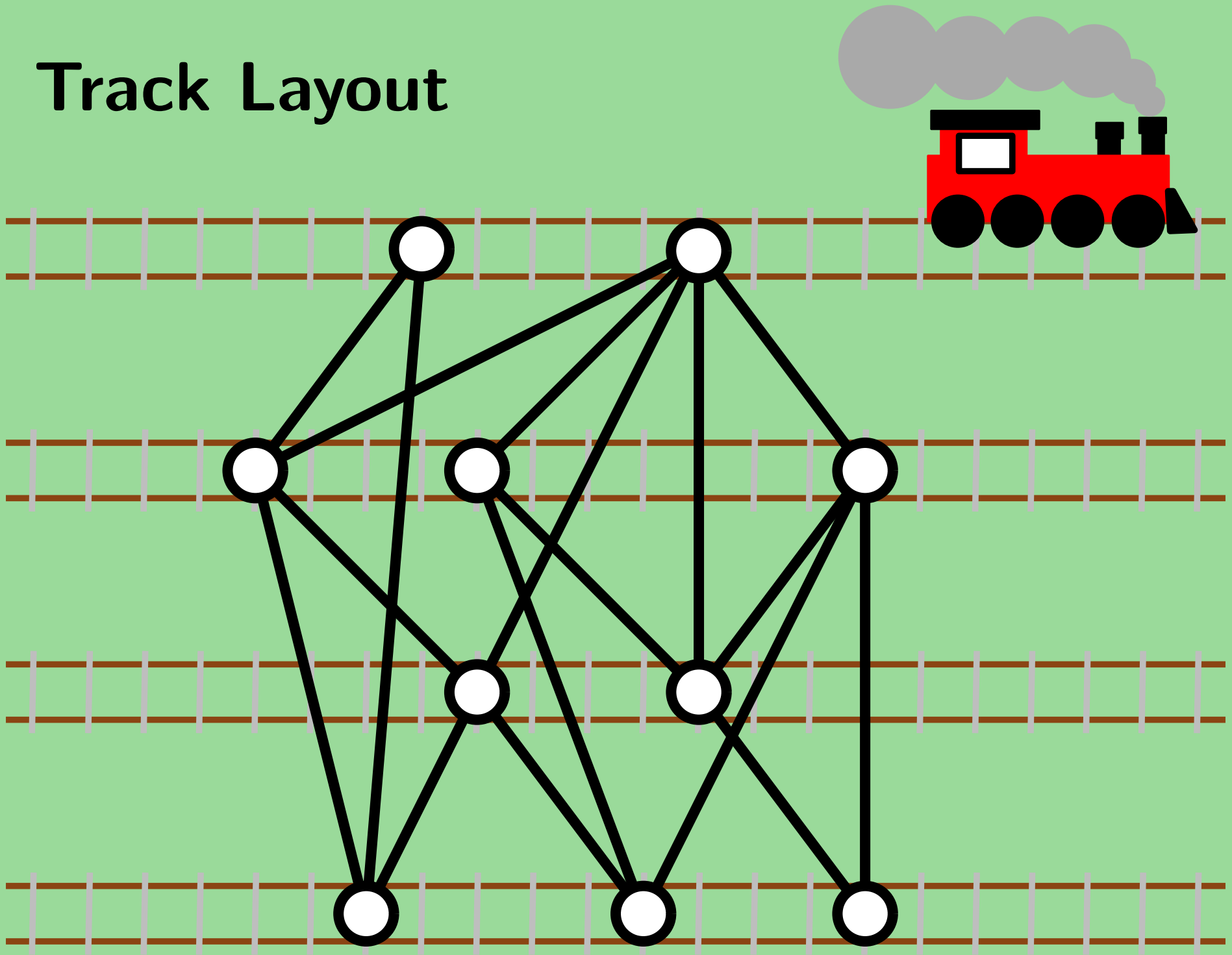
[1]Santa Clara University

[2]University of California, Irvine

[3]University of Ottawa

[4]Monash University

Track Layout



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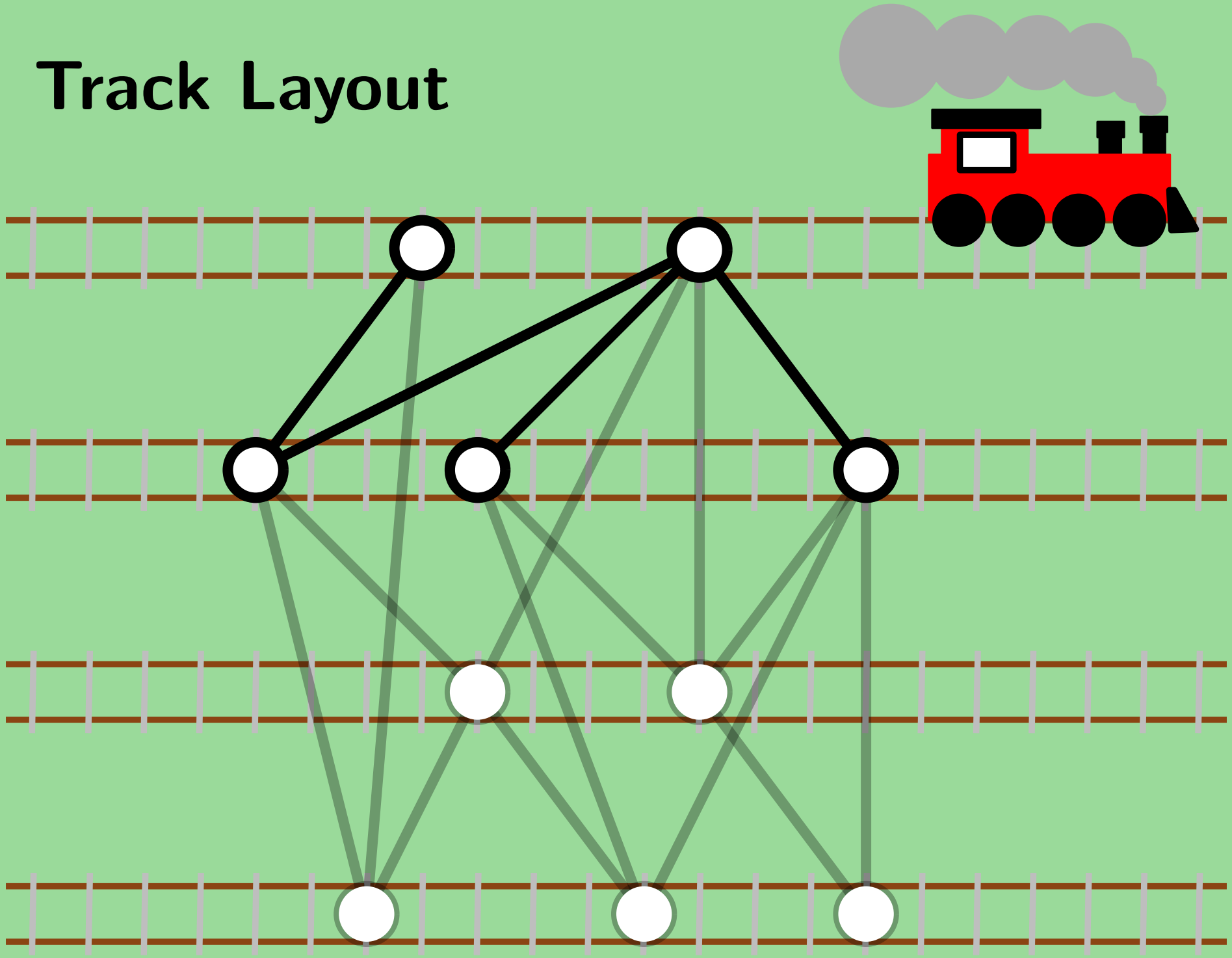

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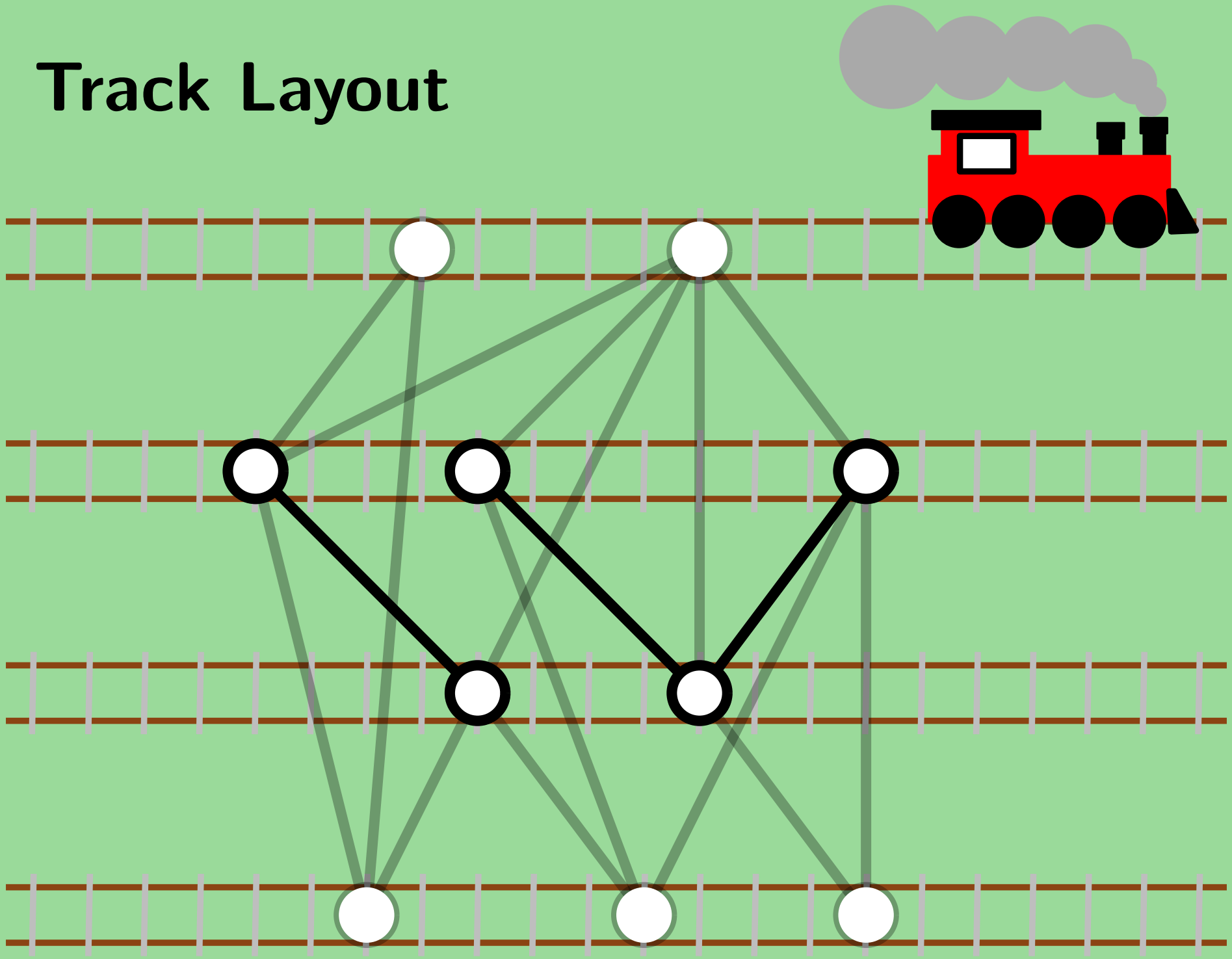

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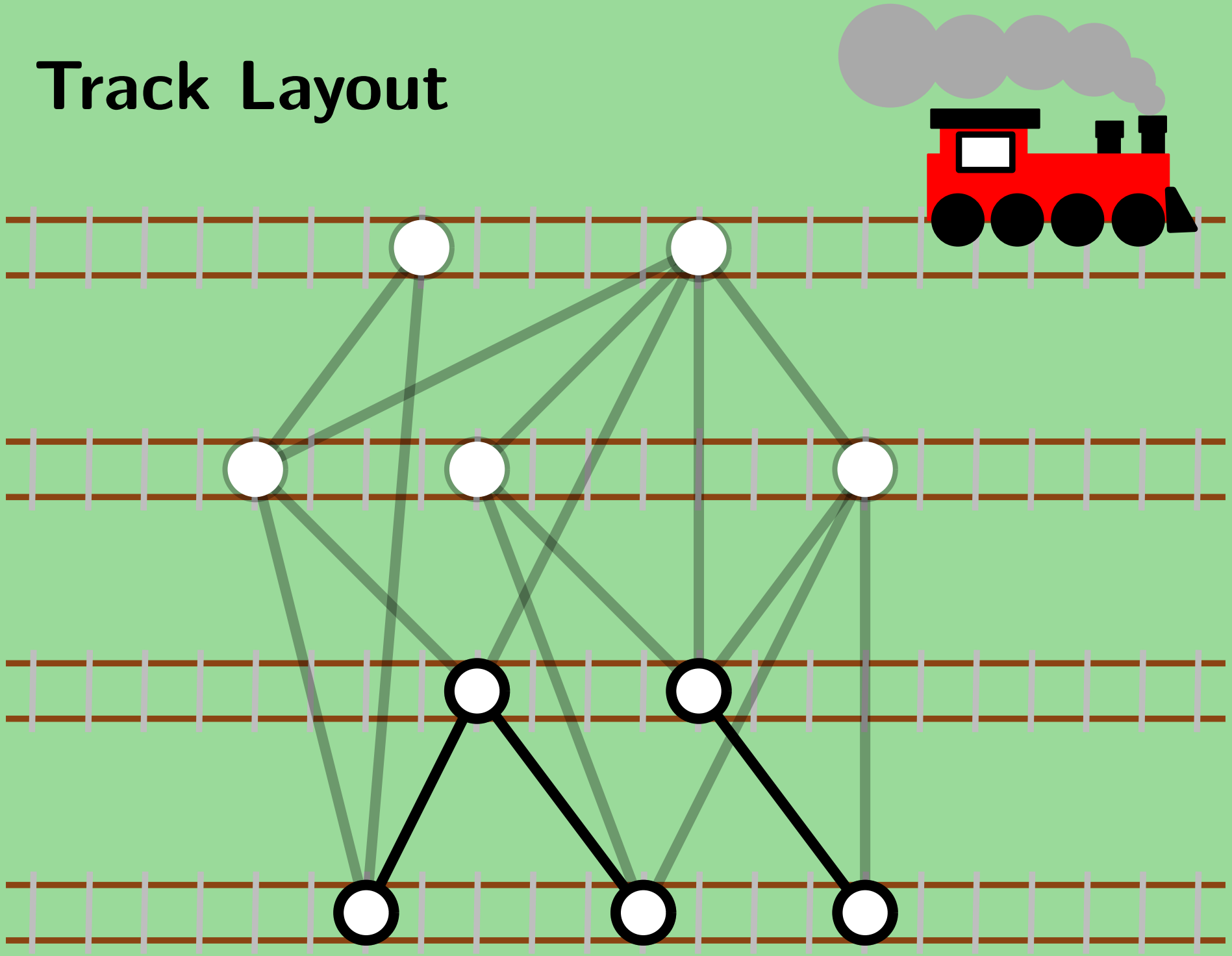

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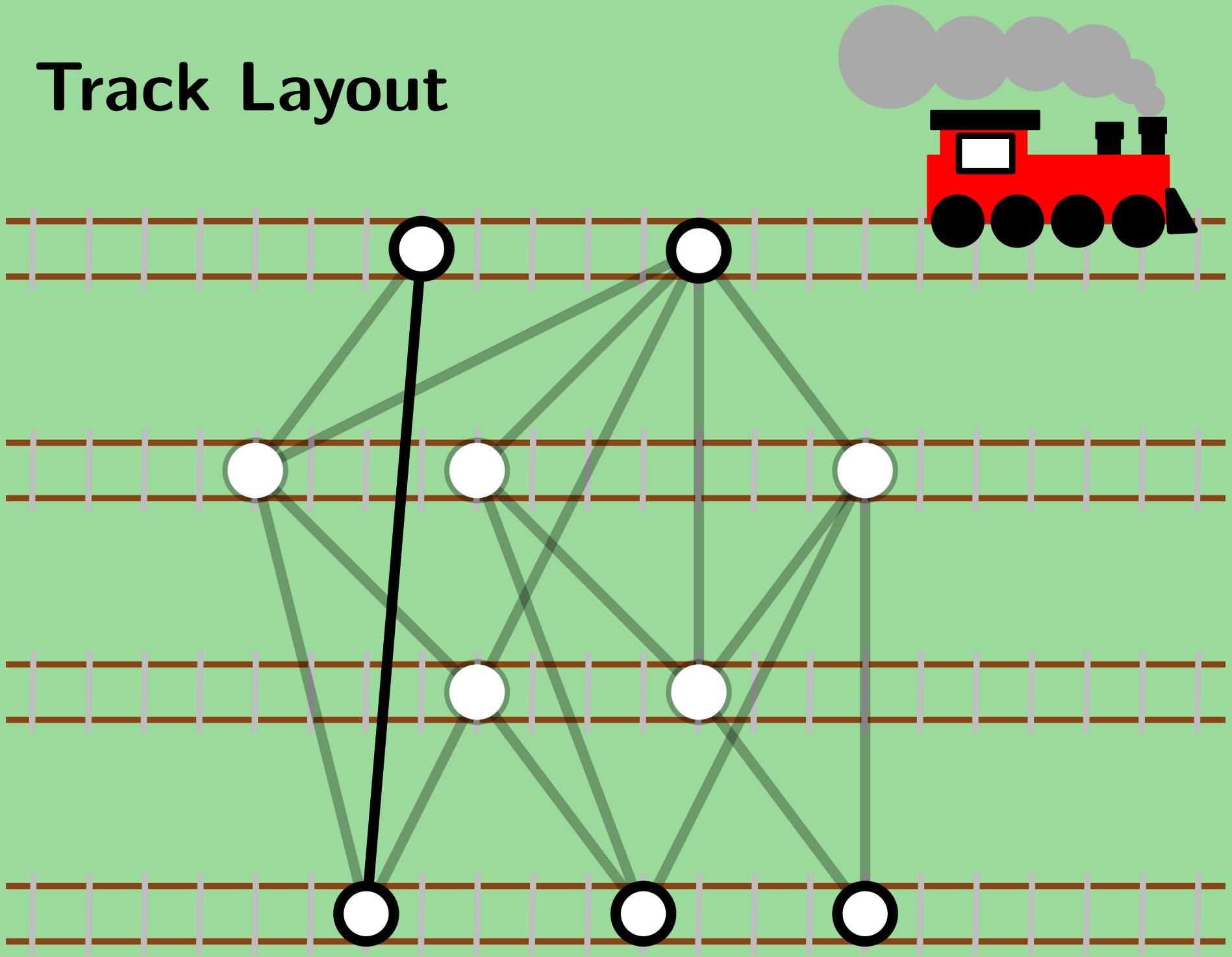

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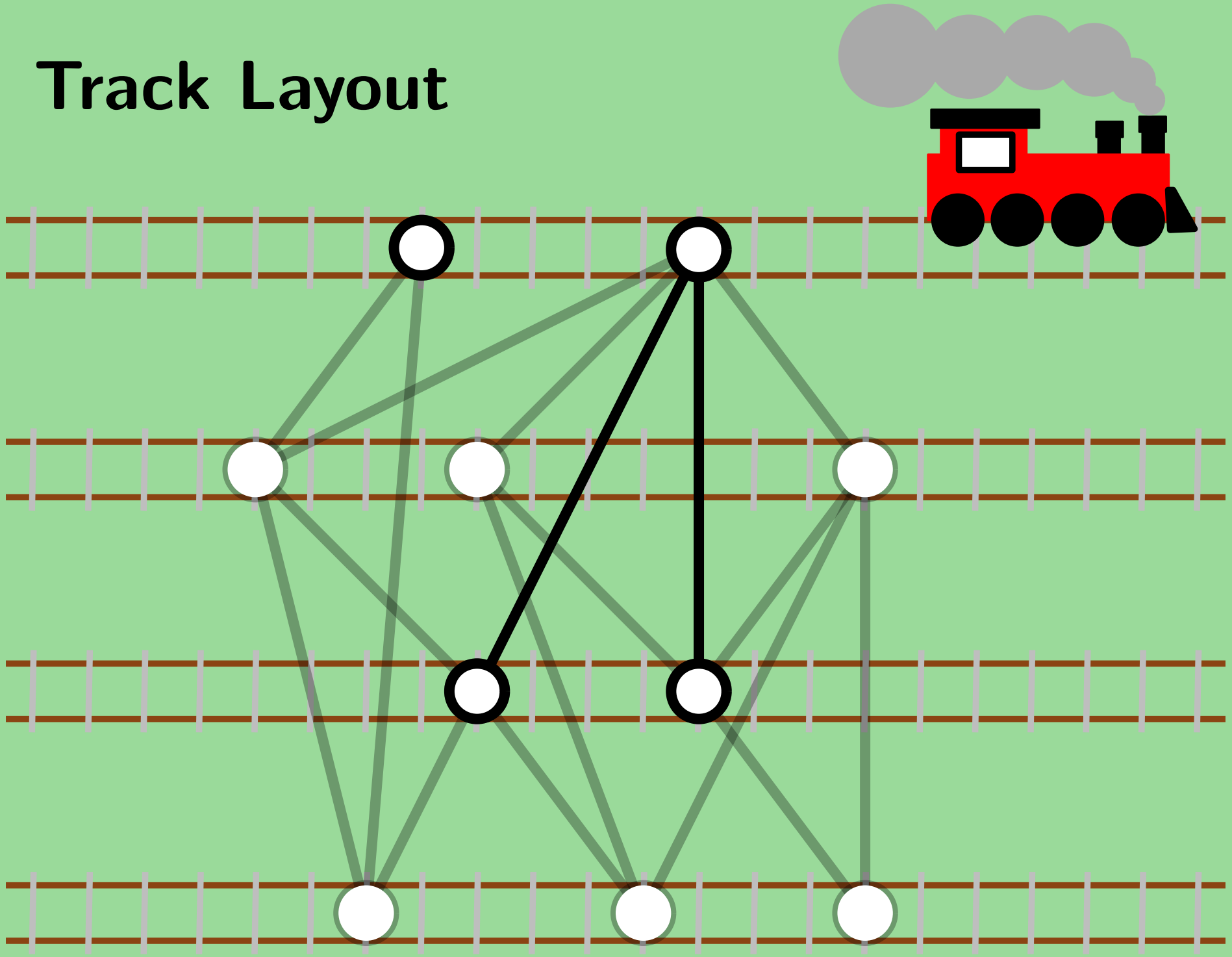

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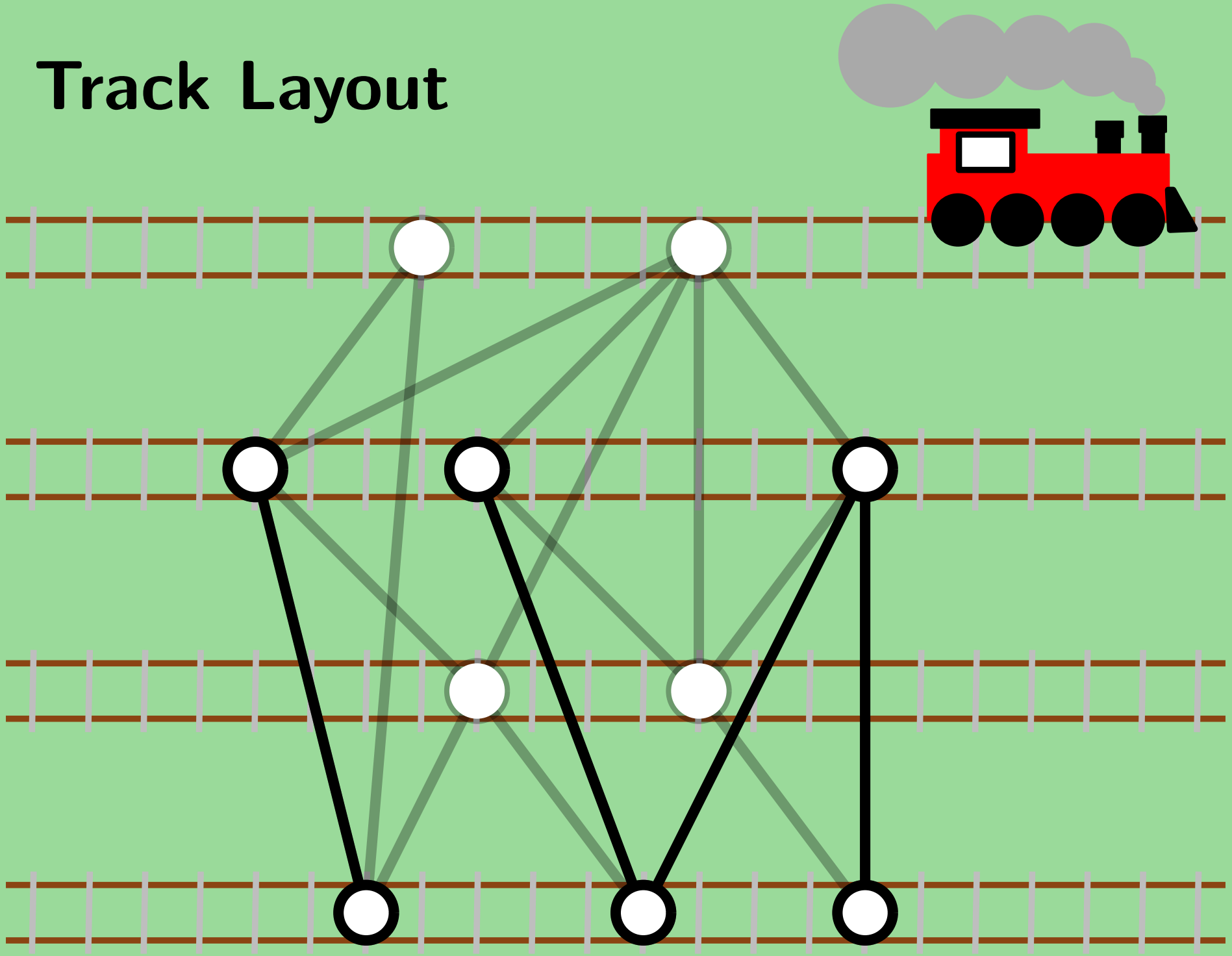

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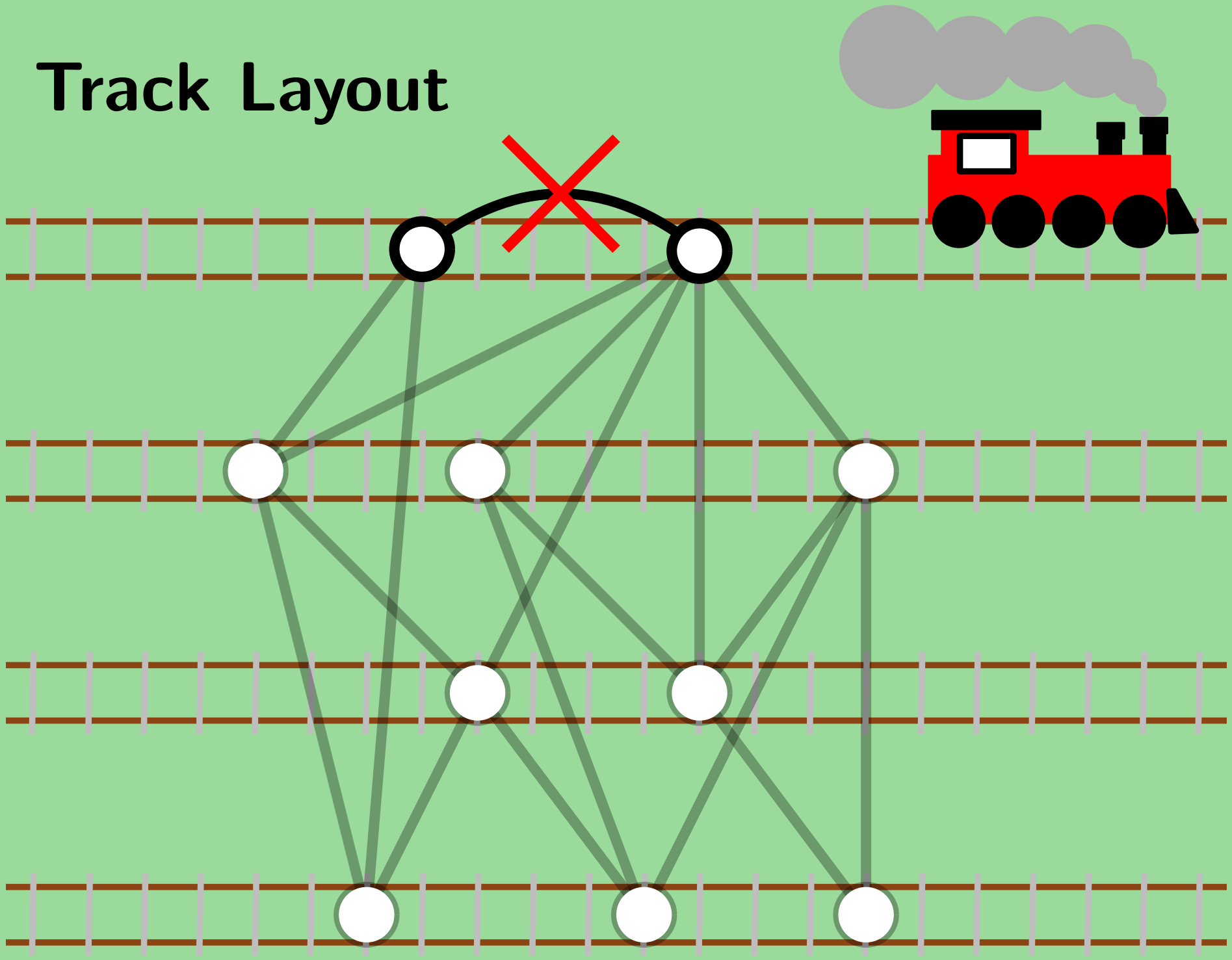

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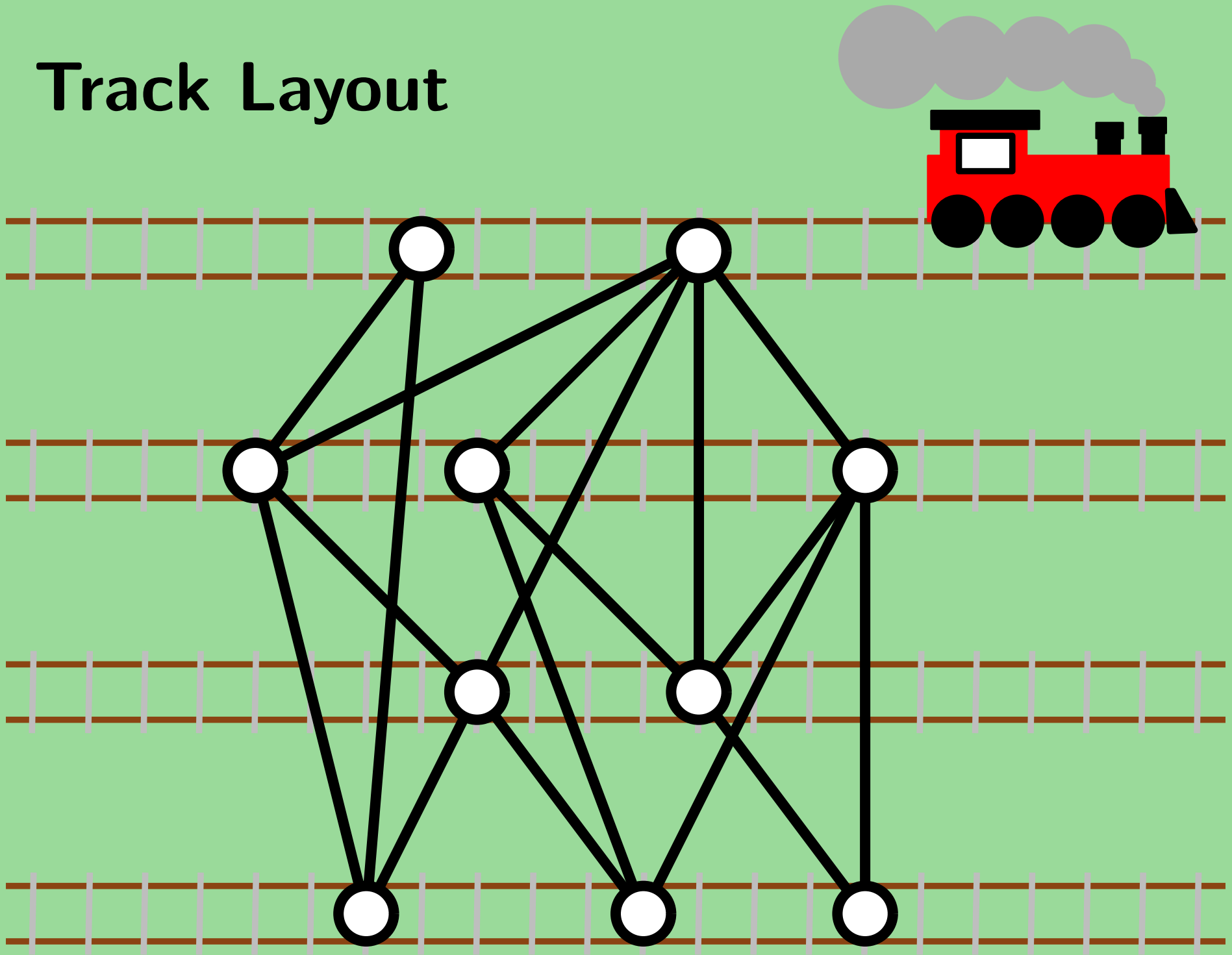

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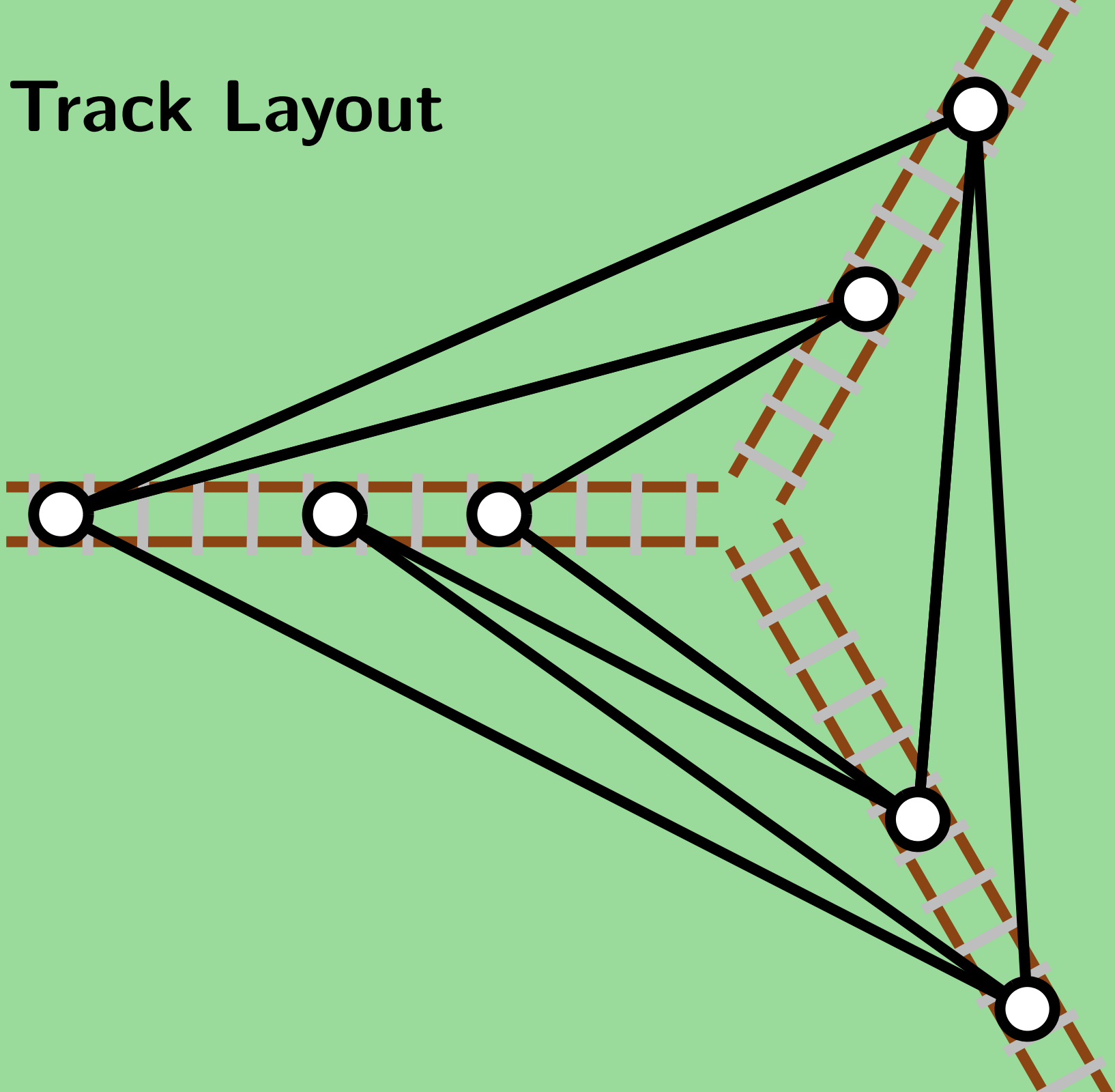
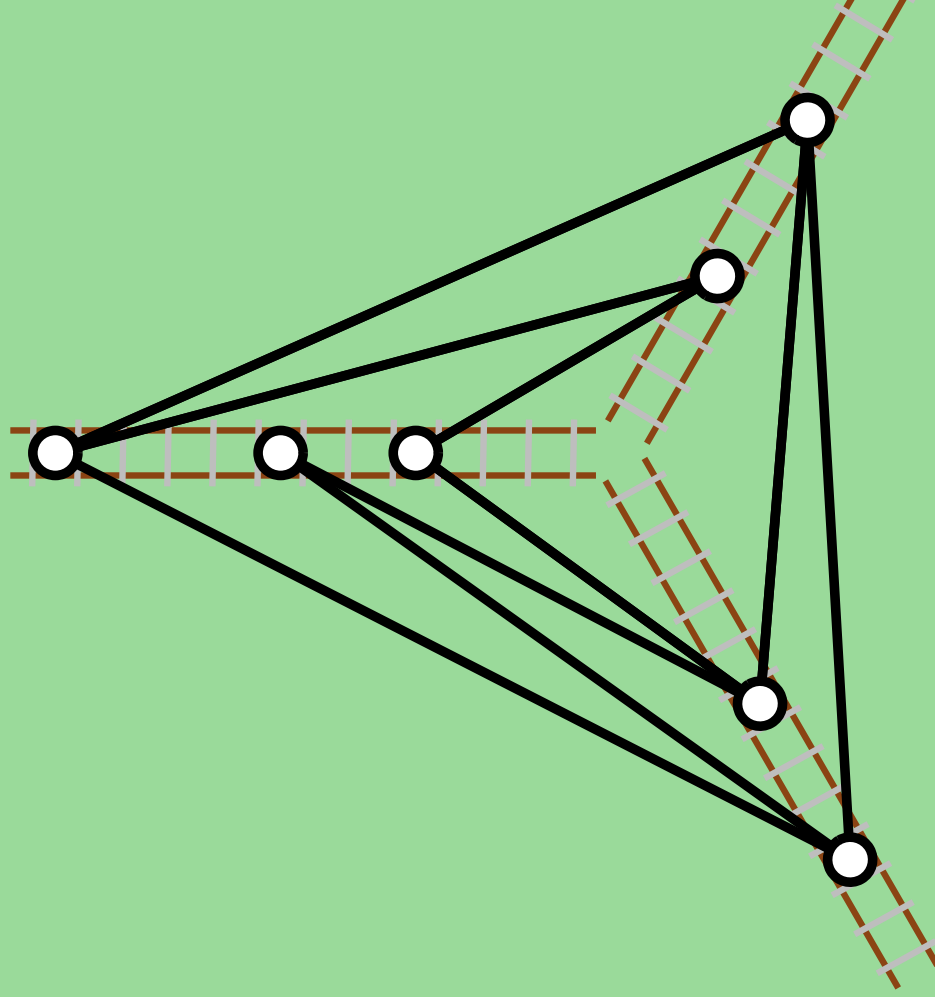


Track Layout

Track layouts find applications in minimizing the volume of 3D drawings

[Di Giacomo, 2004; Di Giacomo and Meijer, 2004; Dujmović and Wood, 2004; Felsner et al, 2003; Hasunuma, 2004]



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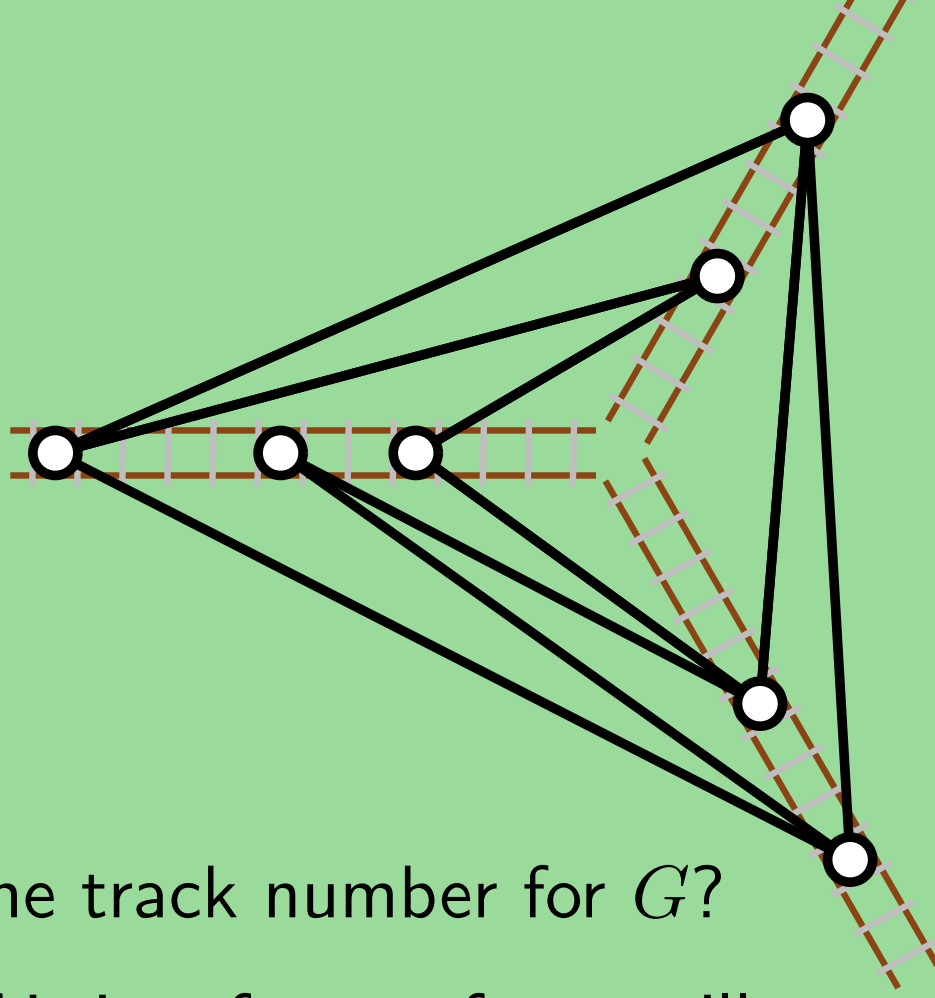
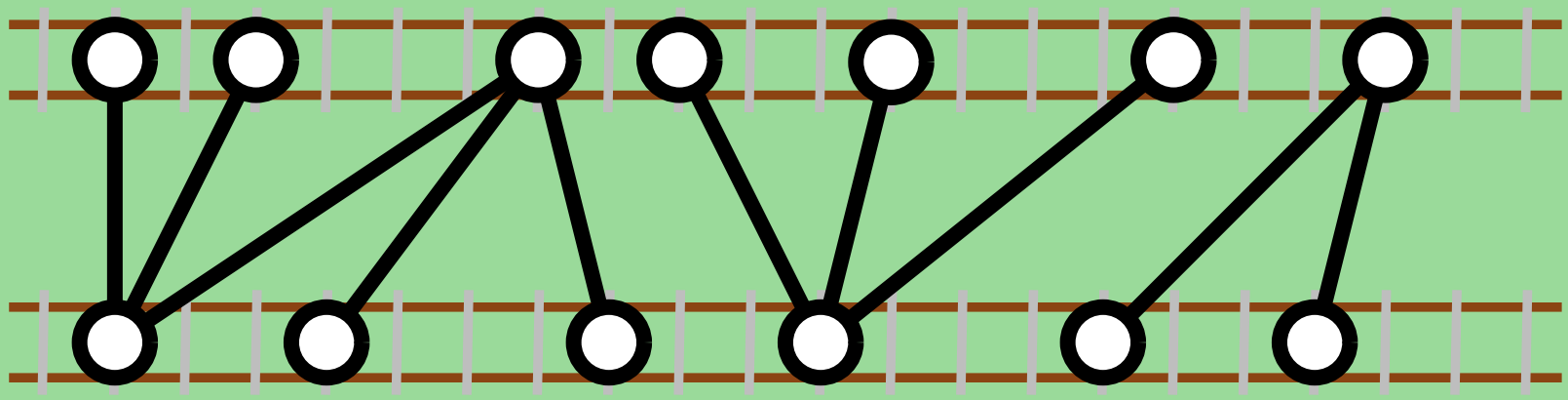


Given a graph G can we compute the track number for G?

A graph is 2-track if and only if it is a forest of caterpillars



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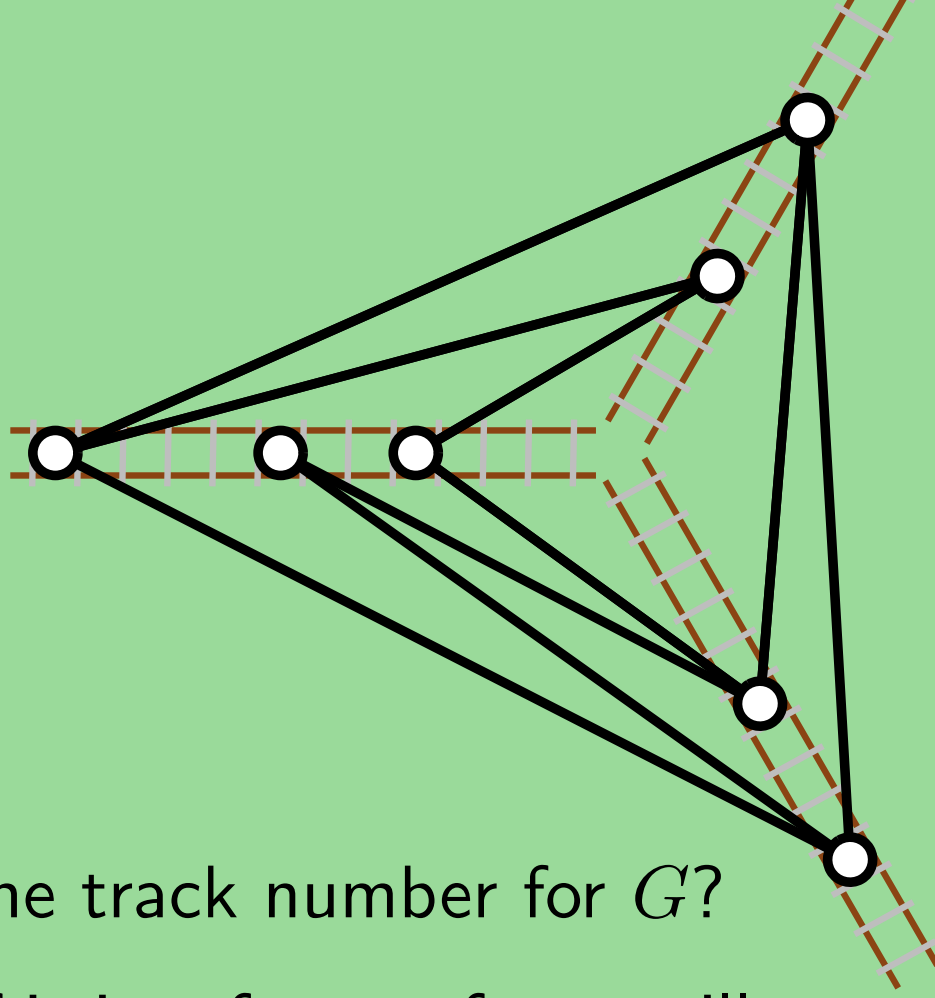

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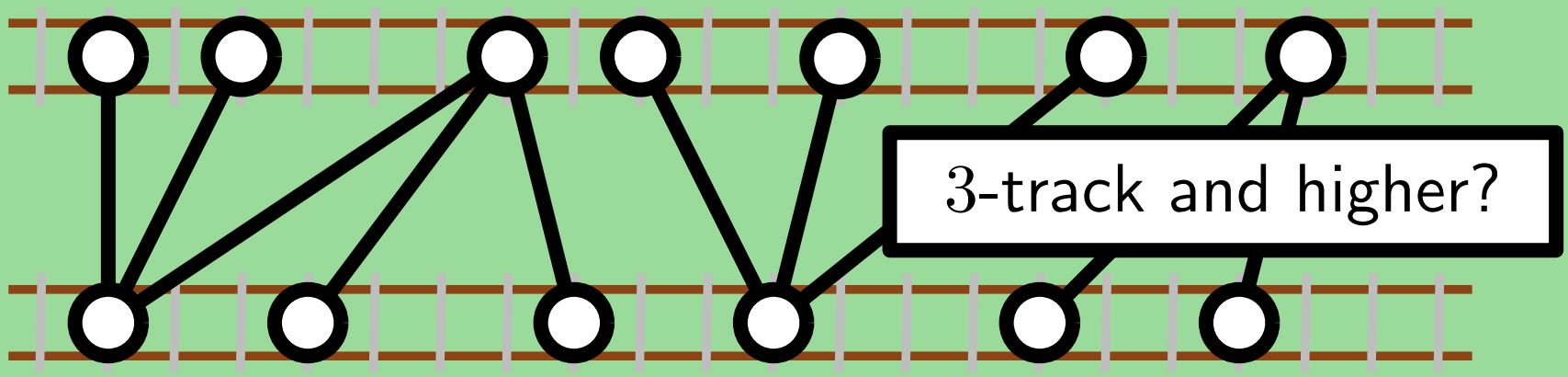


Leveled planarity

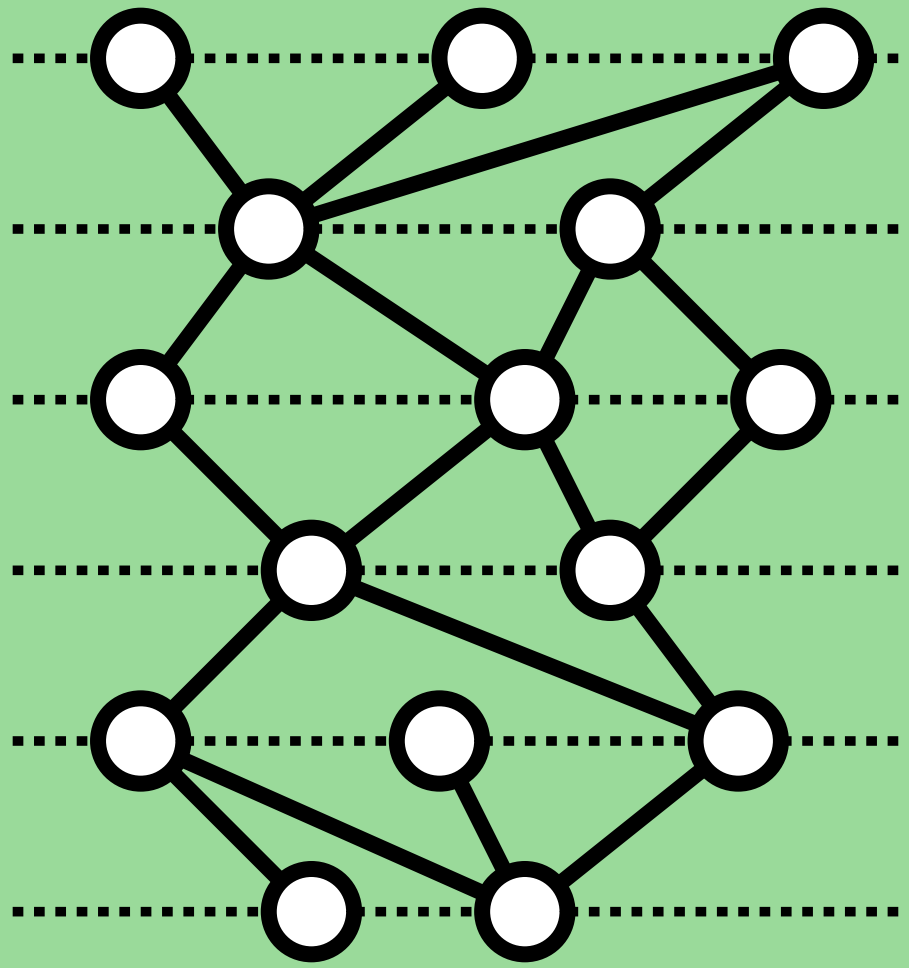
Perfect Sugiyama-style layered drawing

No edge crossing

No dummy vertices

Recognition is NP-complete

[Heath and Rosenberg, 1992]



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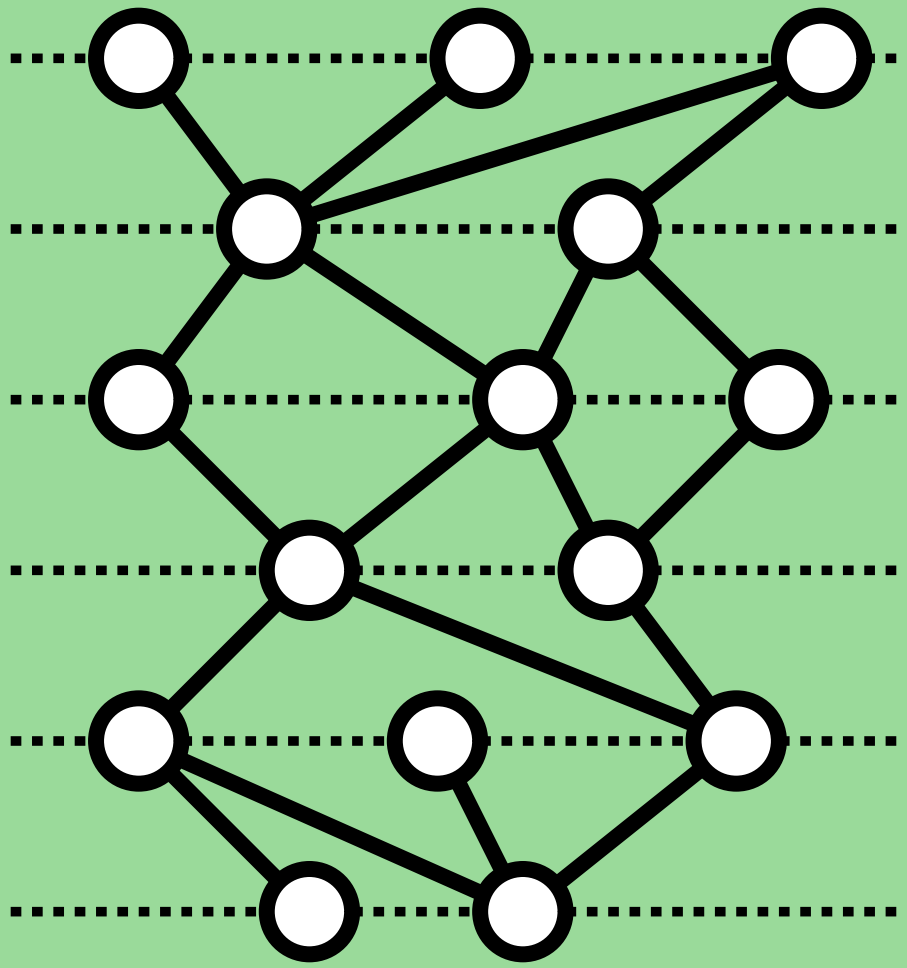
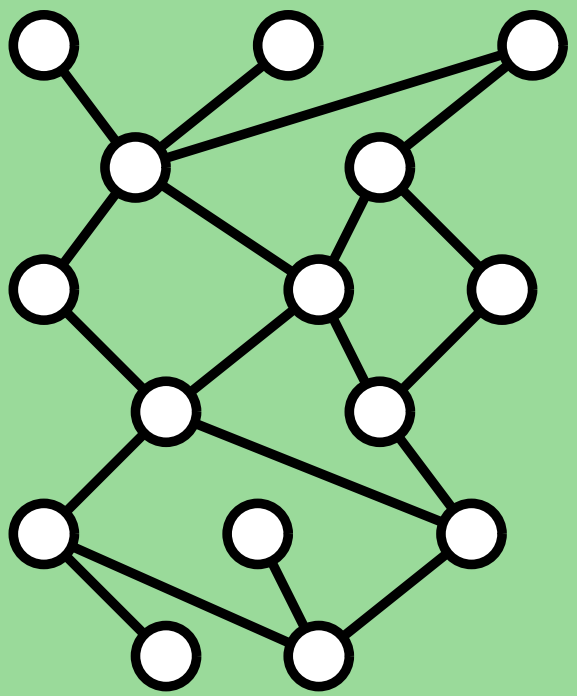


Will show leveled planar graphs are the same as bipartite 3-track graphs

Lemma: Every leveled planar graph has a 3-track layout.



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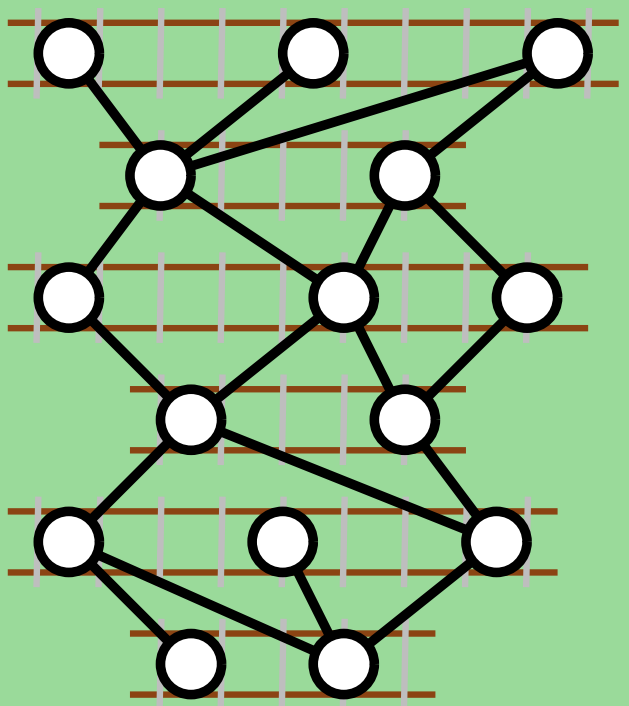

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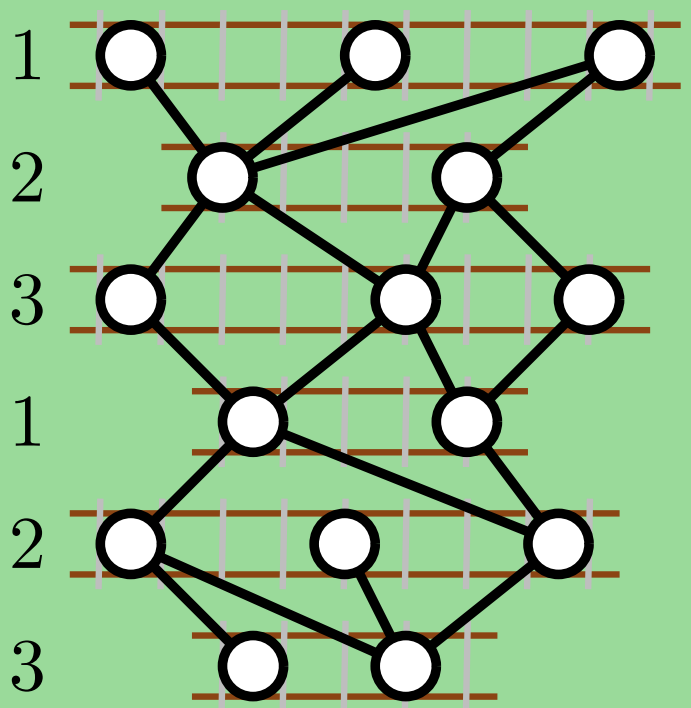

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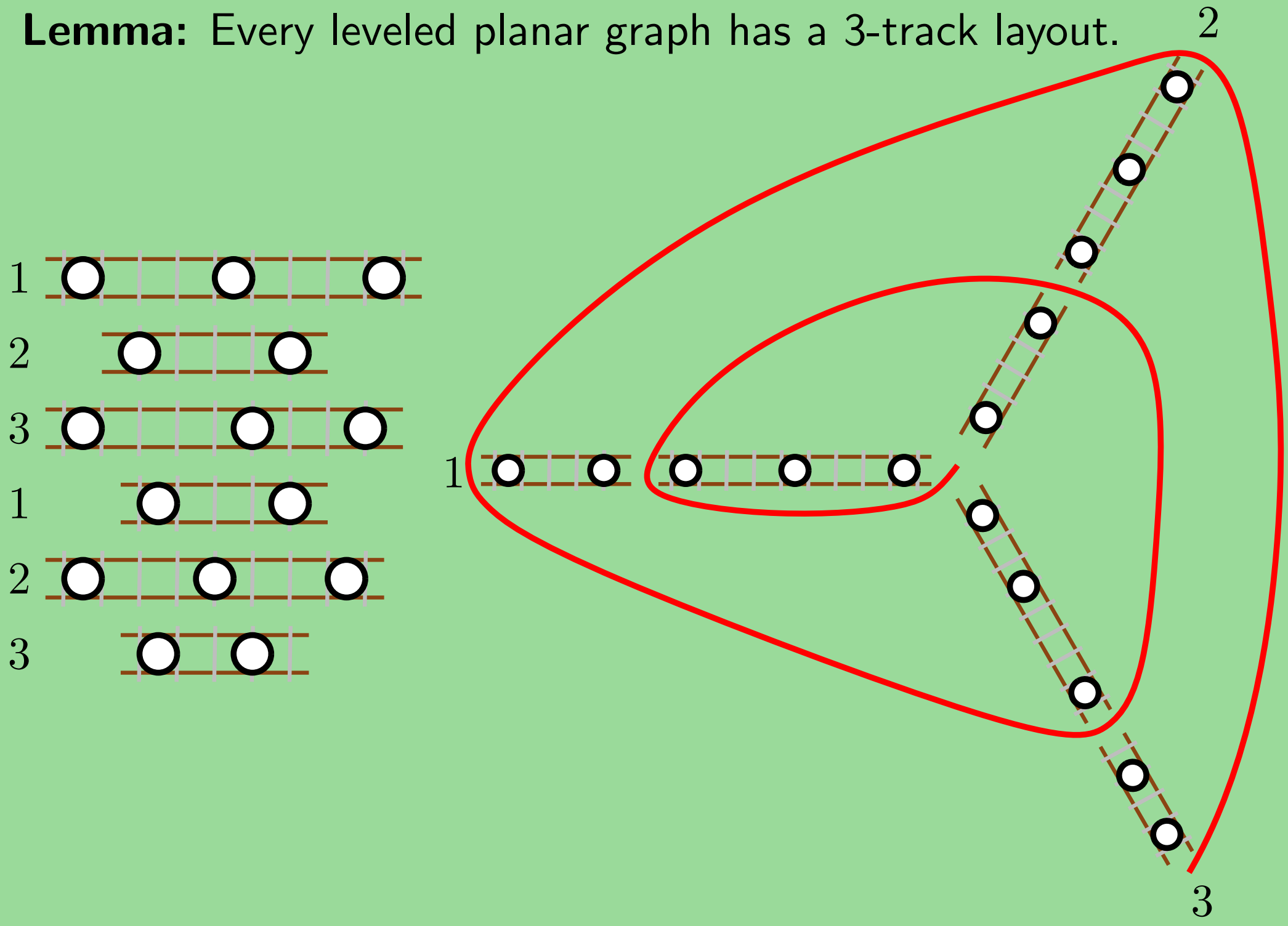

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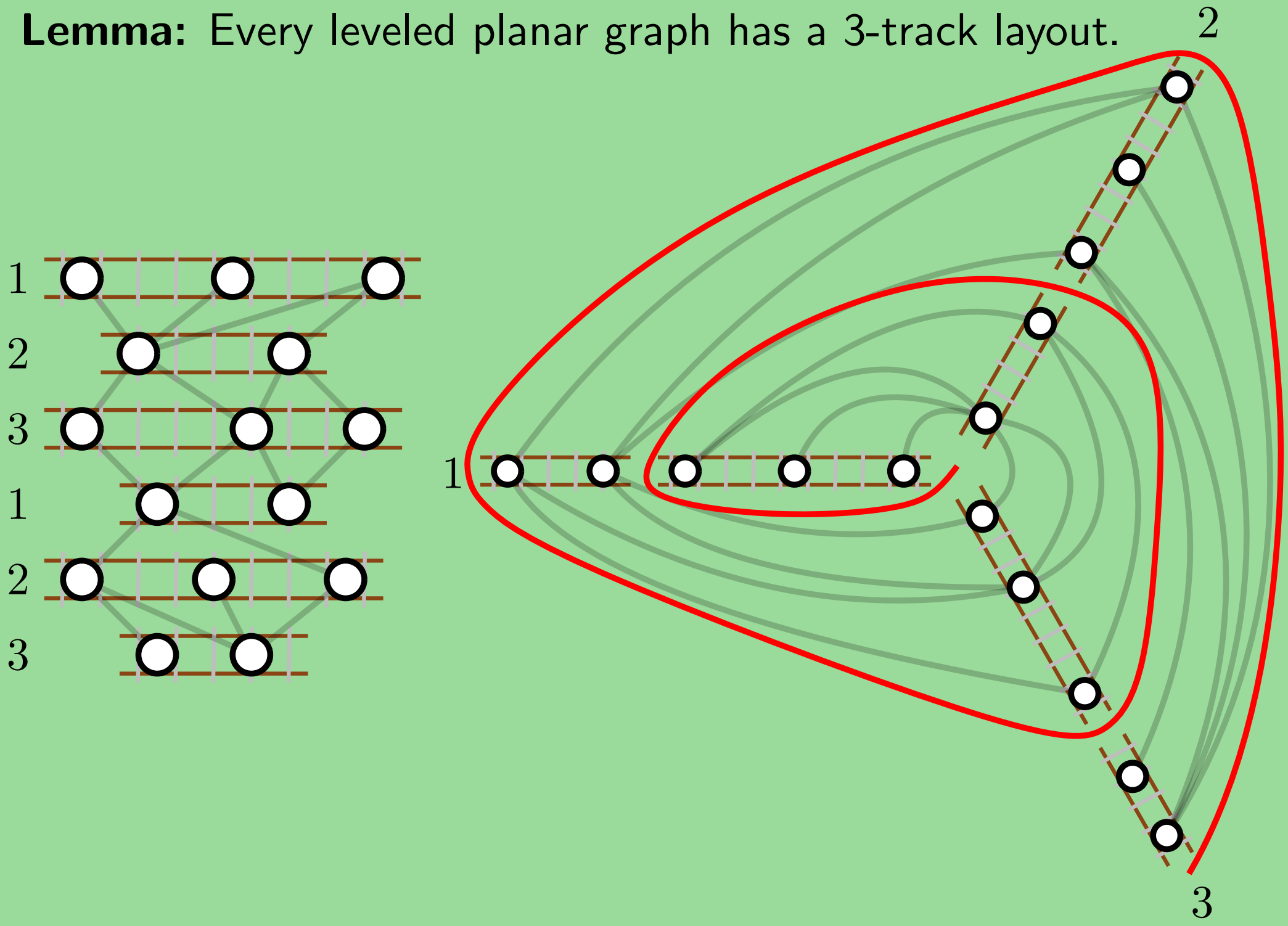

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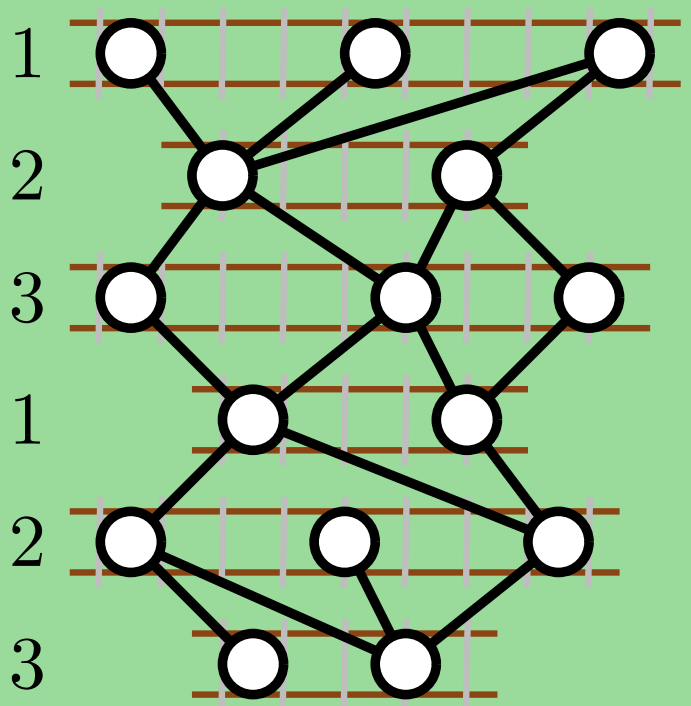
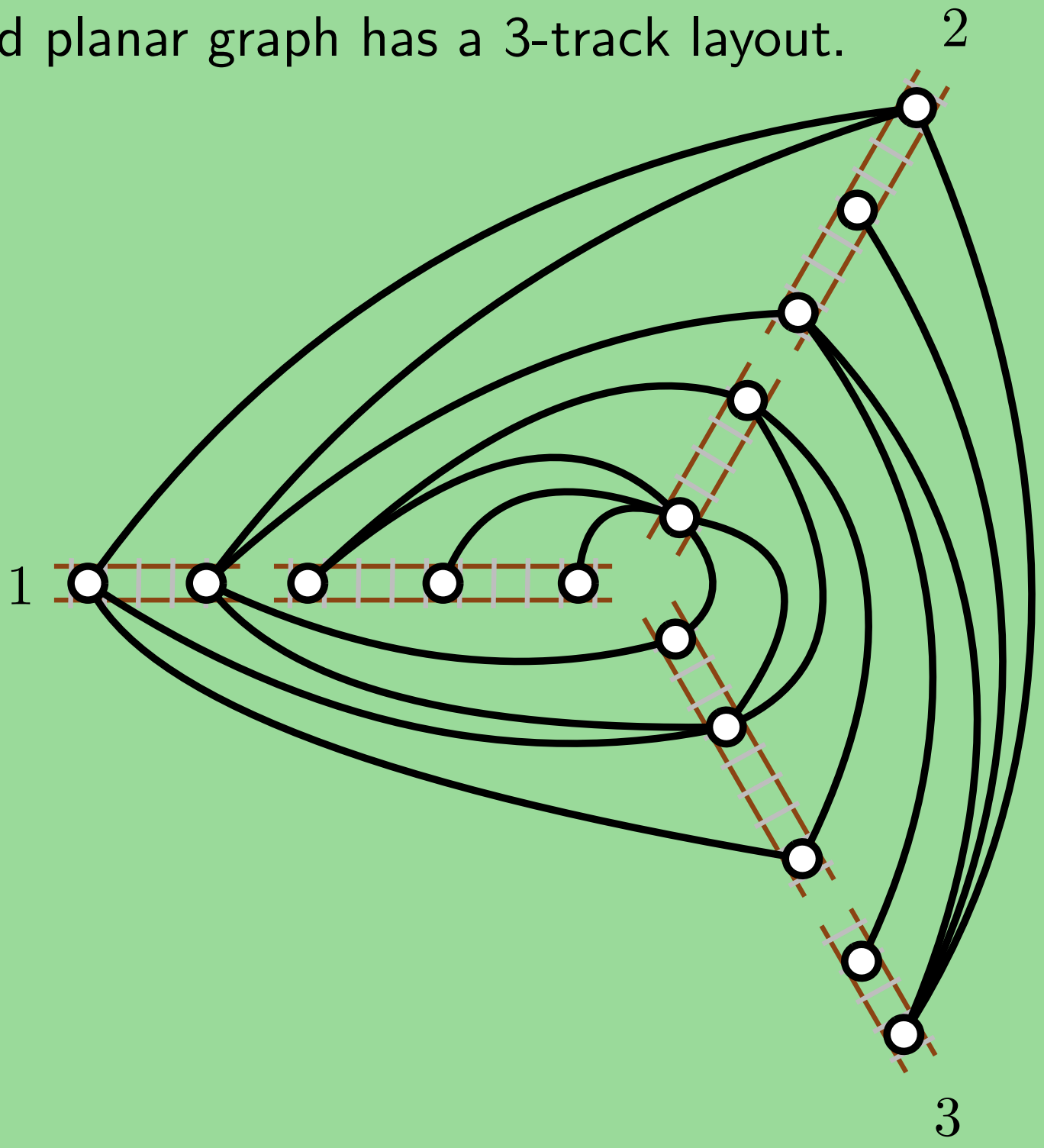
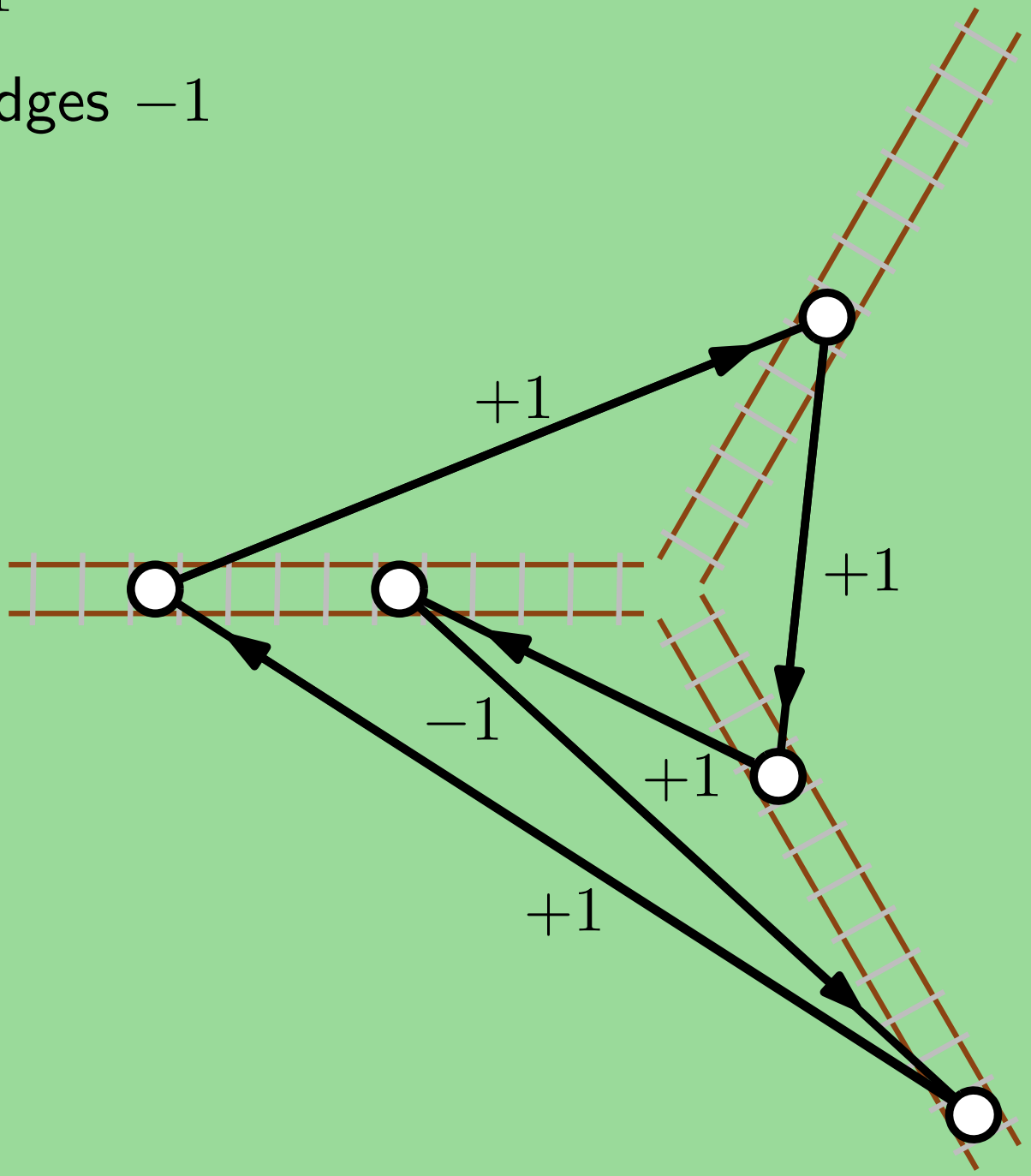




Label clockwise edges $+1$

Label counterclockwise edges -1



Label clockwise edges $+1$

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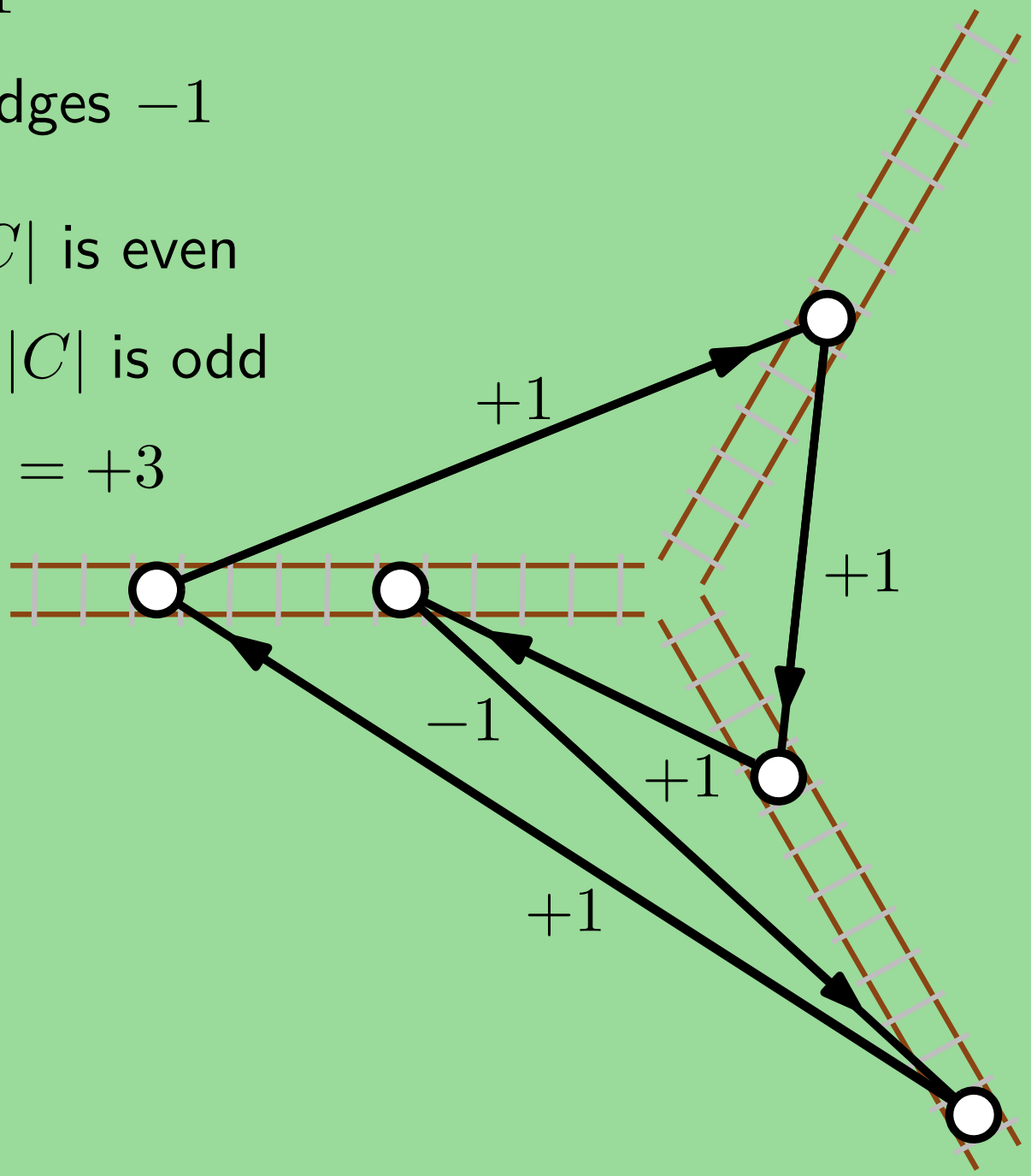
Lemma:

$$\sum_{e \in C} \text{label}(e) = \begin{cases} 0 \text{ if } |C| \text{ is even} \\ \pm 3 \text{ if } |C| \text{ is odd} \end{cases}$$

$$+1 + 1 + 1 - 1 + 1 = +3$$

Proof:



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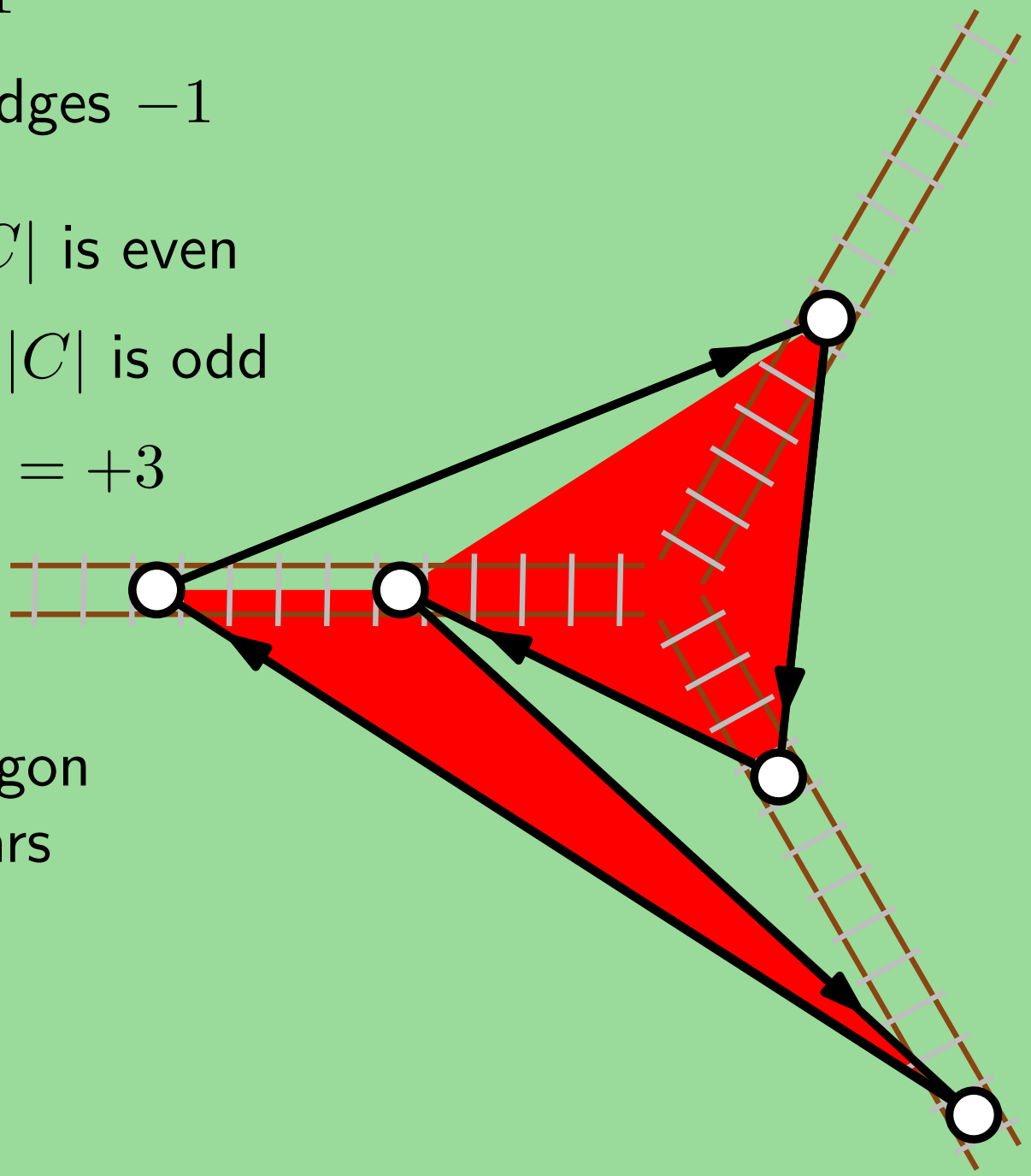

Proof:

The cycle forms a polygon
and has at least two ears

Label clockwise edges $+1$

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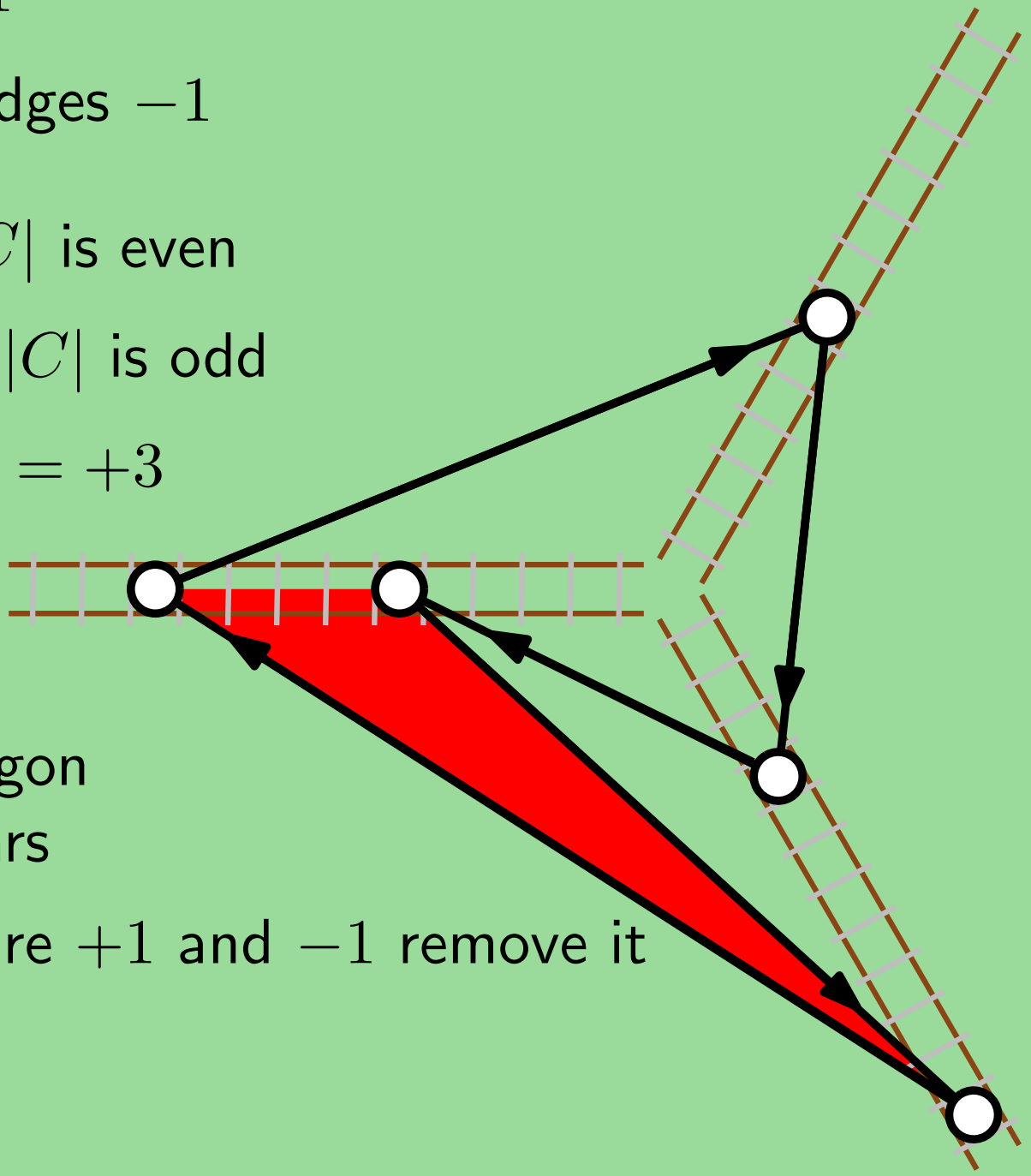

Proof:

The cycle forms a polygon
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If the edges of an ear are $+1$ and -1 remove it

Label clockwise edges $+1$

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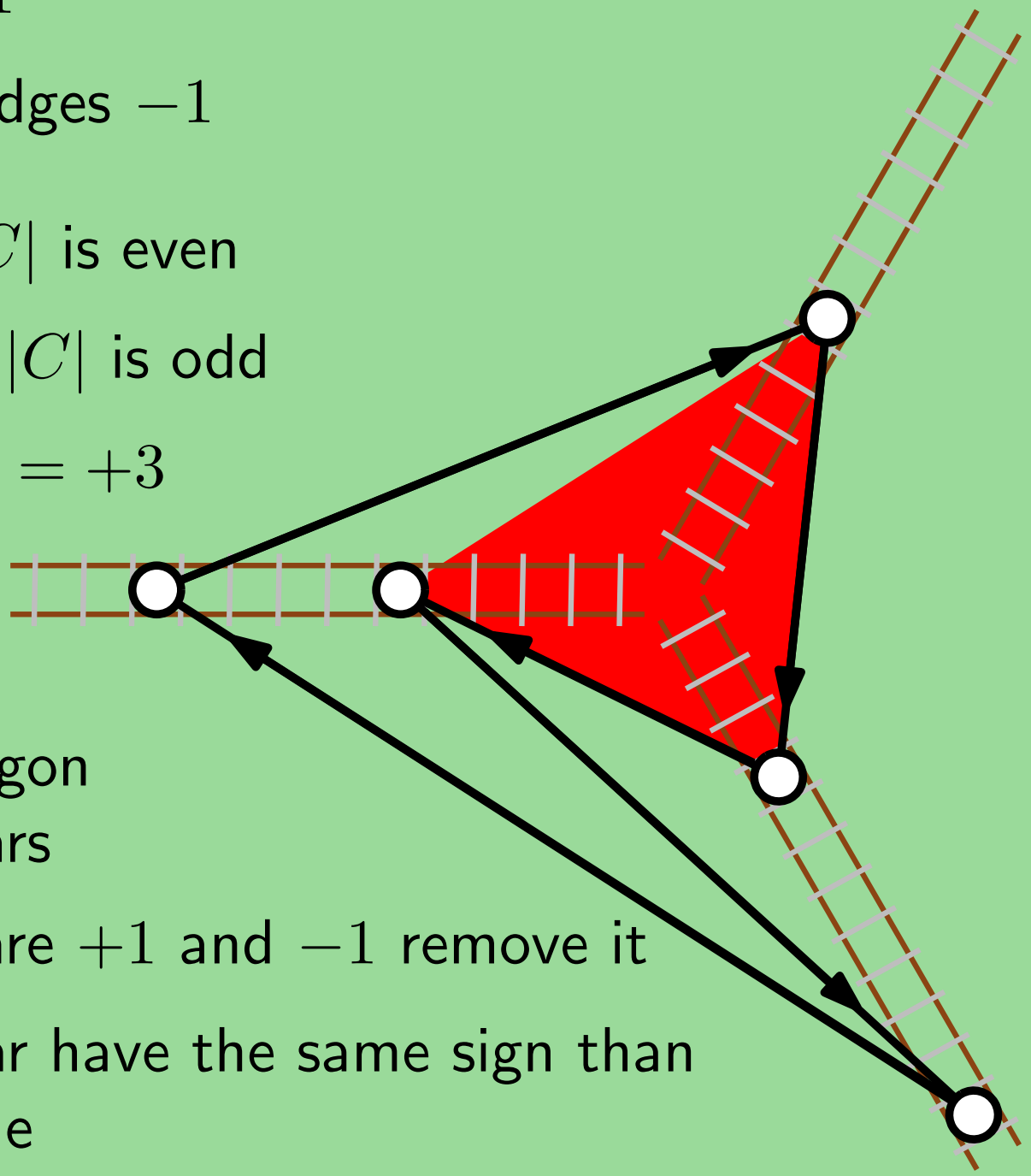

Proof:

The cycle forms a polygon and has at least two ears

If the edges of an ear are $+1$ and -1 remove it

If the edges of every ear have the same sign than the polygon is a triangle

Label clockwise edges $+1$

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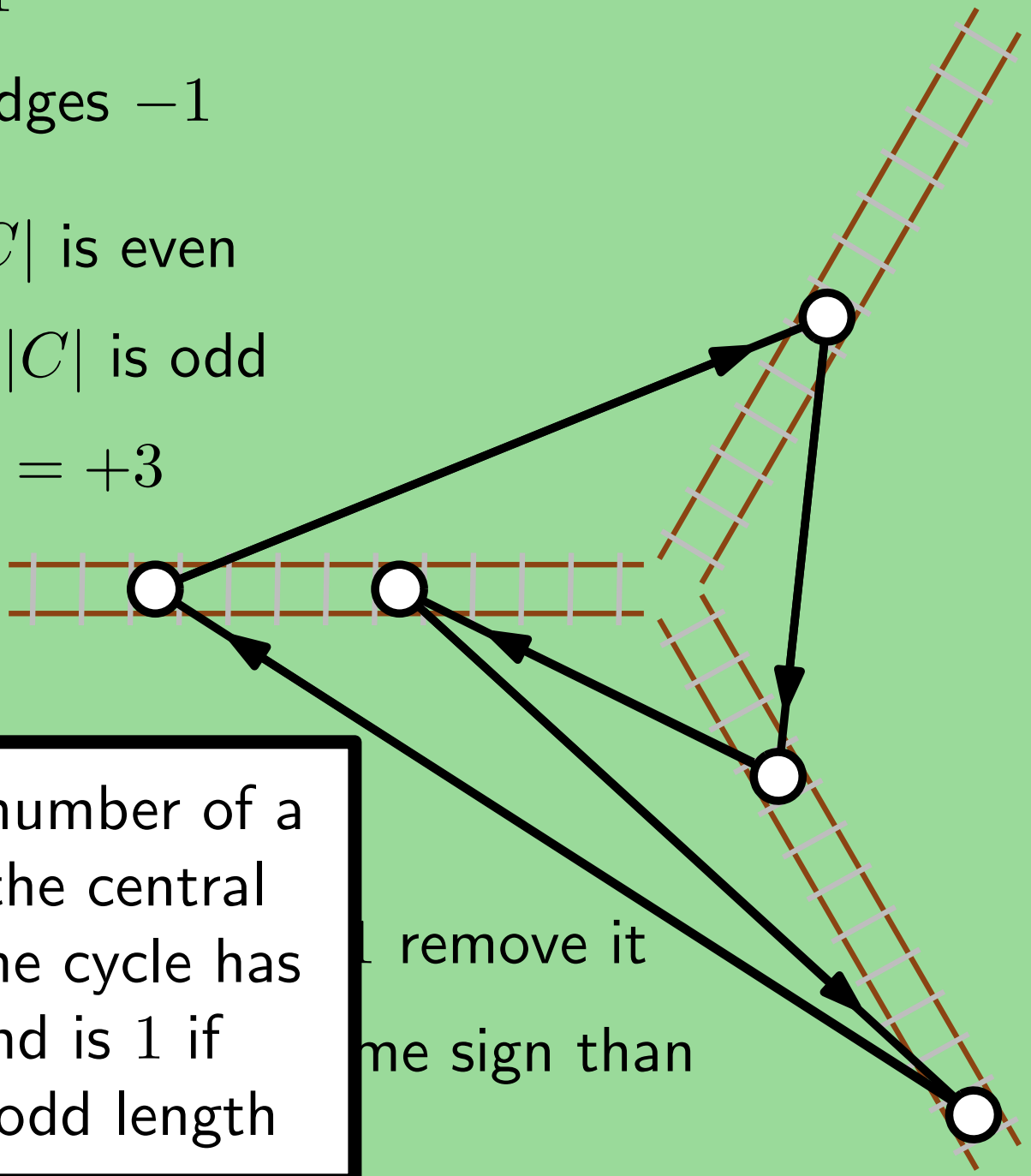

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If the e remove it

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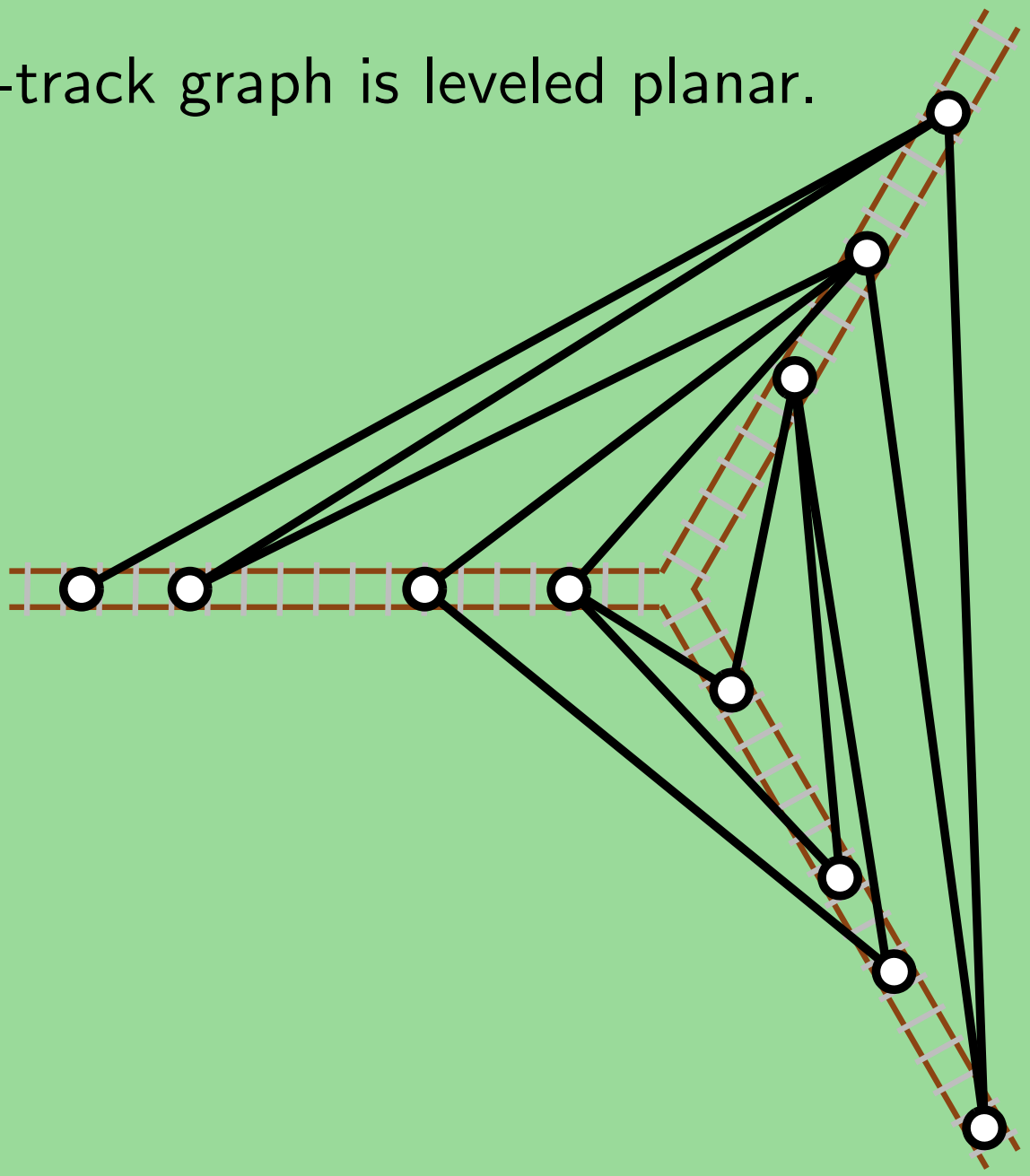
The winding number of a cycle around the central point is 0 if the cycle has even length and is 1 if the cycle has odd length

Lemma: Every bipartite 3-track graph is leveled planar.



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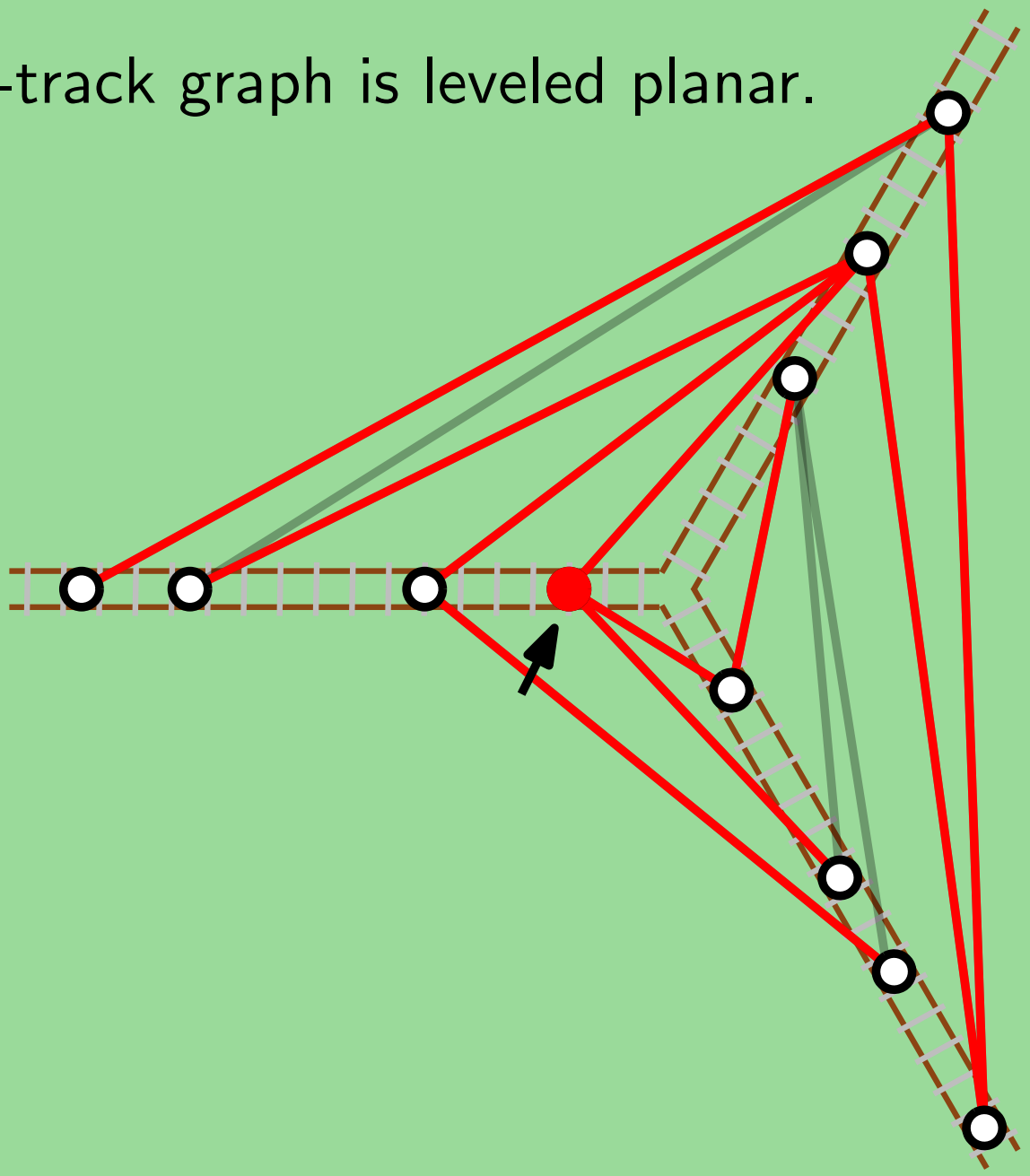

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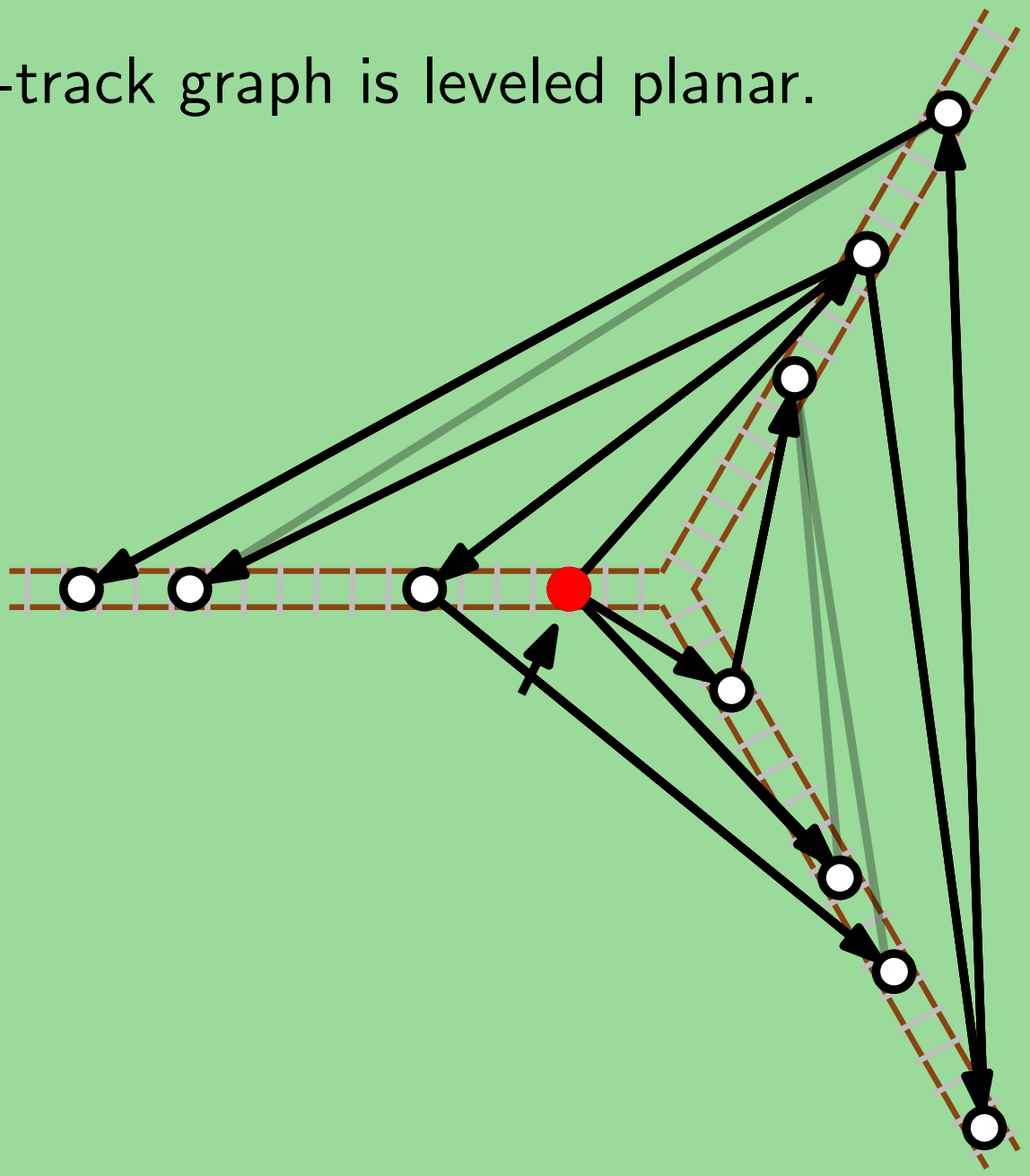

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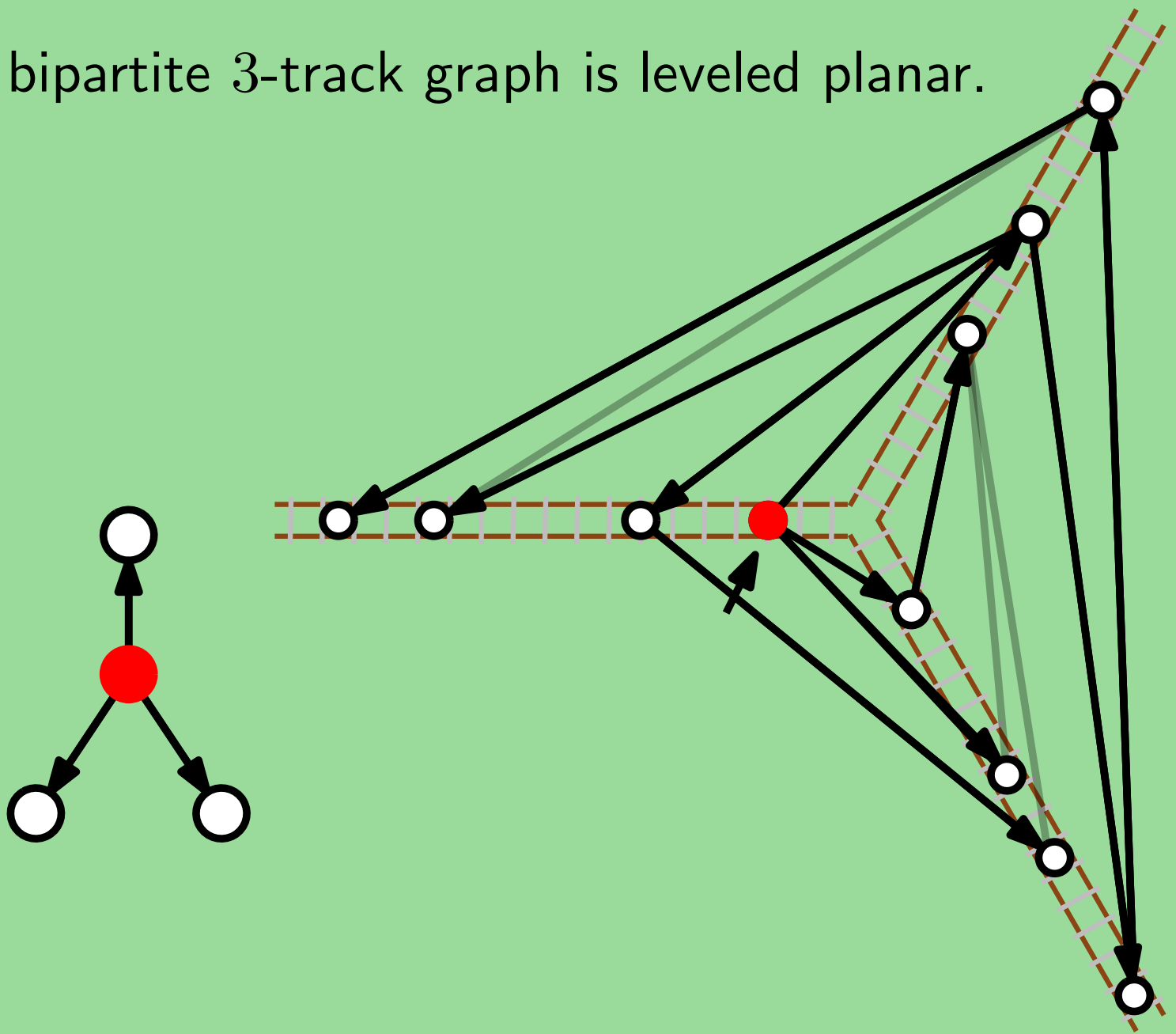

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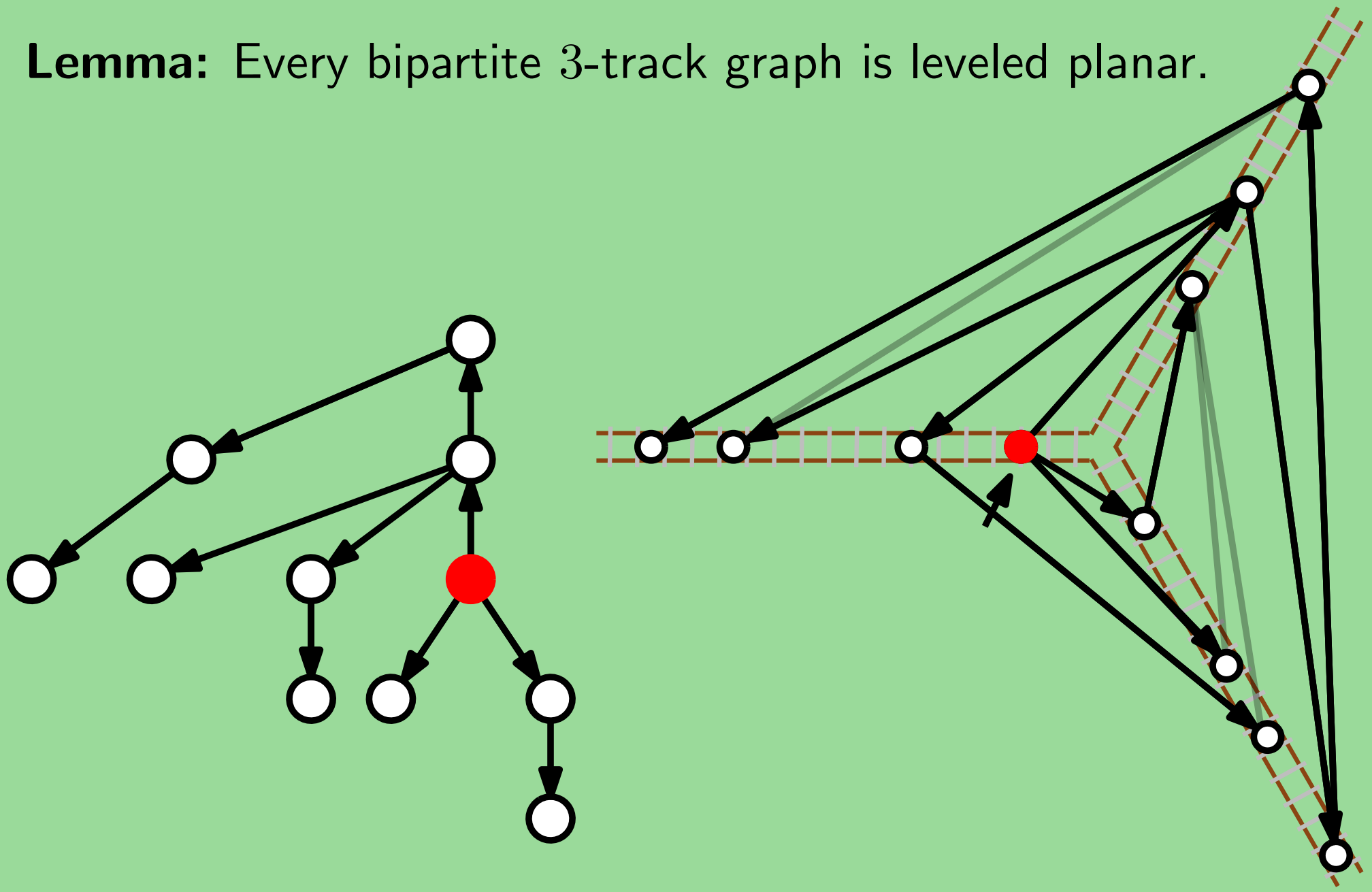

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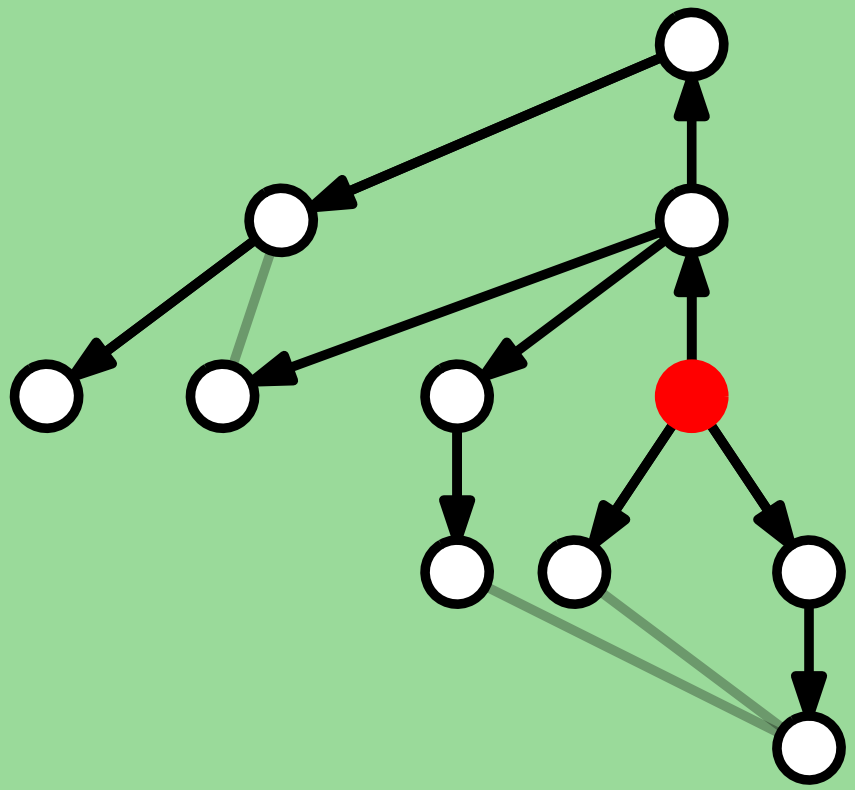

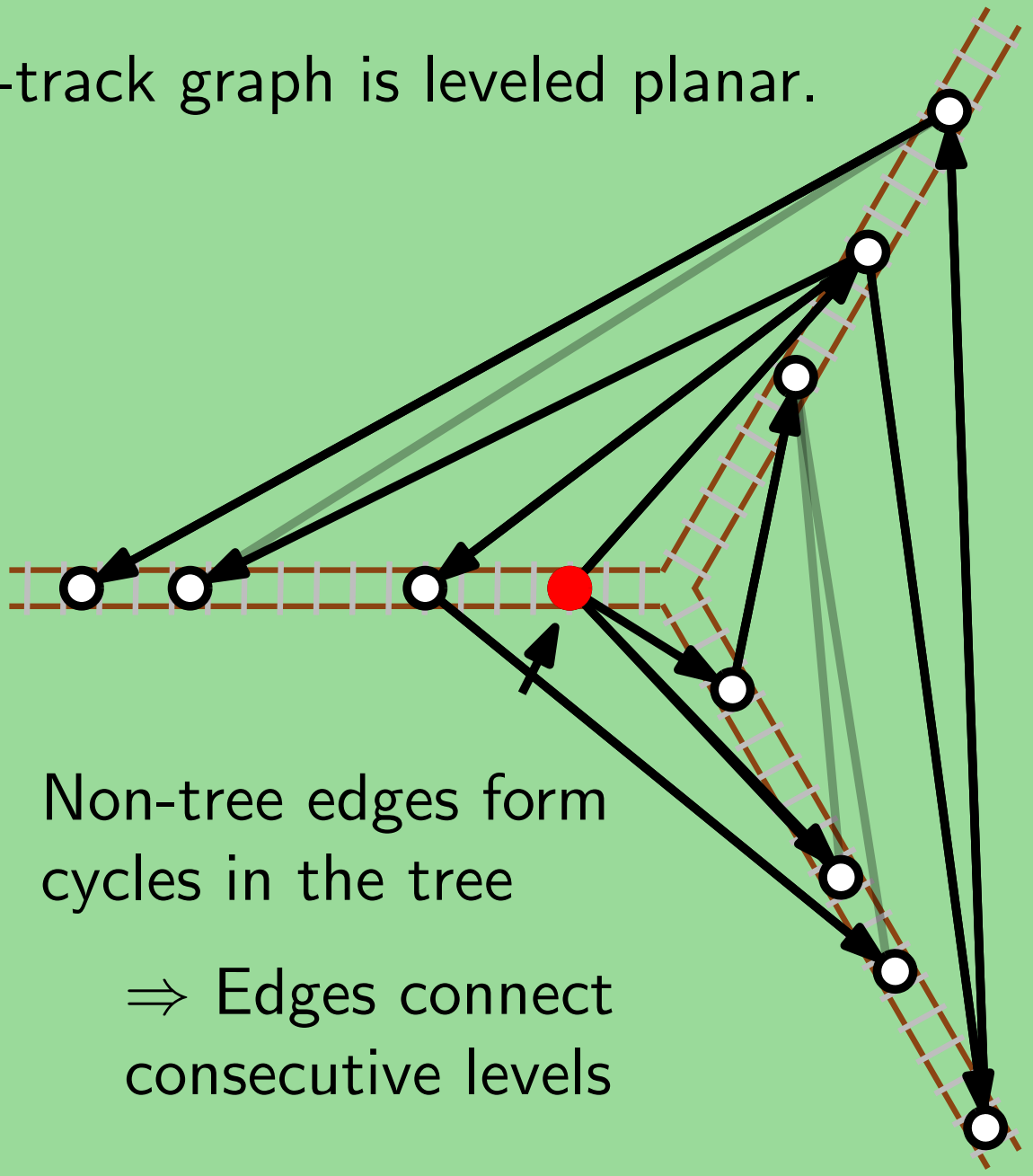

Non-tree edges form cycles in the tree

$\Rightarrow$ Edges connect consecutive levels

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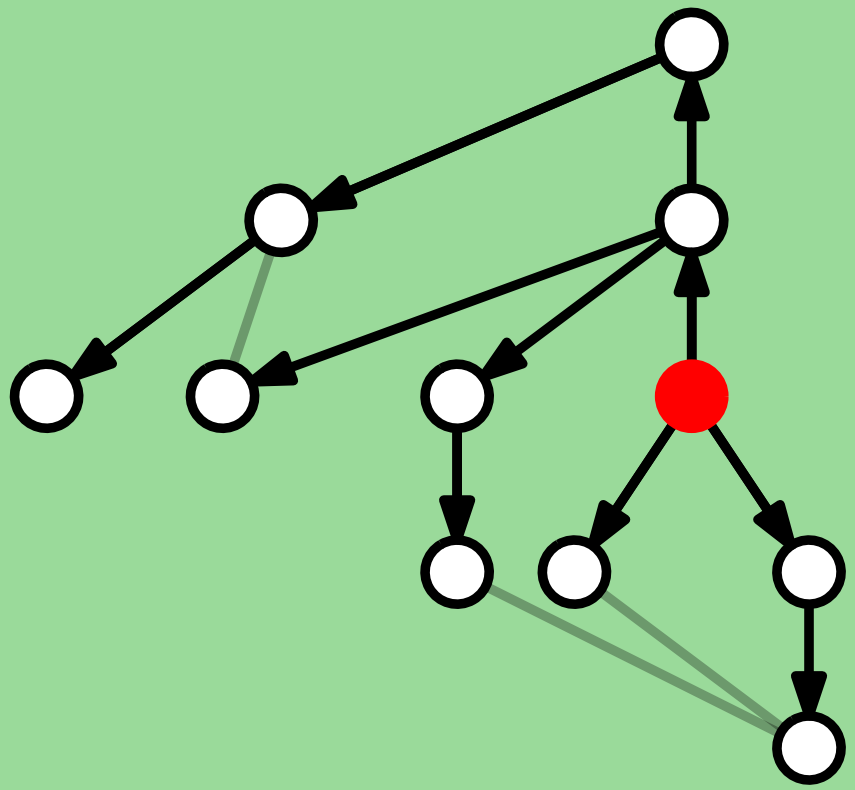
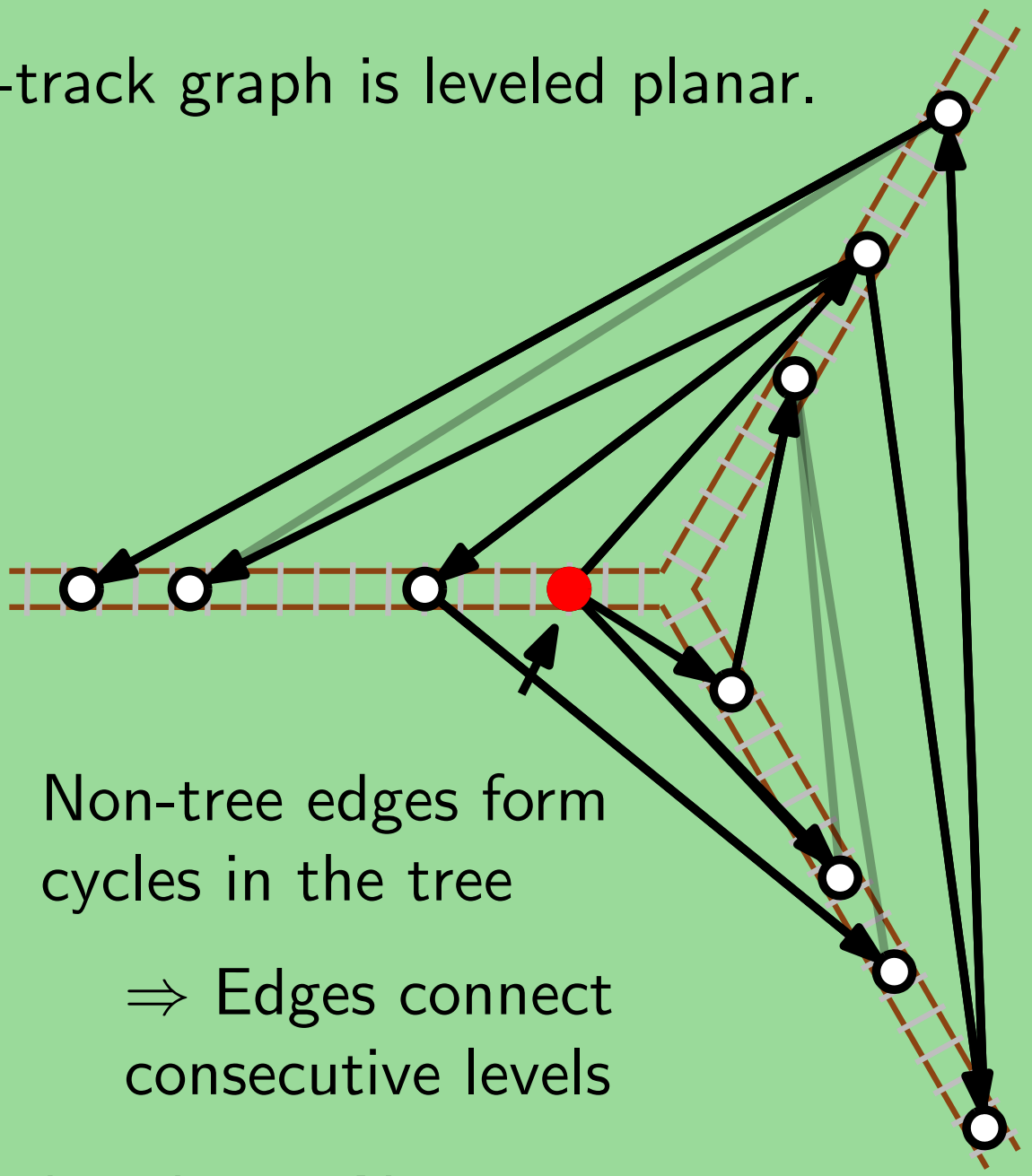




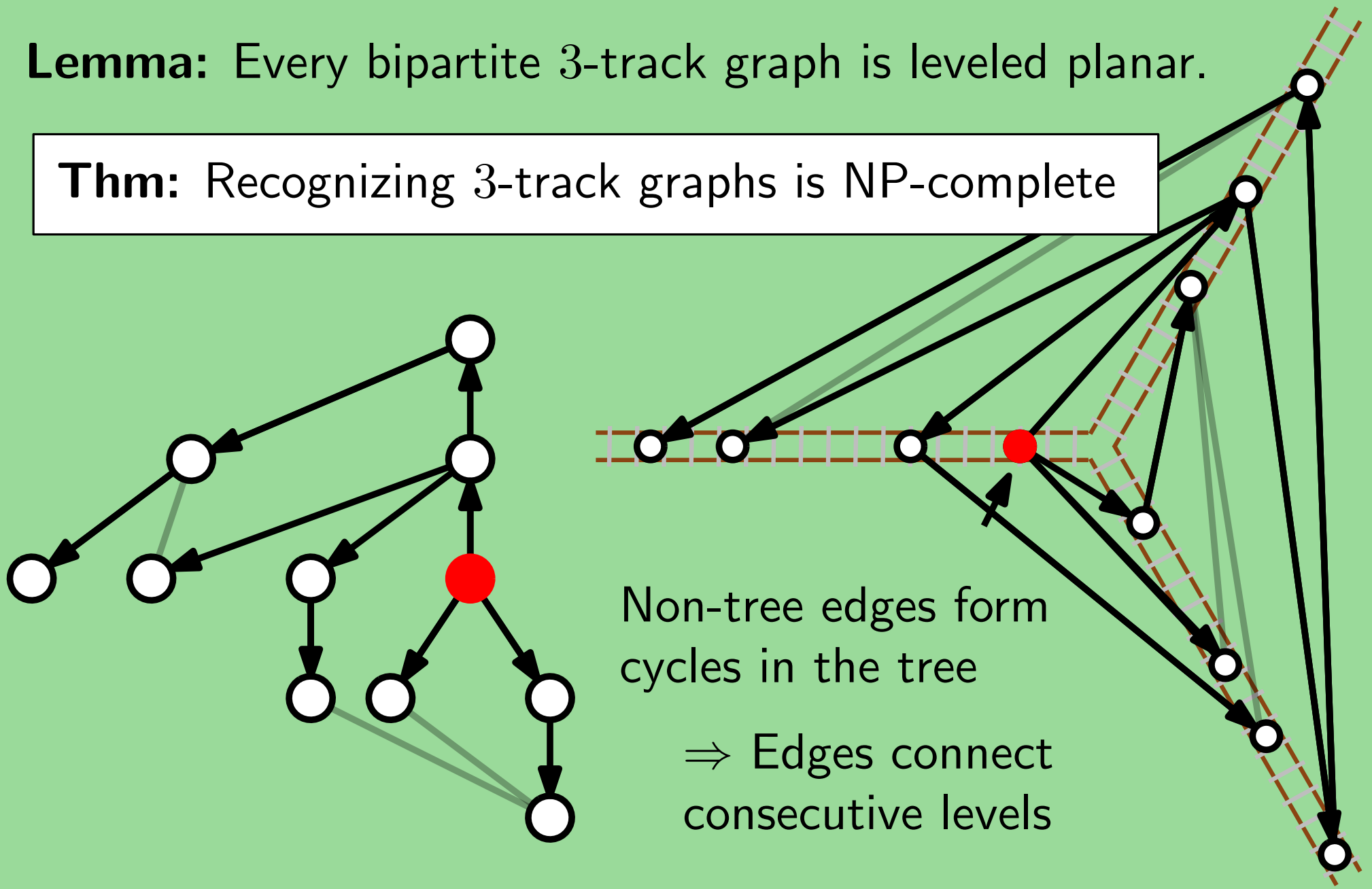
Non-tree edges form cycles in the tree

$\Rightarrow$ Edges connect consecutive levels

Order vertices by their track order $\Rightarrow$ No crossings

Lemma: Every bipartite 3-track graph is leveled planar.

Thm: Recognizing 3-track graphs is NP-complete



Non-tree edges form cycles in the tree

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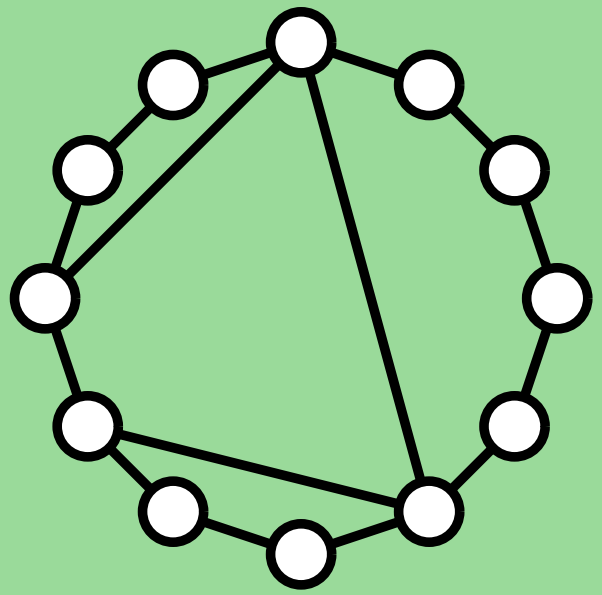
Which 3-track graphs can we recognize and layout 3-track?

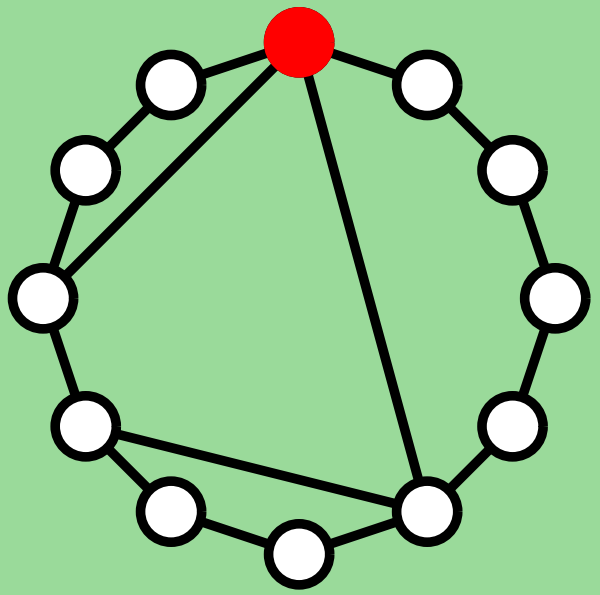
Bipartite Outerplanar Graphs

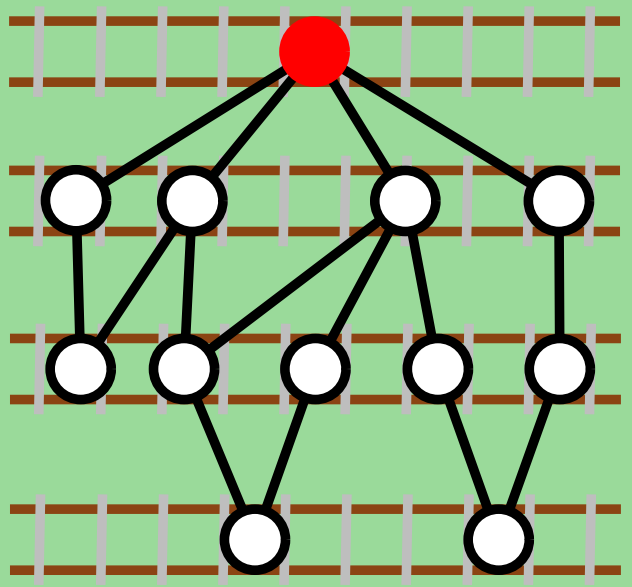


Which 3-track graphs can we recognize and layout 3-track?

Bipartite Outerplanar Graphs

Pick a vertex and perform a breadth first layering





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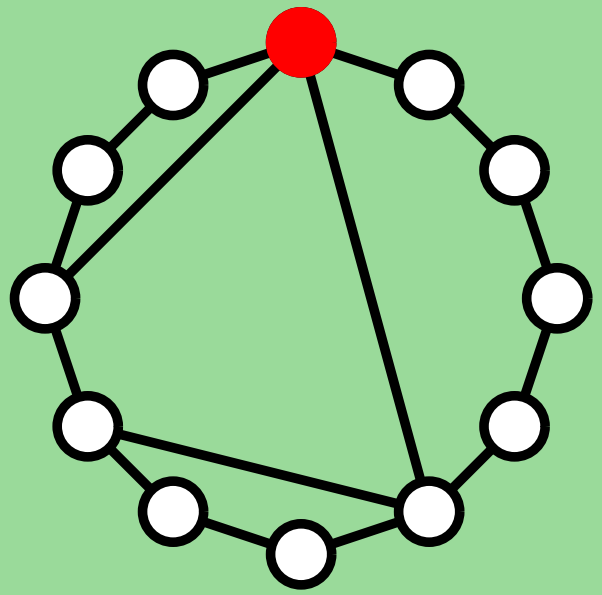
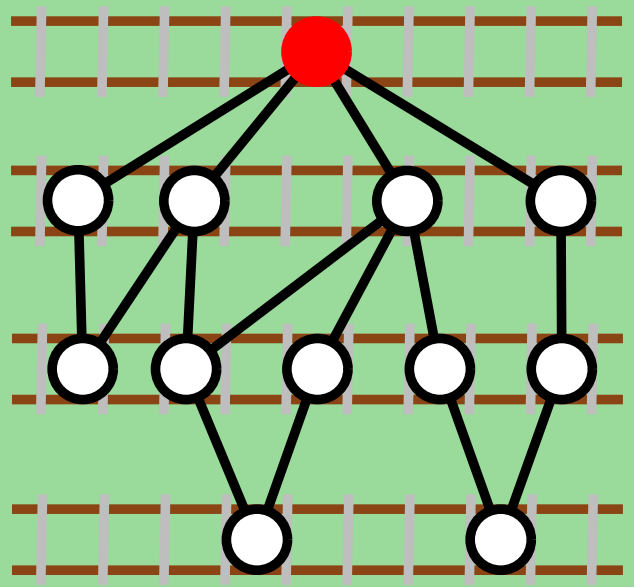
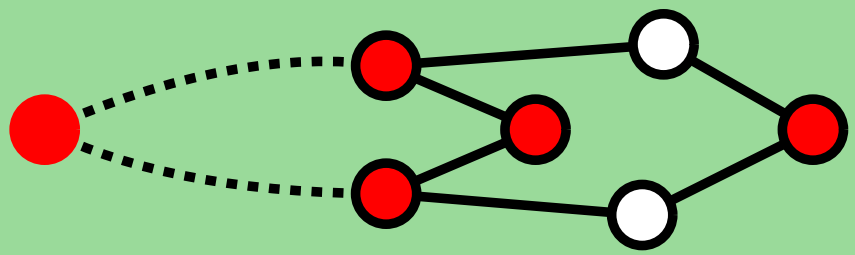




How to layout vertices with a level?

Each face cycle has a closest vertex to the chosen vertex



Otherwise there is a $K_{2,3}$ subdivision

Which 3-track graphs can we recognize and layout 3-track?

Bipartite Outerplanar Graphs

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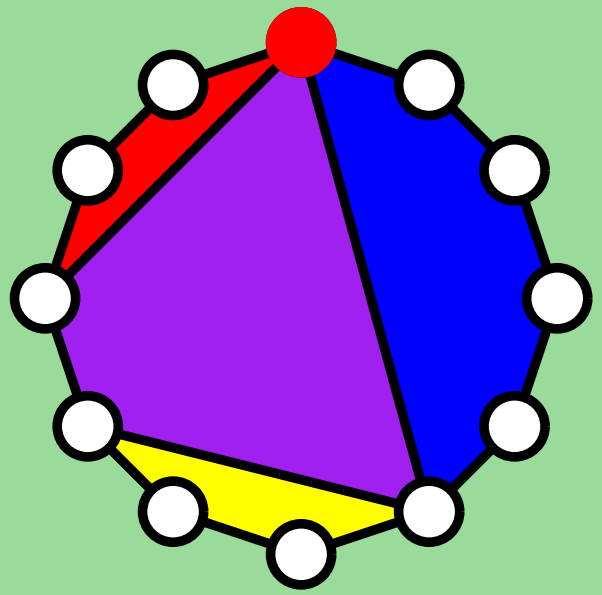
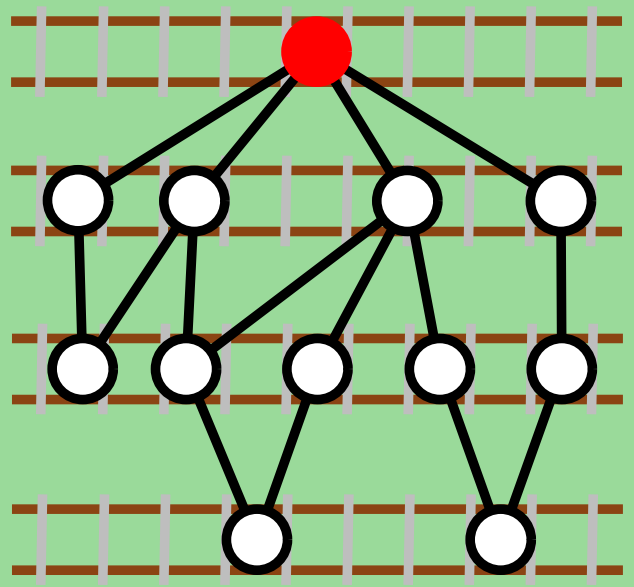




How to layout vertices with a level?

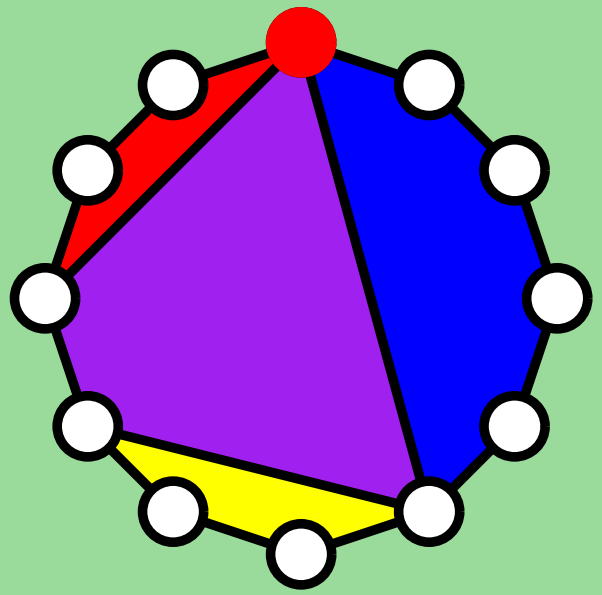
Each face cycle has a closest vertex to the chosen vertex

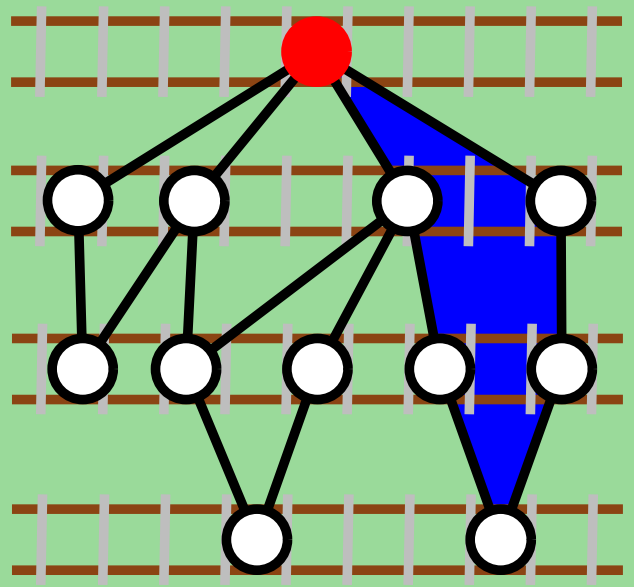
Order vertices of each face cycle independently

Which 3-track graphs can we recognize and layout 3-track?

Bipartite Outerplanar Graphs

Pick a vertex and perform a breadth first layering





How to layout vertices with a level?

Each face cycle has a closest vertex to the chosen vertex

Order vertices of each face cycle independently

Which 3-track graphs can we recognize and layout 3-track?

Bipartite Outerplanar Graphs

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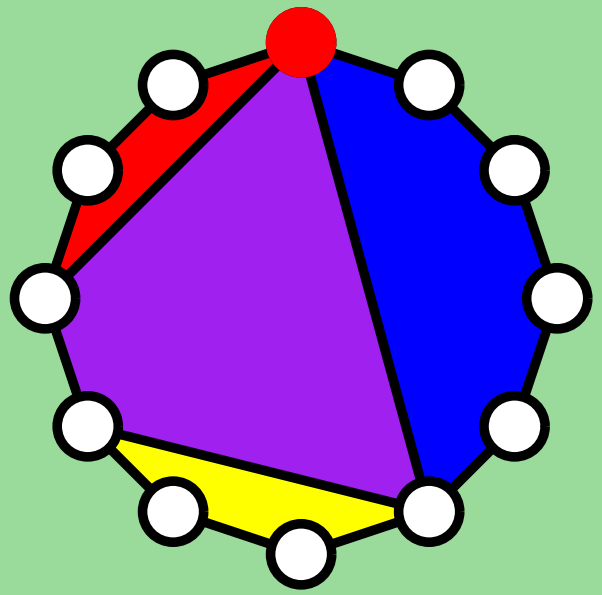

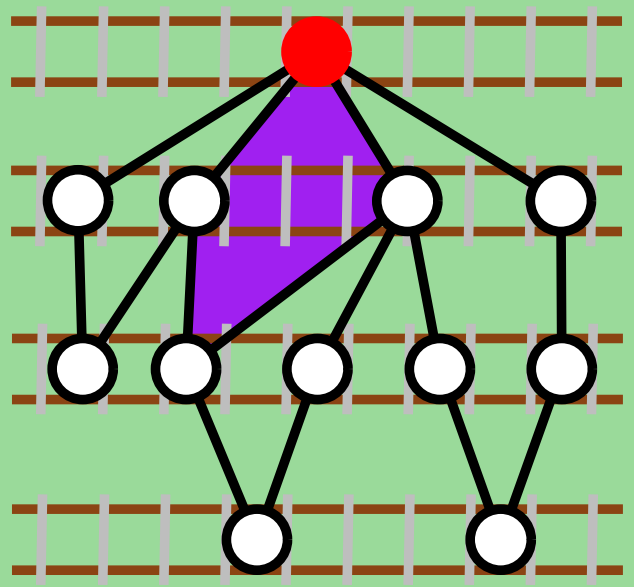

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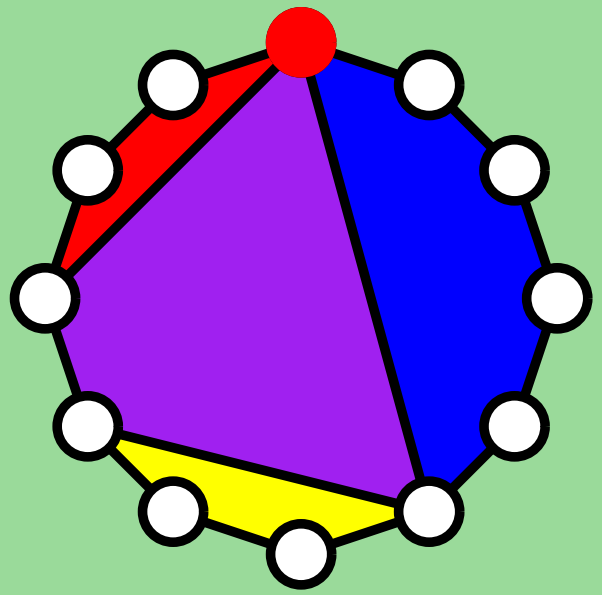

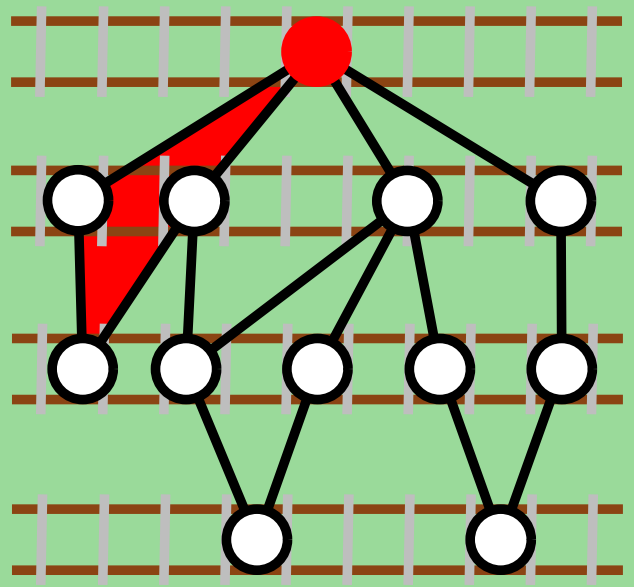

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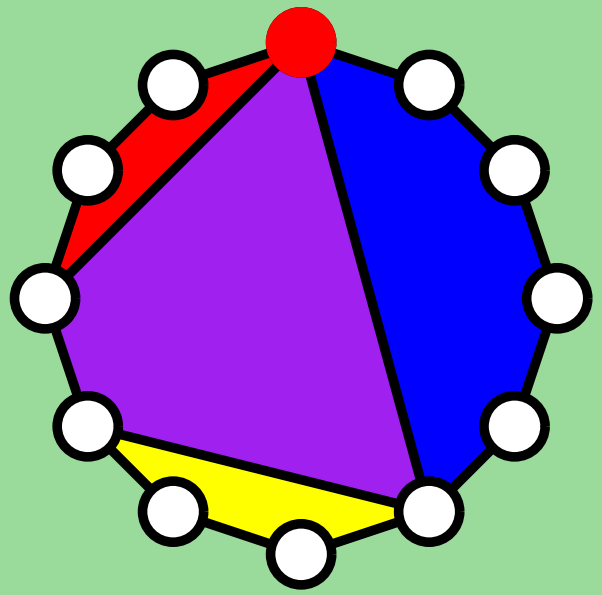

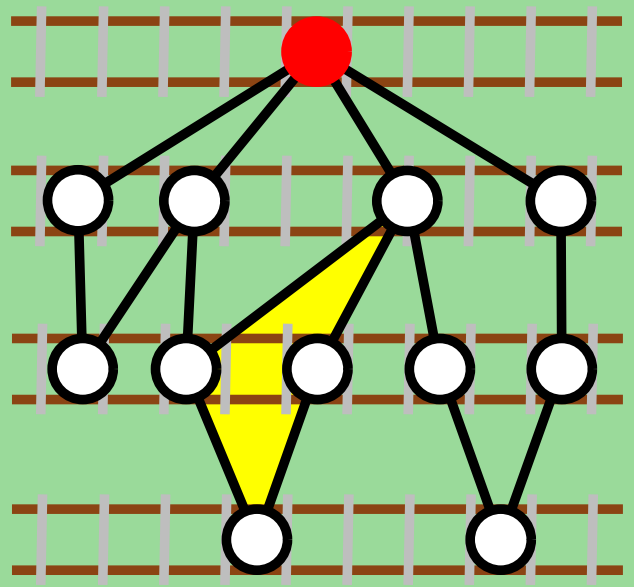

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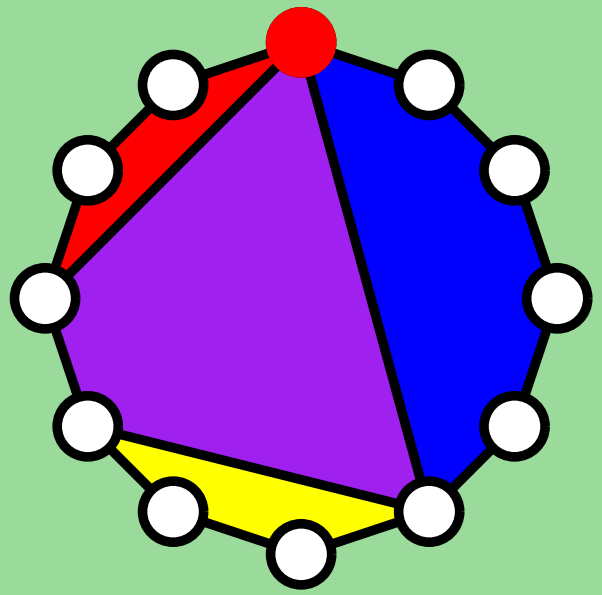

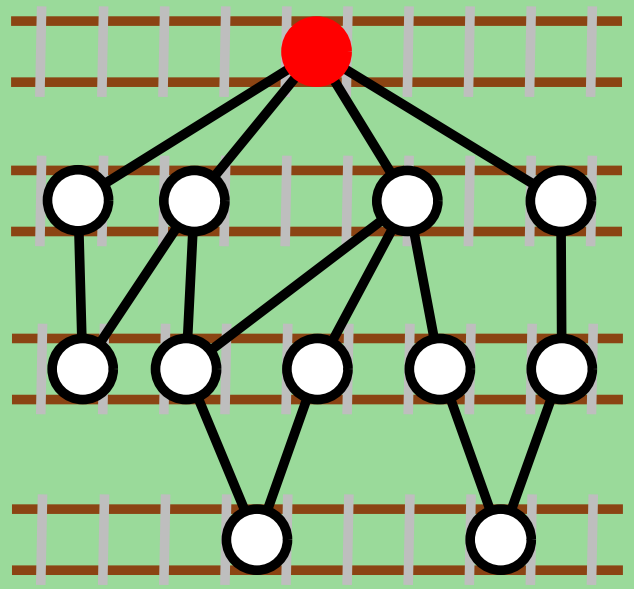

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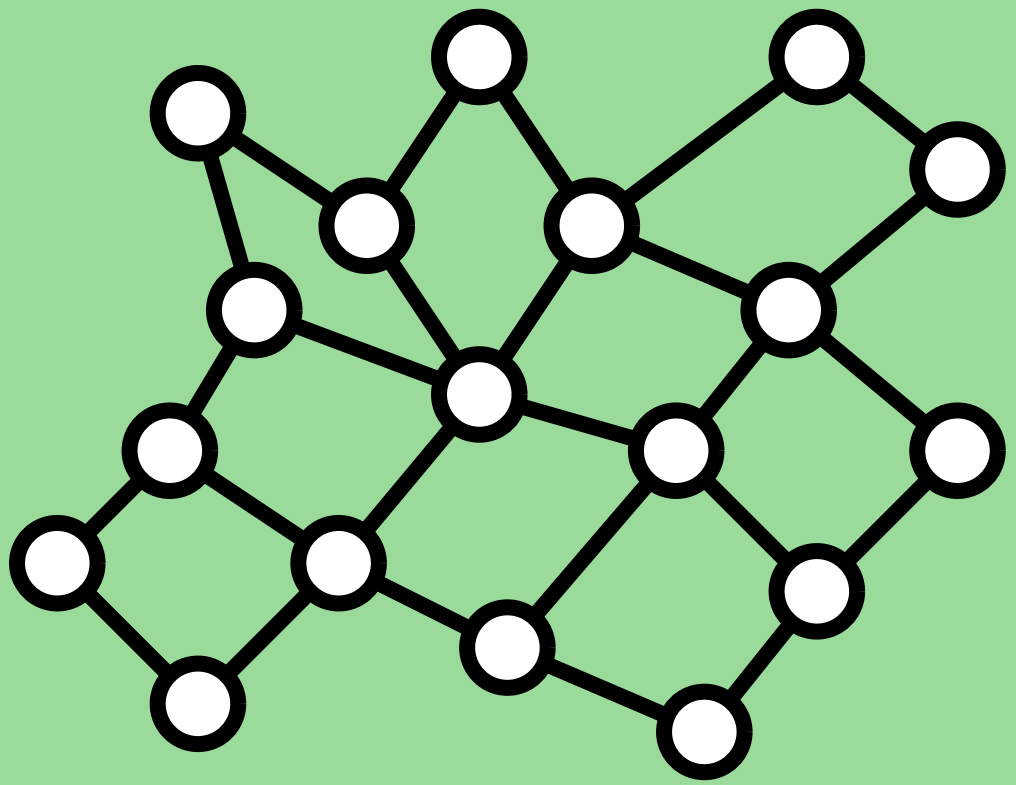
Because every face cycle has a valid layout, the entire graph has a valid layout [Abel et al, 2014]

Squaregraphs

Every bounded face is a 4-cycle and every interior vertex has at least 4 neighbors



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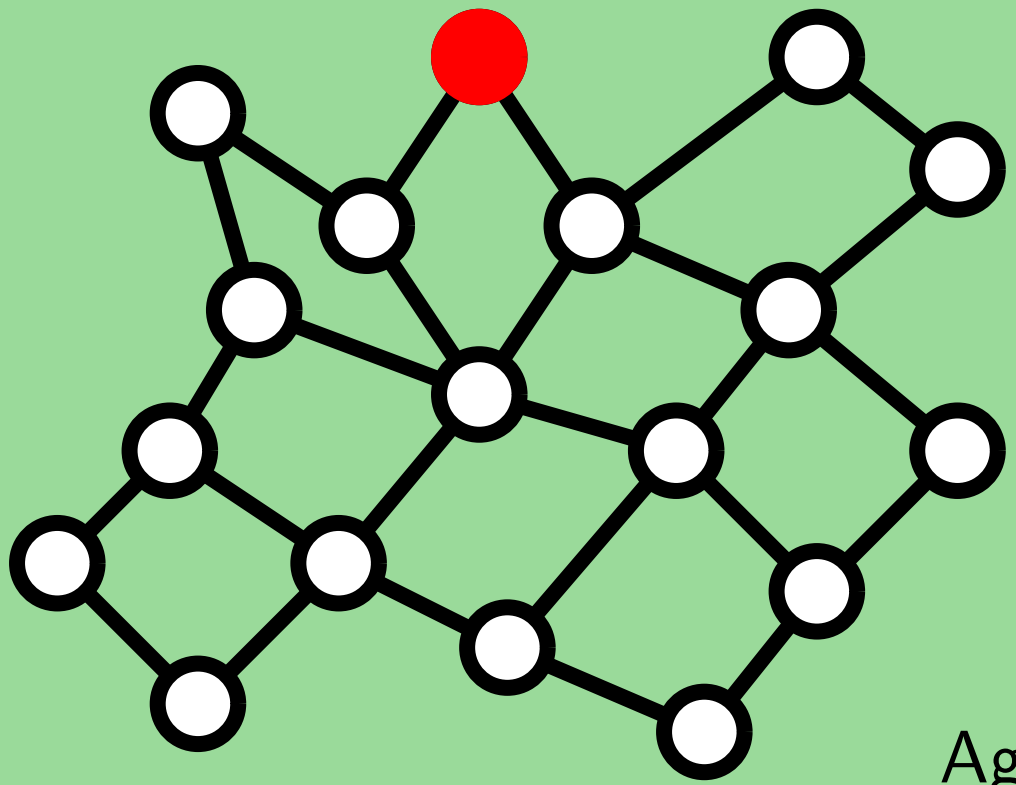

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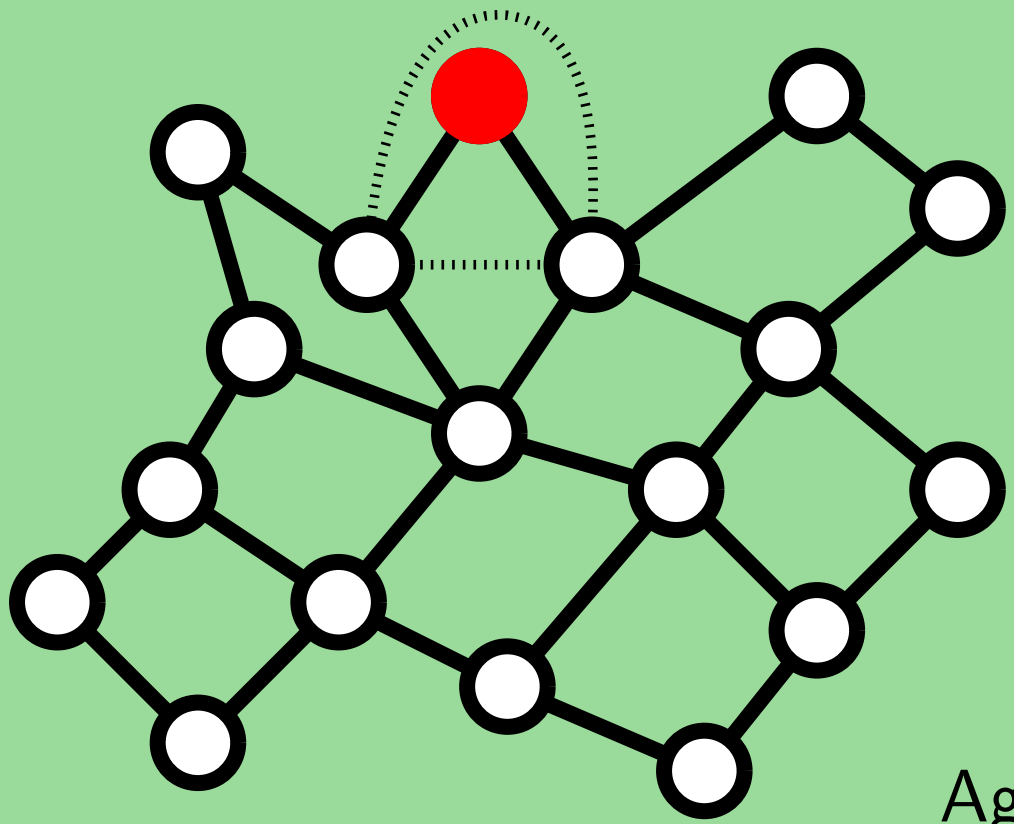

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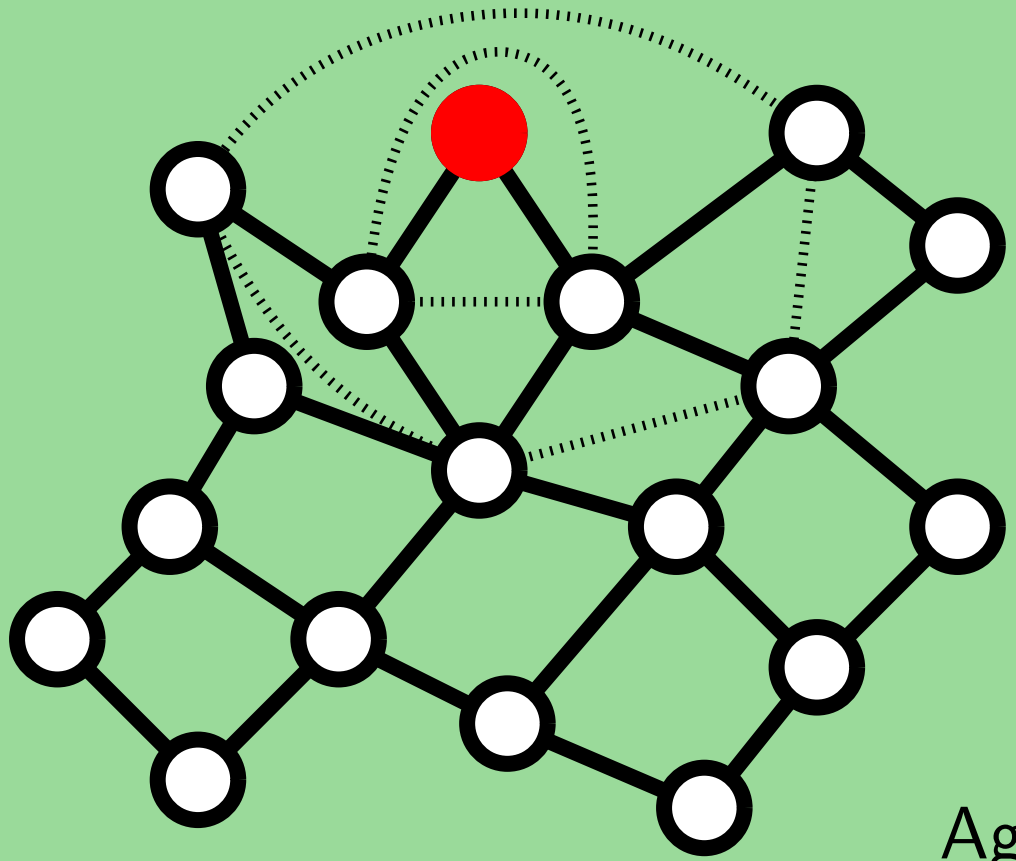

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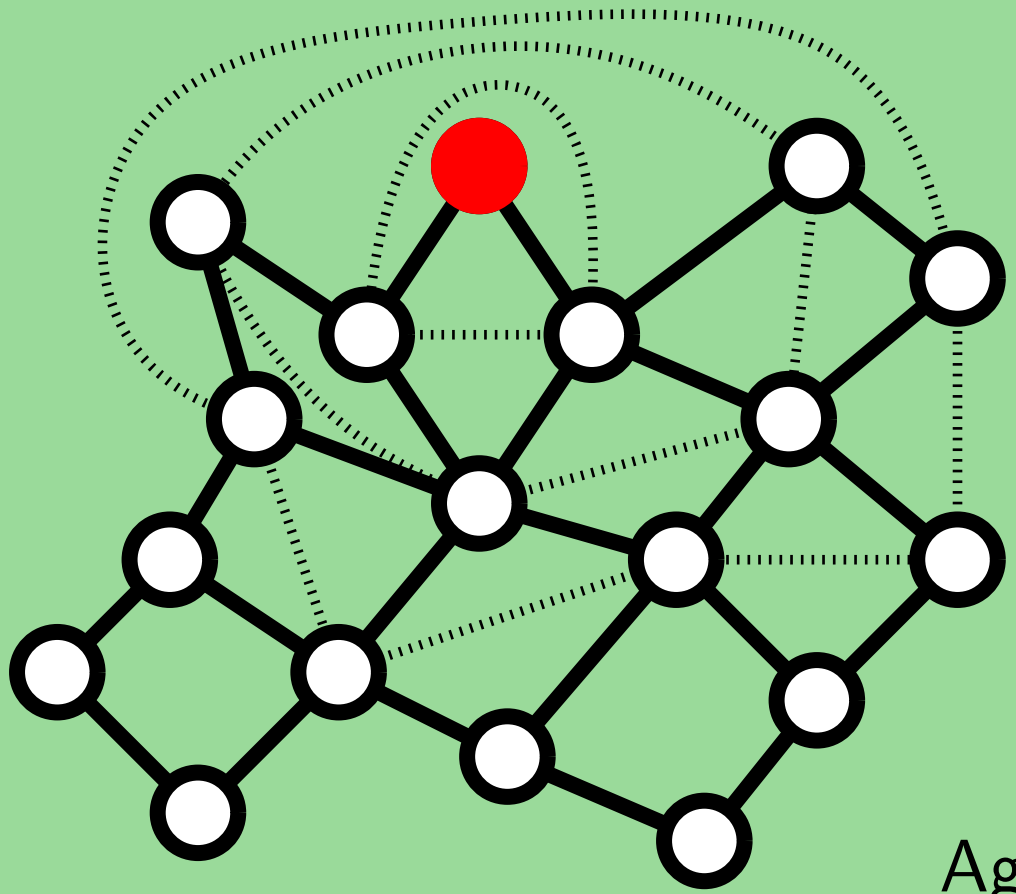

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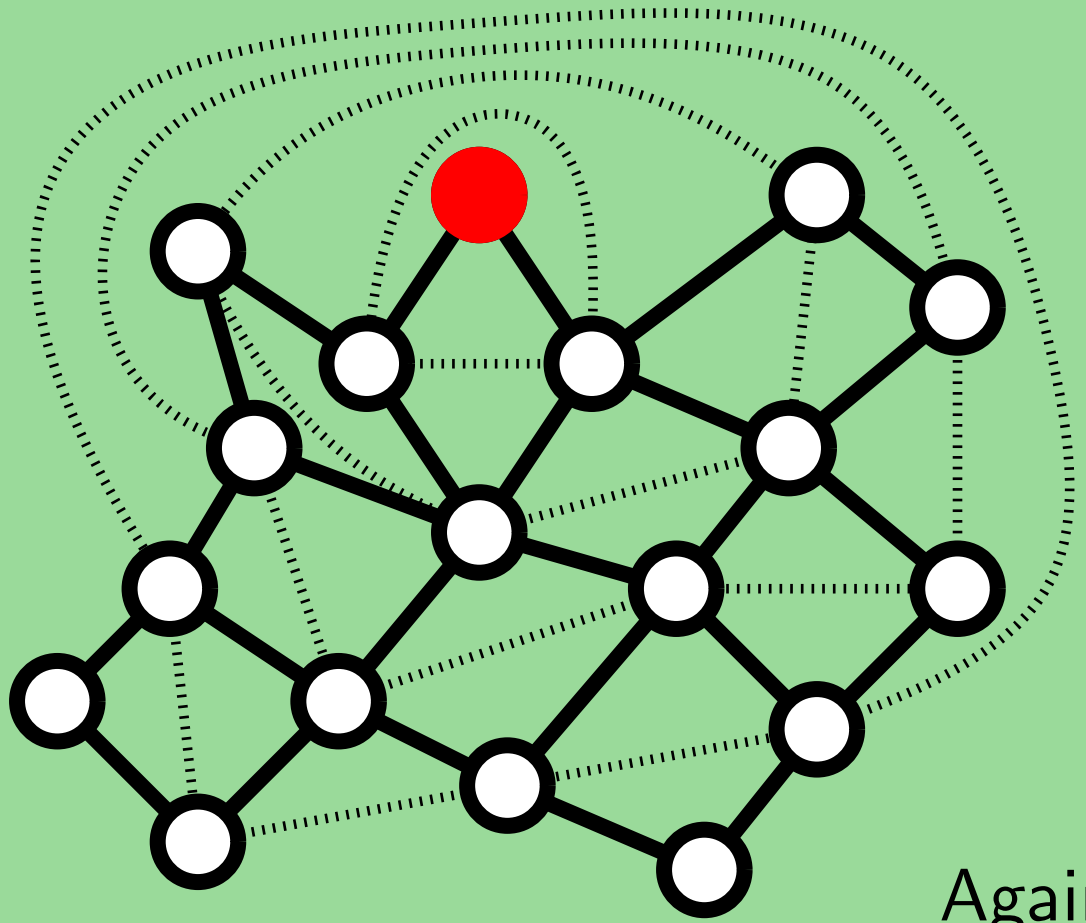

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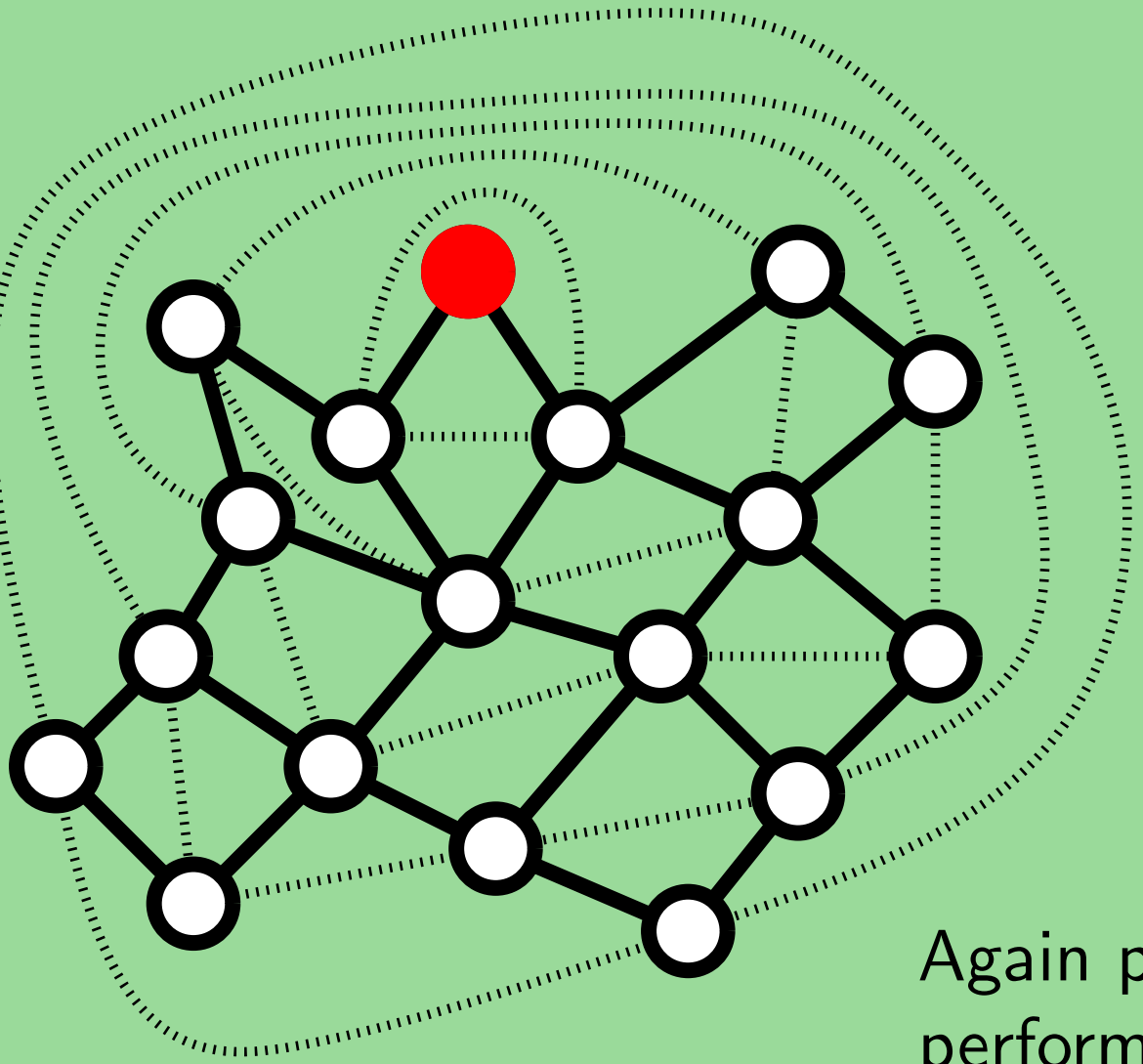

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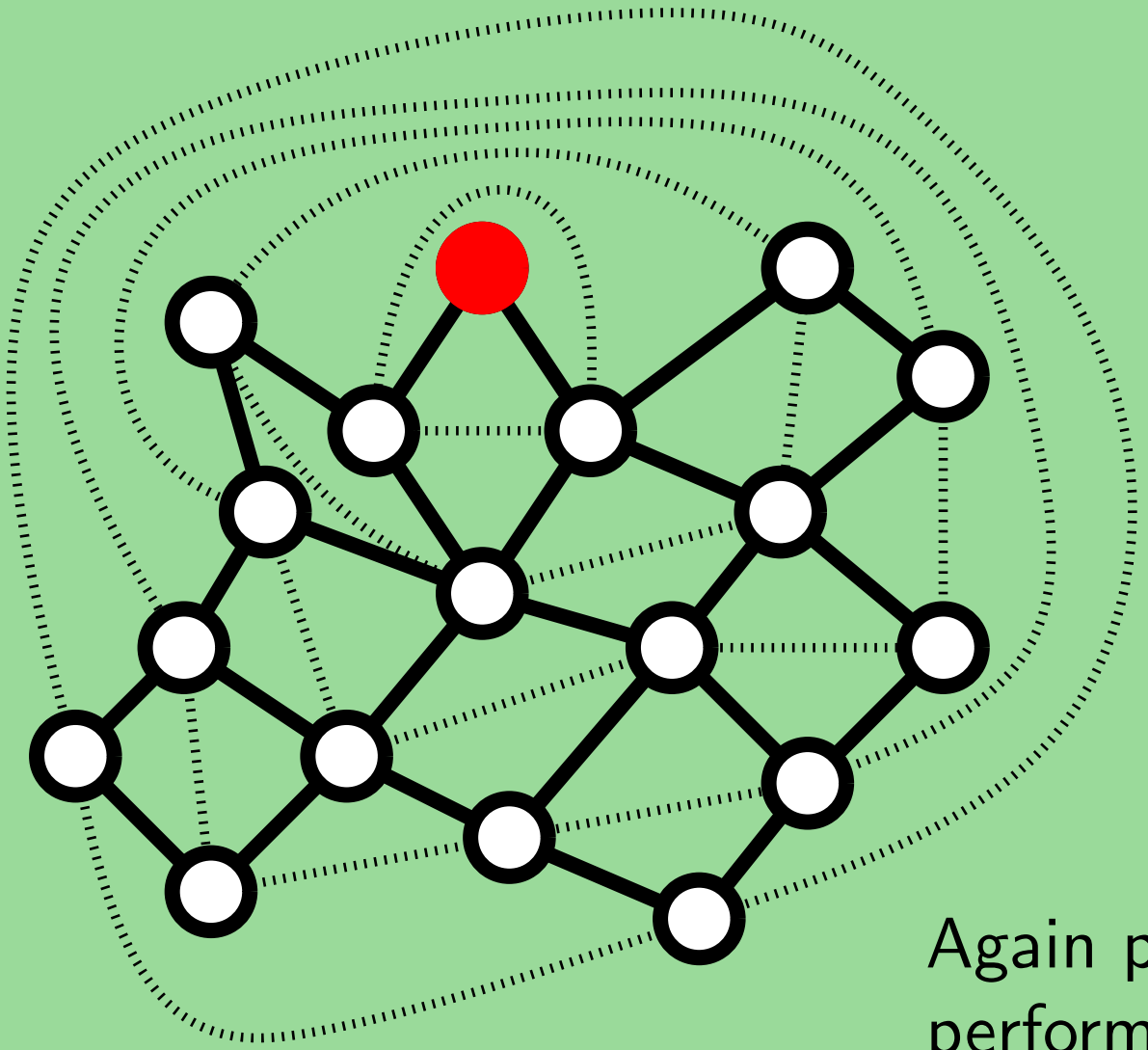

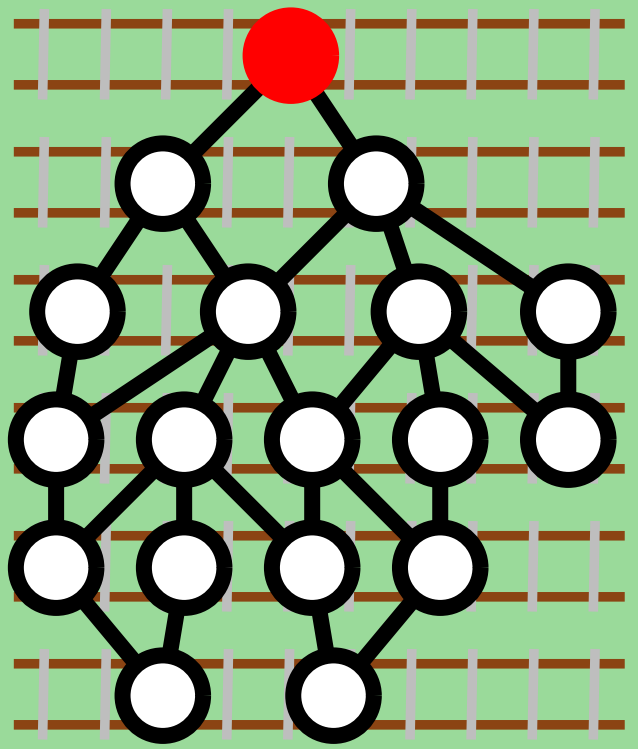

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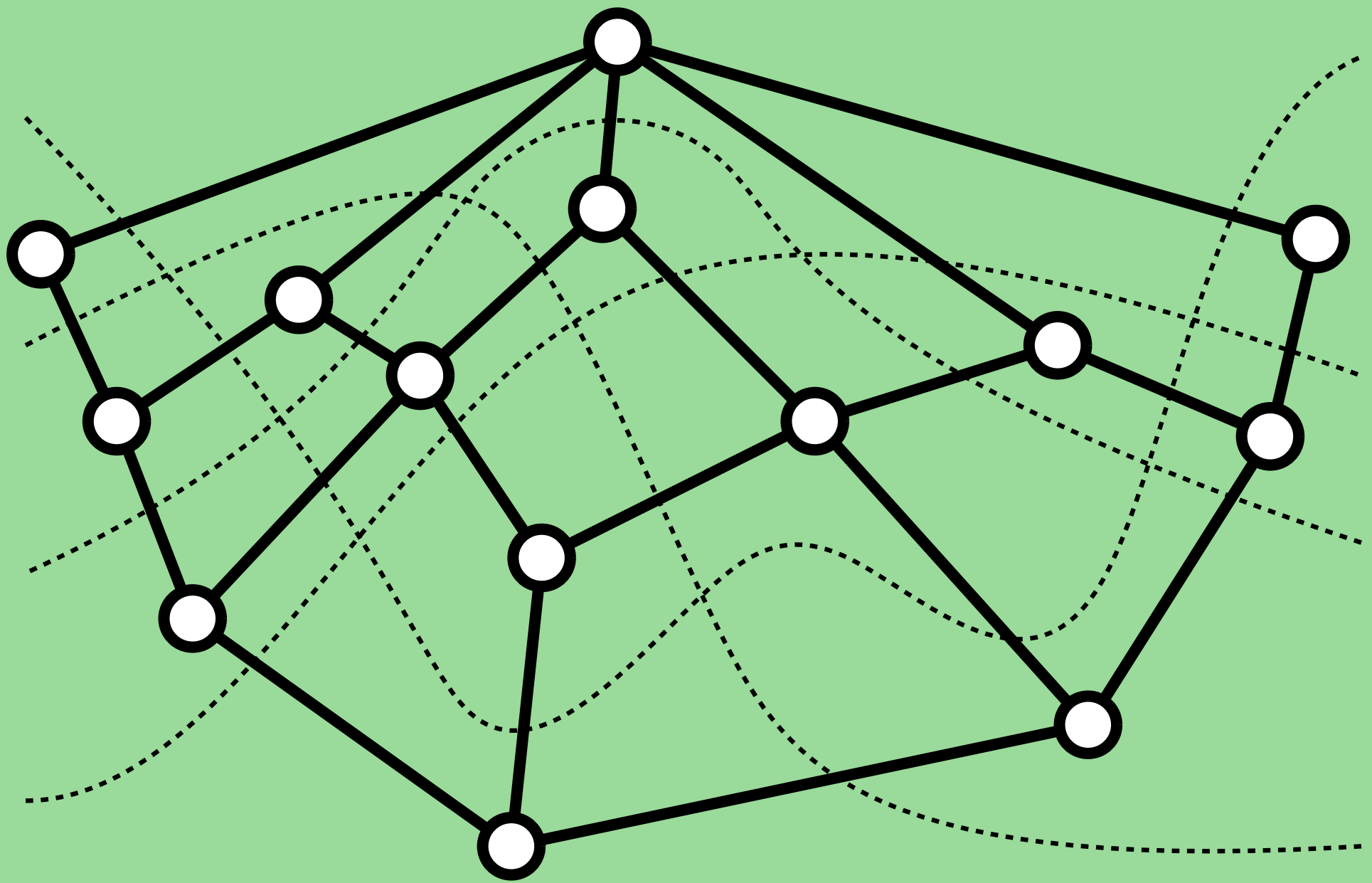
Dual graphs of x-monotone curves

where each curves projection covers the entire x-axis



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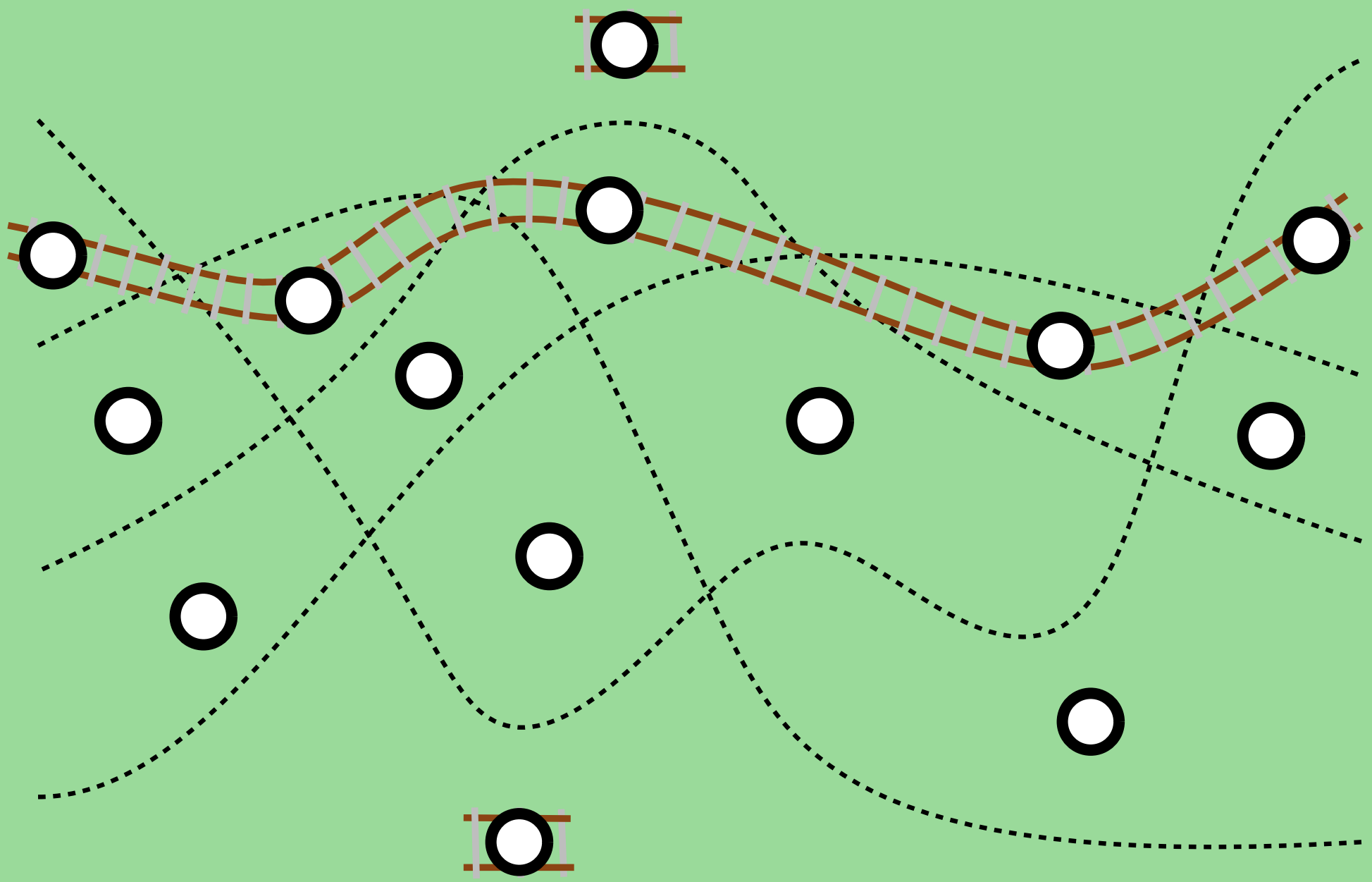

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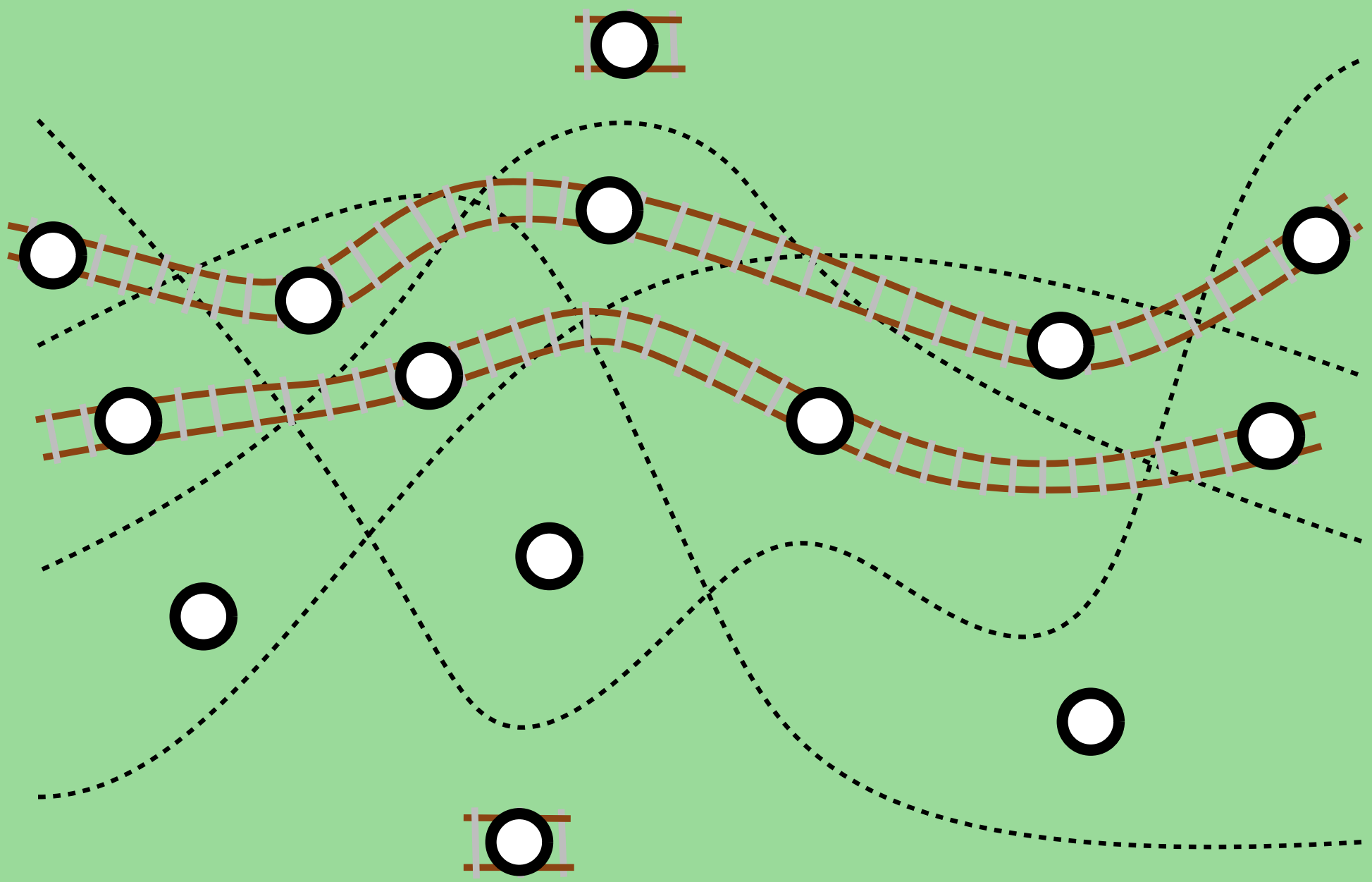

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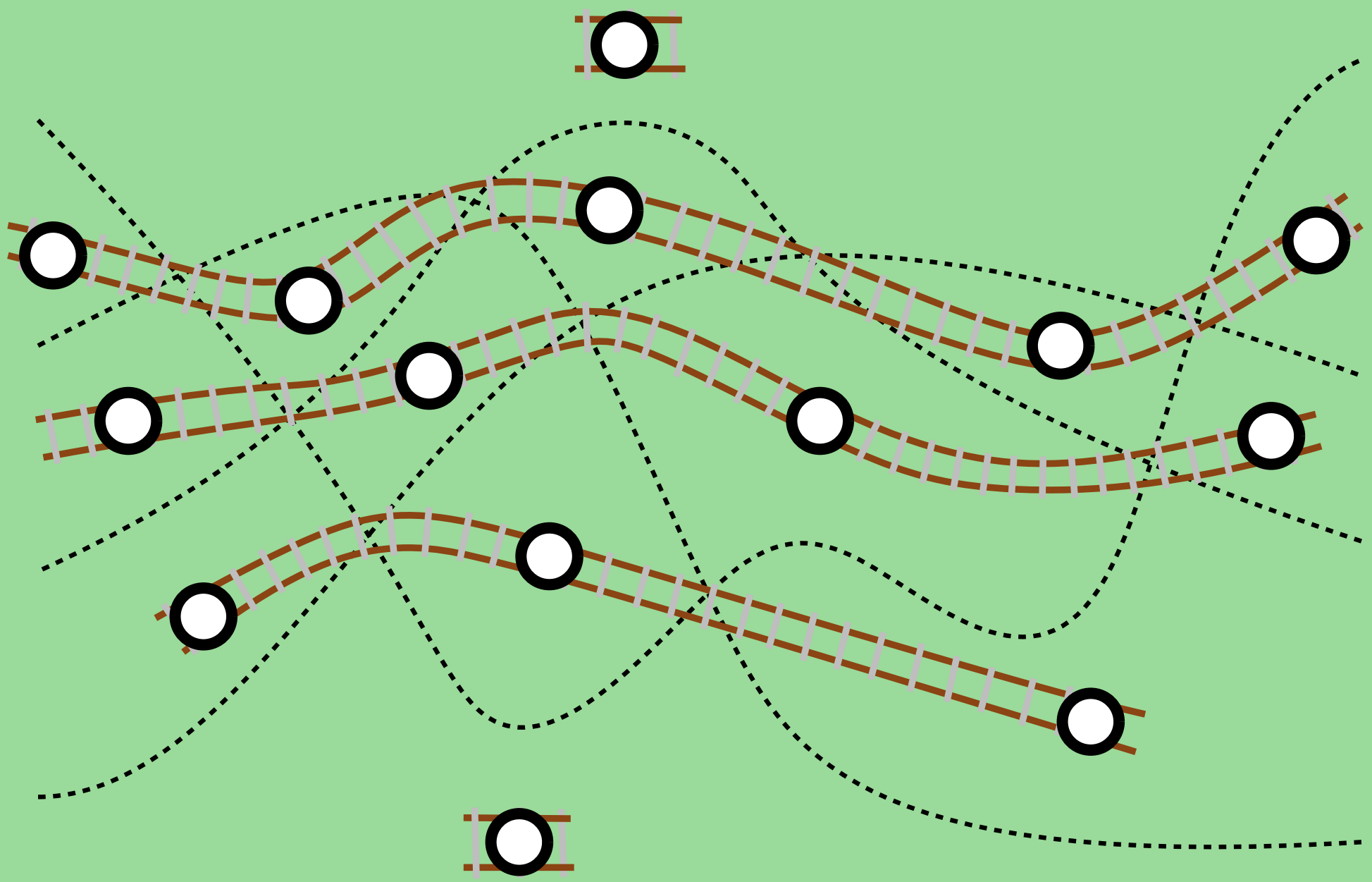

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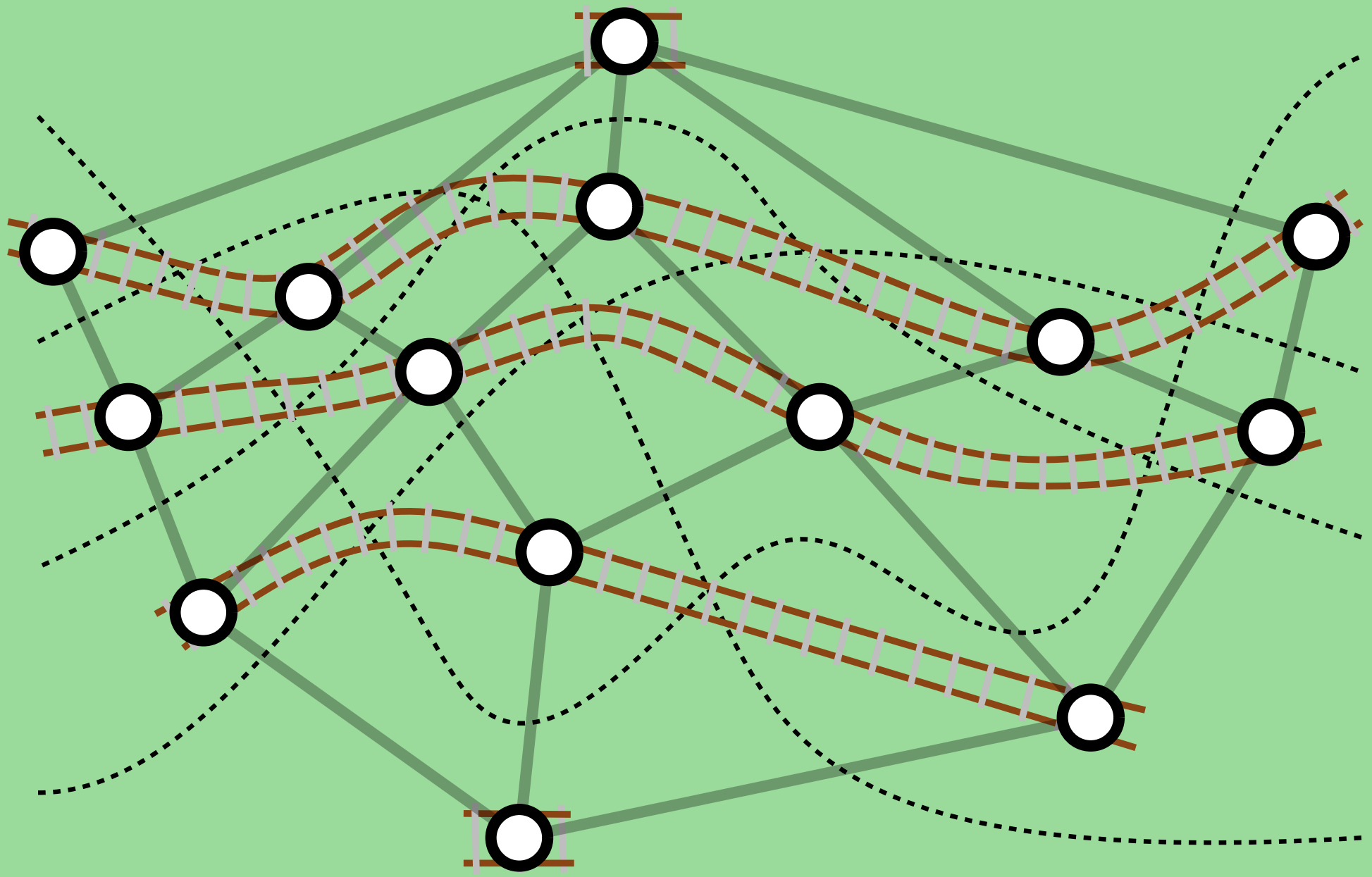


Summary

Recognizing 3-track graphs is NP-complete

-holds for larger track number

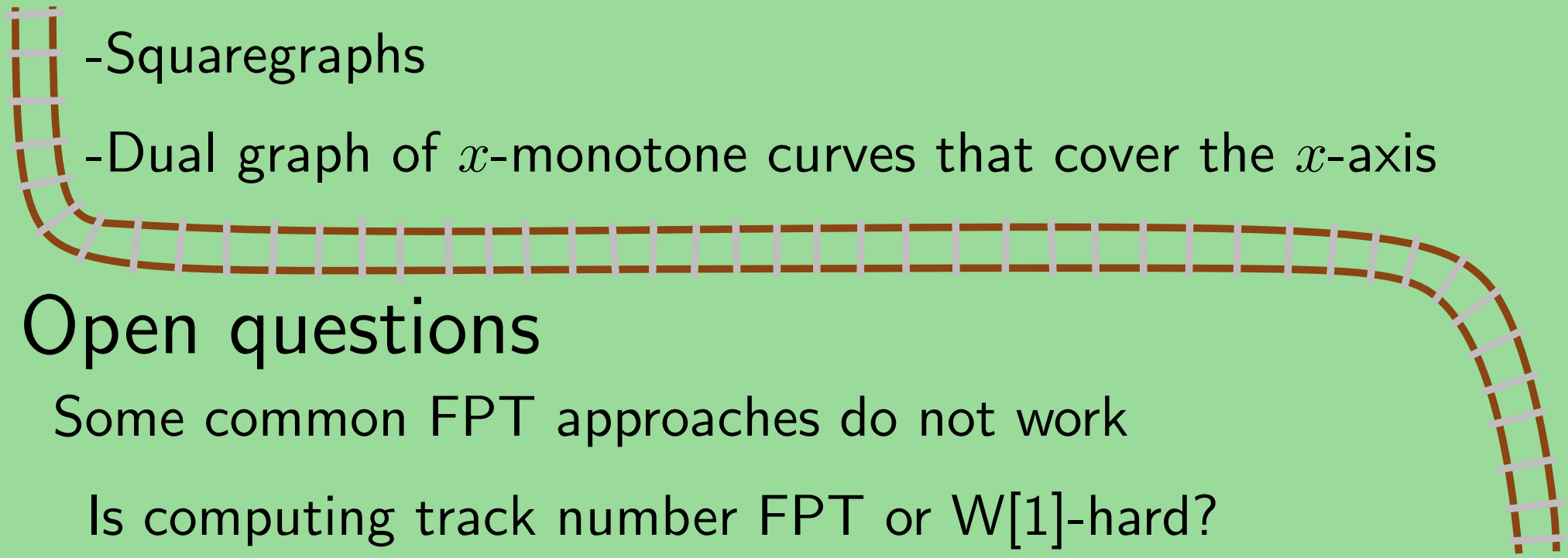
Computing the track number for a graph is NP-hard

3-track layouts for special graph classes

-Bipartite outerplanar graphs

-Squaregraphs

-Dual graph of x-monotone curves that cover the x-axis

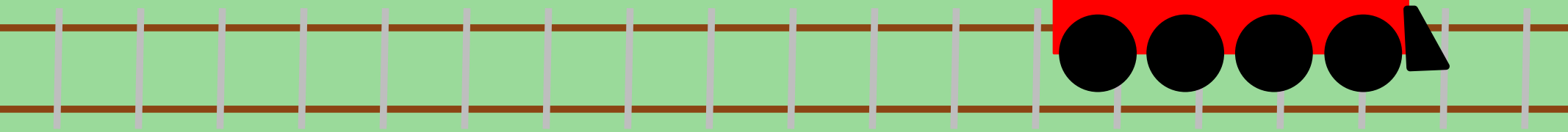


Open questions

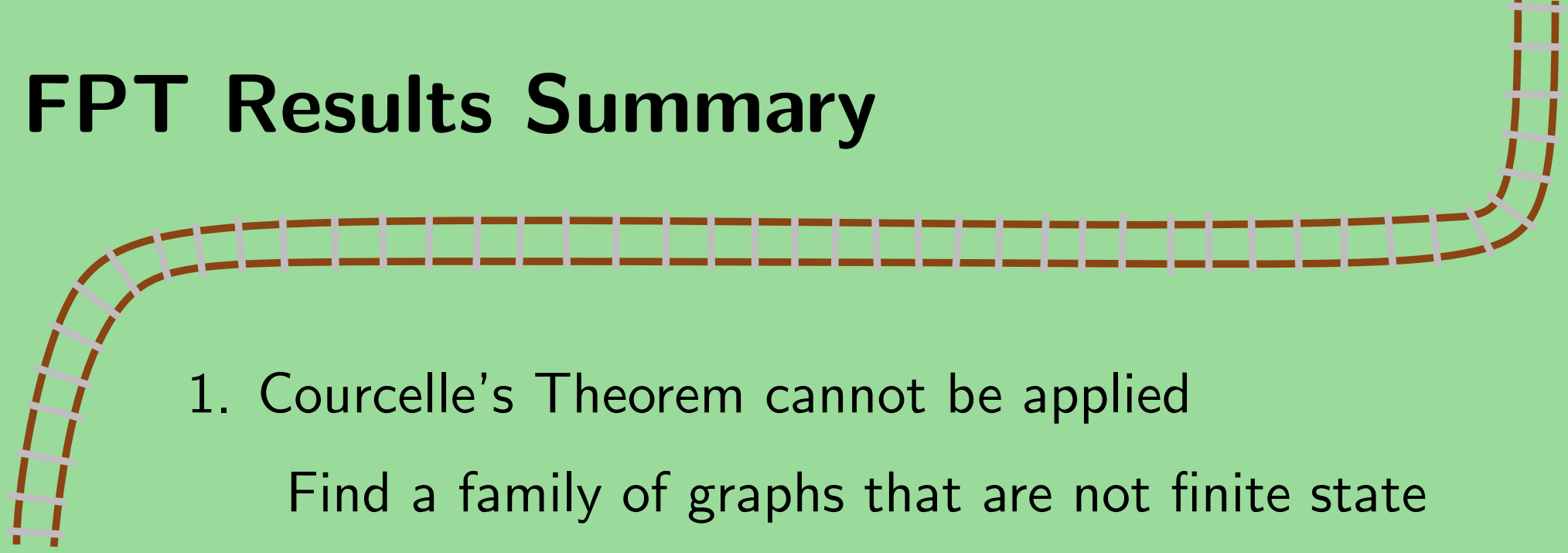
Some common FPT approaches do not work

Is computing track number FPT or W[1]-hard?

Thank you!



FPT Results Summary



1. Courcelle's Theorem cannot be applied

   Find a family of graphs that are not finite state

2. 2-core kernelization for k-almost-tree will not work

   Found pairs of graphs with the same 2-core and different track number

3. Track number is non-uniformly FPT in tree depth

   Using forbidden subgraph testing